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On Intuitionistic fuzzy completely ζ continuous mappings

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Abstract
The objective of this paper is to introduce intuitionistic fuzzy completely ζ continuous functions and to study some of their properties. The relationship between the intuitionistic fuzzy completely ζ continuous functions and few intuitionistic fuzzy continuous mappings are also discussed. Finally we formulate the notion of intuitionistic fuzzy ζ homeomorphism in fuzzy topological space and investigate their characterizations.

Mathematics Subject Classification: 54A40, 03E72.

Keywords: Intuitionistic fuzzy topology, intuitionistic fuzzy point, intuitionistic fuzzy ζ open set, intuitionistic fuzzy completely ζ continuous mapping, intuitionistic fuzzy ζ homeomorphism, intuitionistic fuzzy ζ $T_{1/2}$.

1. Introduction

The concept of fuzzy set was introduced by Zadeh [13] and later Atanassov [2] generalised this idea to intuitionistic fuzzy sets using the notion of fuzzy sets. On the other hand Coker [6] introduced the notion of an intuitionistic fuzzy topological space, fuzzy continuity fuzzy near compactness and some other related concepts. Using the notion of intuitionistic fuzzy sets Joen [9] introduced the concepts of intuitionistic fuzzy α continuity and intuitionistic fuzzy pre continuity. Completely continuous functions and results related to the product in fuzzy topological spaces were introduced in [11] and [3] respectively. In this paper we define the notion of completely ζ continuous functions in intuitionistic fuzzy topological spaces. We discuss characterizations of intuitionistic fuzzy completely ζ continuous functions. We also establish their properties and relationships with other classes of early defined forms of intuitionistic continuous functions. Also we introduce intuitionistic fuzzy ζ homeomorphism and intuitionistic fuzzy Z- ζ homeomorphism. We provide some characterizations of intuitionistic fuzzy ζ homeomorphism

2. Preliminaries

Definition 2.1 [13]: An intuitionistic fuzzy set (IFS, in short) A in X is an object having the form $A=\{x,\mu_A(x),\upsilon_A(x)/x\in X\}$ where the functions $\mu_A:X\to I$ and $\upsilon_A:X\to I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\upsilon_A(x)$) of each element $x\in X$ to the set A on a nonempty set X and $0\le\mu_A(x)+\upsilon_A(x)\le 1$ for each $x\in X$. Obviously every fuzzy set A on a nonempty set X is an IFS's A and B be in the form $A=\{x,\mu_A(x),1-\mu_A(x)/x\in X\}$

Definition 2.2 [2]**:** Let X be a nonempty set and the IFS's A and B be in the form $A = \{x, \mu_A(x), \upsilon_A(x) / x \in X\}$, $B = \{x, \mu_B(x), \upsilon_B(x) / x \in X\}$ and let $A = \{A_j : j \in J\}$ be an arbitrary family of IFS's in X. Then we define

$A \subseteq B$ if and only if $\mu_A(x) \le \mu_B(x)$ and $\upsilon_A(x) \ge \upsilon_B(x)$ for all $x \in X$.

(i) A=B if and only if $A \subseteq B$ and $B \subseteq A$.

(ii) $\overline{A} = \{x, \upsilon_A(x), \mu_A(x) / x \in X\}$.

(iii) $A \cap B = \{x, \mu_A(x) \cap \mu_B(x), \upsilon_A(x) \cup \upsilon_B(x) / x \in X\}$.

(iv) $A \cup B = \{x, \mu_A(x) \cup \mu_B(x), \upsilon_A(x) \cap \upsilon_B(x) / x \in X\}$

(v) $1_{\sim} = \{\langle x,1,0 \rangle x \in X\}$ and $0_{\sim} = \{\langle x,0,1 \rangle x \in X\}$.

Definition 2.3 [6]**:** An intuitionistic fuzzy topology (IFT, in short) on a nonempty set X is a family τ of an intuitionistic fuzzy set (IFS, in short) in X satisfying the following axioms:

(i) $0_{\sim}, 1_{\sim} \in \tau$.

(ii) $A_1 \cap A_2 \in \tau$ for any $A_1, A_2 \in \tau$.

(iii) $\bigcup A_j \in \tau$ for any $A_j : j \in J \subseteq \tau$.

The complement $\overline{A}$ of intuitionistic fuzzy open set (IFOS, in short) in intuitionistic fuzzy topological space (IFTS, in short) (X,τ) is called an intuitionistic fuzzy closed set (IFCS, in short).

Definition 2.4 [6]**:** Let (X,τ) be an IFTS and $A = \{x, \mu_A(x), \upsilon_A(x)\}$ be an IFS in X. Then the fuzzy interior and closure of A are denoted by

(i) $cl(A) = \bigcap$ {K: K is an IFCS in X and $A \subseteq K$}.

(ii) $\text{int}(A) = \bigcup$ {G: G is an IFOS in X and $G \subseteq A$}.

(iii) Note that, for any IFS A in (X,τ), we have $cl(\overline{A}) = \overline{\text{int}(A)}$ and $\text{int}(\overline{A}) = \overline{cl(A)}$.

Definition 2.5 [7]**:** Let A be an IFS in an IFTS (X,τ), then A is

(i) An intuitionistic fuzzy regular open set (IFROS) if $A = \text{int}(cl(A))$.

(ii) An intuitionistic fuzzy semi open set (IFSOS) if $A \subseteq cl(\text{int}(A))$.

(iii) An intuitionistic fuzzy preopen set (IFPOS) if $A \subseteq \text{int}(cl(A))$.

(iv) An intuitionistic fuzzy d open set (IFdOS) if $A \subseteq scl(b\,\text{int}(A)) \cup cl(\text{int}(A))$.

(v) An intuitionistic fuzzy α-open set (IFαOS) if $A \subseteq \text{int}(cl(\text{int}(A)))$.

(vi) An intuitionistic fuzzy β-open set (IFβOS) if $A \subseteq cl(\text{int}(cl((A)))$.

(vii) An intuitionistic fuzzy γ-open set (IFγOS) if $A \subseteq cl(\text{int}(A)) \cup \text{int}(cl(A))$.

The complement of the above said sets are intuitionistic fuzzy regular closed set, intuitionistic fuzzy semiclosed set, intuitionistic fuzzy pre closed set, intuitionistic fuzzy d closed set, intuitionistic fuzzy α-closed set, intuitionistic fuzzy β-closed set, intuitionistic fuzzy γ-closed set, (IFRCS, IFSCS, IFPCS, IFdCS, IFαCS, IFβCS, IFγCS respectively).

Definition 2.6 [2]**:** An IFS $p(\alpha,\beta) = \langle x, C_\alpha, C_{1-\beta} \rangle$ where $\alpha \in (0,1]$, $\beta \in [0,1)$ and $\alpha + \beta \le 1$ is called an intuitionistic fuzzy point (IFP) in X.

Note that an IFP $p(\alpha,\beta)$ is said to belong to an IFS $A = \langle X, \mu_A, \upsilon_A \rangle$ of X denoted by $p(\alpha,\beta) \in A$ if $\alpha \le \mu_A$ and $\beta \ge \upsilon_A$.

Definition 2.7 [10]**:** Let $p(\alpha,\beta)$ be an IFP of an IFTS (X,τ). An IFS A of X is called an intuitionistic fuzzy neighbourhood (IFN) of $p(\alpha,\beta)$ if there exists an IFOS B in X such that $p(\alpha,\beta) \in B \subseteq A$.

Definition 2.8 [10]**:** Two IFSs are said to be q-coincident (A_qB) if and only if there exists an element $x \in X$ such that $\mu_A(x) > \upsilon_B(x)$ or $\upsilon_A(x) < \mu_B(x)$.

Definition 2.9 [6]**:** Let X and Y be two IFTSs. Let $A = \{\langle X, \mu_A(x), \upsilon_A(x) \rangle : x \in X\}$ and $B = \{\langle Y, \mu_B(y), \upsilon_B(y) \rangle y \in Y\}$ be IFSs of X and Y respectively. Then is an IFS $A \times B$ of $X \times Y$ defined by $A \times B(x,y) = \langle (X,Y), \min(\mu_A(x), \mu_B(y)), \max(\upsilon_A(x), \upsilon_B(y)) \rangle$.

Definition 2.10 [12]**:** Let A be an IFTS (X,τ). Then A is called an intuitionistic fuzzy ζ open set(IFζ OS, in short) in X if $A \subseteq bcl(\text{int}(A))$.

Definition 2.11 [12]**:** Let A be an IFTS (X,τ). Then A is called an intuitionistic fuzzy ζ closed set (IFζ CS, in short) in X if $b\,\text{int}(cl(A)) \subseteq$ A.

Definition 2.12 [12]**:** Let $f : X \to Y$ from an IFTS X into an IFTS Y. Then f is said to be an

(i) Intuitionistic fuzzy continuous [4] if $f^{-1}(B) \in IFO(X)$ for every $B \in \kappa$.

(ii) Intuitionistic fuzzy semi-continuous [6] if $f^{-1}(B) \in IFSO(X)$ for every $B \in \kappa$.

(iii) Intuitionistic fuzzy pre continuous [6] if $f^{-1}(B) \in IFPO(X)$ for every $B \in \kappa$.

(iv) Intuitionistic fuzzy d continuous [6] if $f^{-1}(B) \in IFdO(X)$ for every $B \in \kappa$.

(v) Intuitionistic fuzzy α-continuous [6] if $f^{-1}(B) \in IF\alpha O(X)$ for every $B \in \kappa$.

(vi) Intuitionistic fuzzy β-continuous [6] if $f^{-1}(B) \in IF\beta O(X)$ for every $B \in \kappa$.

(vii) Intuitionistic fuzzy γ-continuous [6] if $f^{-1}(B) \in IF\gamma O(X)$ for every $B \in \kappa$.

(viii) Intuitionistic fuzzy ζ continuous (IFζ cont, in short) [11] if $f^{-1}(B) \in IF\zeta OS(\mathrm{X})$ for every $B \in \kappa$.

Definition 2.13 [8]: Let f be a bijection mapping from IFTS (X,τ) into an IFTS(Y,κ). Then f is said to be intuitionistic fuzzy homeomorphism (IF homeomorphism, in short) if f and f^{-1} are continuous mappings.

Definition 2.14 [7]: Let $f_1 : X_1 \to Y_1$ and $f_2 : X_2 \to Y_2$. The product $f_1 \times f_2 : X_1 \times X_2 \to Y_1 \times Y_2$ is defined by $f_1 \times f_2(x_1, x_2) = (f_1(x_1), f_2(x_2))$ for every $(X_1, X_2) \in X_1 \times X_2$.

2.1. Intuitionistic Fuzzy Completely ζ Continuous Mappings

Definition 3.1 A mapping $f : X \to Y$ from an IFTS X into an IFTS Y is called an intuitionistic fuzzy completely ζ continuous (IFcζ continuous, for short) mapping if $f^{-1}(B)$ is an IFROS in X, for every IFζ OS B in Y.

Theorem 3.2

(i) Every IFcζ continuous mapping is an IF continuous.

(ii) Every IFcζ continuous mapping is an IFζ continuous.

(iii) Every IFcζ continuous mapping is an IFS continuous.

(iv) Every IFcζ continuous mapping is an IFP continuous.

(v) Every IFcζ continuous mapping is an IFd continuous.

(vi) Every IFcζ continuous mapping is an IFα continuous.

(vii) Every IFcζ continuous mapping is an IFβ continuous.

(viii) Every IFcζ continuous mapping is an IFγ continuous.

The proof is immediate.

The converse of the above statements may not be true as seen from the following examples:

Example 3.3: Let $X = \{a,b\}$, $Y = \{u,v\}$ $G_1 = \{\langle y,(0.6,0.7),(0,0.1)\rangle\}$.

Then $\tau = \{0_\sim, 1_\sim, G_1\}$ and $\kappa = \{0_\sim, 1_\sim, G_1\}$ are IFT on X and Y respectively.

Define a mapping $f : (X,\tau) \to (Y,\kappa)$ by $f(a) = u$ and $f(b) = v$.

Then f is an IF continuous mapping. But f is not an IFcζ continuous mapping, Since $B = \{\langle y,(0.6,0.7),(0,0.1)\rangle\}$ is an IFζ OS in Y but $f^{-1}(B) = \{\langle x,(0.6,0.7),(0,0.1)\rangle\}$ is not an IFROS in X.

Example 3.4: Let $X = \{a,b\}$, $Y = \{u,v\}$ $G_1 = \{\langle x,(0.2,0.3),(0.4,0.6)\rangle, \langle x,(0.3,0.7),(0.1,0.5)\rangle\}$, $G_2 = \{\langle y,(0.3,0.7),(0.1,0.5)\rangle\}$. Then $\tau = \{0_\sim, 1_\sim, G_1, G_2\}$ and $\kappa = \{0_\sim, 1_\sim, G_2\}$ are IFT on X and Y respectively.

Define a mapping $f : (X,\tau) \to (Y,\kappa)$ by $f(a) = u$ and $f(b) = v$.

Then f is an IFζ continuous mapping. But f is not an IFc ζ continuous mapping, SinceG_2 is an IFζ OS in Y but $f^{-1}(G_2) = \{\langle x,(0.3,0.7),(0.1,0.5)\rangle\}$ is not an IFROS in X.

Example 3.5: Let $X = \{a,b\}$, $Y = \{u,v\}$ $G_1 = \{\langle x,(0.6,0.7),(0.4,0.2)\rangle\}$, $G_2 = \{\langle y,(0.6,0.7),(0.4,0.2)\rangle\}$

Then $\tau = \{0_\sim, 1_\sim, G_1\}$ and $\kappa = \{0_\sim, 1_\sim, G_2\}$ are IFT on X and Y respectively.

Define a mapping $f : (X,\tau) \to (Y,\kappa)$ by $f(a) = u$ and $f(b) = v$.

Then f is an IFS continuous mapping. But f is not an IFcζ continuous mapping, SinceG_2 is an IFζ OS in Y but $f^{-1}(G_2) = \{\langle x,(0.6,0.7),(0.4,0.2)\rangle\}$ is not an IFROS in X.

Example 3.6: Let $X = \{a,b\}$, $Y = \{u,v\}$

$G_1 = \{\langle x, (0.7,0.8),(0.3,0.2)\rangle\}$,
$G_2 = \{\langle y, (0.6,0.7),(0.4,0.3)\rangle\}$,
$G_3 = \{\langle y, (0.5,0.4),(0.5,0.6)\rangle\}$

Then $\tau = \{0_\sim, 1_\sim, G_1, G_2\}$ and $\kappa = \{0_\sim, 1_\sim, G_3\}$ are IFT on X and Y respectively.
Define a mapping $f : (X, \tau) \to (Y, \kappa)$ by $f(a) = u$ and $f(b) = v$.
Then f is an IFP continuous mapping. But f is not an IFcζ continuous mapping, SinceG$_3$is an IFζ OS in Y but $f^{-1}(G_3) = \{\langle x, (0.5,0.4), (0.5,0.6)\rangle\}$ is not an IFROS in X.

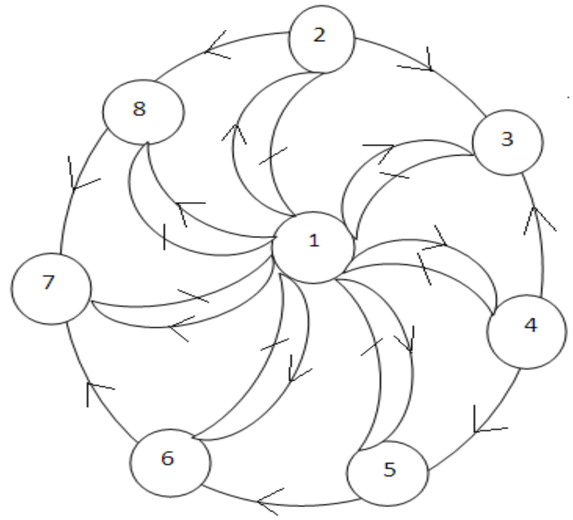
Example 3.7: Let $X = \{a, b\}$, $Y = \{u, v\}$
$G_1 = \{\langle x, (0.7,0.8),(0.3,0.2)\rangle\}$,
$G_2 = \{\langle x, (0.6,0.7),(0.4,0.3)\rangle\}$,
$G_3 = \{\langle y, (0.5,0.4),(0.5,0.6)\rangle\}$

Then $\tau = \{0_\sim, 1_\sim, G_1, G_2\}$ and $\kappa = \{0_\sim, 1_\sim, G_3\}$ are IFT on X and Y respectively.
Define a mapping $f : (X, \tau) \to (Y, \kappa)$ by $f(a) = u$ and $f(b) = v$.
Then f is an IFd continuous mapping. But f is not an IFcζ continuous mapping, SinceG$_3$ is an IFζ OS in Y but $f^{-1}(G_3) = \{\langle x, (0.5,0.4), (0.5,0.6)\rangle\}$ is not an IFROS in X.
In the above example f is an IF β continuous mapping. But f is not an IFcζ continuous mapping, SinceG$_3$ is an IFζ OS in Y but $f^{-1}(G_3) = \{\langle x, (0.5,0.4), (0.5,0.6)\rangle\}$ is not an IFROS in X.
In the above example f is an IF γ continuous mapping. But f is not an IFcζ continuous mapping, SinceG$_3$ is an IFζ OS in Y but $f^{-1}(G_3) = \{\langle x, (0.5,0.4), (0.5,0.6)\rangle\}$ is not an IFROS in X.

Example 3.8: Let $X = \{a, b\}$, $Y = \{u, v\}$
$G_1 = \{\langle x, (0.6,0.7),(0.4,0.2)\rangle\}$,
$G_2 = \{\langle y, (0.6,0.7),(0.4,0.2)\rangle\}$

Then $\tau = \{0_\sim, 1_\sim, G_1\}$ and $\kappa = \{0_\sim, 1_\sim, G_2\}$ are IFT on X and Y respectively.
Define a mapping $f : (X, \tau) \to (Y, \kappa)$ by $f(a) = u$ and $f(b) = v$.
Then f is an IFd continuous mapping. But f is not an IFcζ continuous mapping, SinceG$_3$ is an IFζ OS in Y but $f^{-1}(G_2) = \{\langle x, (0.6,0.7), (0.4,0.2)\rangle\}$ is not an IFROS in X.

1. IFcζ continuous



2. IF continuous
3. IFP continuous
4. IF α continuous
5. IFS continuous
6. IFd continuous
7. IF β continuous
8. IF γ continuous

2.2. The above figure represents the relationship between IFcζ continuous and other continuous mappings.

Theorem 3.9: A mapping $f : X \to Y$ from an IFTS X into an IFTS Y is an IFcζ continuous mapping if and only if $f^{-1}(B)$ is an IFRCS in X, for each IFζ CS in Y.

Proof: Let B be an IFζ CS in Y. Then $\overline{B}$ is an IFζ OS in Y. Since f is anIFcζ continuous mapping, $f^{-1}(\overline{B})$ is an IFROS in X. But $f^{-1}(\overline{B}) = \overline{f^{-1}(B)}$, hence $f^{-1}(B)$ is an IFRCS in X.

Converse: Let B be any IFζ CS in Y. Then $\overline{B}$ is an IFζ OS in Y. By hypothesis $f^{-1}(\overline{B})$ is an IFRCS in X. From $f^{-1}(\overline{B}) = \overline{f^{-1}(B)}$, it follows that $f^{-1}(B)$ is an IFROS in X. Therefore f is an IFcζ continuous mapping.

Theorem 3.10: If a mapping $f : X \to Y$ is an IFcζ continuous, then for each IFP $p(\alpha, \beta) \in X$ and for every IFN A of $f(p(\alpha, \beta))$, there exists an IFROS B$\subseteq X$ such that $p(\alpha, \beta) \in B \subseteq f^{-1}(A)$.

Proof: Let $p(\alpha, \beta) \in X$ and let A be an IFN of $f(p(\alpha, \beta))$. Then there exists an IFOS C in Y such that $f(p(\alpha, \beta)) \in C \subseteq A$. Since every IFOS is an IFζ OS, C is an IFζ OS in Y.

Hence by hypothesis, $f^{-1}(C)$ is an IFROS in X and $p(\alpha,\beta)\in f^{-1}(C)$. Now let $f^{-1}(C)=B.$ Therefore $p(\alpha,\beta)\in B=f^{-1}(C)\subseteq f^{-1}(A).$

Theorem 3.11: If a mapping $f:X\to Y$ is an IFcζ continuous, then for each IFP $p(\alpha,\beta)\in X$ and for every IFN A of $f(p(\alpha,\beta))$, there exists an IFROS B$\subseteq X$ such that $p(\alpha,\beta)\in B$ and $f(B)\subseteq A.$

Proof: Let $p(\alpha,\beta)\in X$ and let A be an IFN of $f(p(\alpha,\beta))$. Then there exists an IFOS C in Y such that $f(p(\alpha,\beta))\in C\subseteq A.$ Since every IFOS is an IFζ OS, C is an IFζ OS in Y.

Hence by hypothesis, $f^{-1}(C)$ is an IFROS in X and $p(\alpha,\beta)\in f^{-1}(C)$. Now let $f^{-1}(C)=B.$ Therefore $p(\alpha,\beta)\in B=f^{-1}(C)\subseteq f^{-1}(A).$

Thus $f(B)\subseteq f(f^{-1}(A))\subseteq A.$ That is $f(B)\subseteq A$.

Theorem 3.12: If a mapping $f:X\to Y$ is an IFcζ continuous, then $\operatorname{int}(cl(f^{-1}(\operatorname{int}(B))))\subseteq f^{-1}(B)$ for every IFS B in Y.

Proof: Let $B\subseteq Y$ be an IFS. Then int(B) is an IFOS in Y and hence an IFζ OS in Y. By hypothesis, $f^{-1}(\operatorname{int}(B))$ is an IFROS in X. Hence $\operatorname{int}(cl(f^{-1}(\operatorname{int}(B))))=f^{-1}(\operatorname{int}(B))\subseteq f^{-1}(B)$.

Theorem 3.13: A mapping $f:X\to Y$ is an IFcζ continuous mapping then the following are equivalent:

(i) Forany IFζ OS A in Y and for any IFP $p(\alpha,\beta)\in X$, if $f(p(\alpha,\beta))_q A$, then $p(\alpha,\beta)_q \operatorname{int}(f^{-1}(A)).$

(ii) For any IFζ OS A in Y and for any $p(\alpha,\beta)\in X$, if $f(p(\alpha,\beta))_q A$, then there exists an IFOS B such that $p(\alpha,\beta)_q B$ and $f(B)\subseteq A.$

Proof: $(i)\Rightarrow(ii)$. Let $A\subseteq Y$ be an IFζ OS and let $p(\alpha,\beta)\in X$. Let $f(p(\alpha,\beta))_q A$. Then $p(\alpha,\beta)_q(f^{-1}(A))$ (i) implies that $p(\alpha,\beta)_q \operatorname{int}(f^{-1}(A))$ where $\operatorname{int}(f^{-1}(A))$ is an IFOS in X. Let $B=\operatorname{int}(f^{-1}(A))$. Since $\operatorname{int}(f^{-1}(A))\subseteq f^{-1}(A)$, $B\subseteq f^{-1}(A)$. Then $f(B)\subseteq f(f^{-1}(A))\subseteq A$.

$(ii)\Rightarrow(i)$. Let $A\subseteq Y$ be an IFζ OS and let $p(\alpha,\beta)\in X$. Suppose $f(p(\alpha,\beta))_q A$, then by (ii) there exists an IFOS B in X such that $p(\alpha,\beta)_q B$ and $f(B)\subseteq A$. Now $B\subseteq f^{-1}(f\ (B))\subseteq f^{-1}(A)$. That is $B=\operatorname{int}(B)\subseteq \operatorname{int}(f^{-1}(A))$. Therefore, $p(\alpha,\beta)_q B$ implies $p(\alpha,\beta)_q \operatorname{int}(f^{-1}(A))$.

Theorem 3.14: Let $f:X\to Y$ be a mapping. Then the following are equivalent.

(i) f is an IFcζ continuous mapping.

(ii) $f^{-1}(B)$ is an IFROS in X for every IFζ OS B in Y.

(iii) For every IFP $p(\alpha,\beta)\in X$ and for every IFζ OS B in Y such that $f(p(\alpha,\beta))\in B$ there exists an IFROS A in X such that $p(\alpha,\beta)\in A$ and $f(A)\subseteq B.$

Proof: $(i)\Rightarrow(ii)$ is obvious.

$(ii)\Rightarrow(iii)$. Let $p(\alpha,\beta)\in X$ and $B\subseteq Y$ such that F. This implies $p(\alpha,\beta)\in f^{-1}(B)$. Since B is an IF$\zeta$ OS B in Y. By hypothesis $f^{-1}(B)$ is an IFROS in X. Let $A=f^{-1}(B)$. Then $p(\alpha,\beta)\subseteq f^{-1}(f(p(\alpha,\beta)))\in f^{-1}(B)=A$. Therefore $p(\alpha,\beta)\in A$ and $f(A)=f(f^{-1}(B))\subseteq B$. This implies $f(A)\subseteq B.$

$(iii)\Rightarrow(i)$. Let $B\subseteq Y$ be an IFζ OS. Let $p(\alpha,\beta)\in X$ and $f(p(\alpha,\beta))\in B$. By hypothesis, there exists an IFROS C in X such that $p(\alpha,\beta)\in C$ and $f(C)\subseteq B$. This implies $C\subseteq f^{-1}(f(C))\subseteq f^{-1}(B)$. Therefore, $p(\alpha,\beta)\in C\subseteq f^{-1}(B)$. That is $f^{-1}(B)=U_{p(\alpha,\beta)\in f^{-1}(B)}p(\alpha,\beta)\subseteq U_{p(\alpha,\beta)\in f^{-1}(B)}C\subseteq f^{-1}(B)$. This implies $f^{-1}(B)=U_{p(\alpha,\beta)\in f^{-1}(B)}C$. Since union of IFROSs is IFROS. Hence f is an IFcζ continuous mapping.

Theorem 3.15: Let $f_1:(X,\tau)\to(Y,\kappa)$ and $f_2:(X,\tau)\to(Y,\kappa)$ be any two IFcζ continuous mappings. Then the mapping $(f_1,f_2):(X,\tau)\to(Y\times Y,\kappa\times\kappa)$ is also an IFcζ continuous mapping.

Proof: Let $A\times B$ be an IFζ OS of $Y\times Y$.

Then $(f_1,f_2)^{-1}(A\times B)(x)=(A\times B)(f_1(x),f_2(x))$

$$= \langle x, \min(\mu_A(f_1(x)), \mu_B(f_2(x))), \max(\upsilon_A(f_1(x)), \upsilon_B(f_2(x)))\rangle$$

$$= \langle x, \min(f_1^{-1}(\mu_A)(x), f_2^{-1}(\mu_B)(x)), \max(f_1^{-1}(\upsilon_A)(x), f_2^{-1}(\upsilon_B)(x))\rangle$$

$$= (f_1^{-1}(A) \cap f_2^{-1}(B))(x).$$

Since f_1 and f_2 are an IFcζ continuous mapping, $f^{-1}(A)$ and $f^{-1}(B)$ are IFROS in X. Since the intersection of two IFROS is an IFROS. Therefore $f_1^{-1}(A) \cap f_2^{-1}(B)$ is an IFROS in X. Hence (f_1, f_2) is an anIFcζ continuous mapping.

Theorem 3.16: Let $f : X \to Y$ and $g : Y \to Z$ be any mappings. If f and g are IFcζ continuous, then gof is also IFcζ continuous.

2.3. Intuitionistic Fuzzy ζ Homeomorphisms

In this section we introduce intuitionistic fuzzy ζ homeomorphism and study some of its properties.

Definition 4.1: A bijection mapping $f : (X, \tau) \to (Y, \kappa)$ is called an intuitionistic fuzzy ζ homeomorphism (IFζ homeomorphism, in short) if f and f^{-1} are IFζ continuous mappings.

Example 4.2: Let $X = \{a, b\}$, $Y = \{u, v\}$

$G_1 = \{\langle x, (0.2, 0.2), (0.6, 0.7)\rangle\}$,

$G_2 = \{\langle y, (0.4, 0.7), (0.4, 0.2)\rangle\}$

Then $\tau = \{0_\sim, 1_\sim, G_1\}$ and $\kappa = \{0_\sim, 1_\sim, G_2\}$ are IFT on X and Y respectively.

Define a bijectionmapping $f : (X, \tau) \to (Y, \kappa)$ by $f(a) = u$ and $f(b) = v$.

Then f is IFζ continuous mapping and f^{-1} is also IFζ continuous mapping. Therefore f is an IFζ homeomorphism.

Theorem 4.3: Every IF homeomorphism is an IFζ homeomorphism.

Example 4.4: Let $X = \{a, b\}$, $Y = \{u, v\}$

$G_1 = \{\langle x, (0.3, 0.2), (0.6, 0.7)\rangle\}$,

$G_2 = \{\langle y, (0.5, 0.4), (0.4, 0.2)\rangle\}$

Then $\tau = \{0_\sim, 1_\sim, G_1\}$ and $\kappa = \{0_\sim, 1_\sim, G_2\}$ are IFT on X and Y respectively.

Define a bijection mapping $f : (X, \tau) \to (Y, \kappa)$ by $f(a) = u$ and $f(b) = v$. Then f is an IFζ homeomorphism but not an IF homeomorphism, since f and f^{-1} are not IF continuous mappings.

Definition 4.5: An IFTS (X, τ) is said to be intuitionistic fuzzyζ $T_{1/2}$ space(IFζ $T_{1/2}$, in short) if every IFζ OS in X is an IFOS in X.

Theorem 4.6: Let $f : (X, \tau) \to (Y, \kappa)$ be an IFζ homeomorphism, then f is an IF homeomorphism if X and Y are IFζ $T_{1/2}$space.

Proof: Let B be an IFOS in Y. Then $f^{-1}(B)$ is an IFζ OS in X. Since X is an IFζ $T_{1/2}$space, $f^{-1}(B)$ is an IFOS in X. Hence f is an IF continuous mapping. By hypothesis $f^{-1} : (Y, \kappa) \to (X, \tau)$ is an IFζ continuous mapping. Let A be an IFOS in X. Then $(f^{-1})^{-1}(A) = f(A)$ is an IFζ OS in Y. Since Y is an IFζ $T_{1/2}$space, $f(A)$ is an IFOS in. Hence f^{-1} is an IF continuous mapping. Therefore the mapping f is an homeomorphism.

Definition 4.7 [11]**:** Let f be a mapping from IFTS (X, τ) into an IFTS (Y, κ). Then f is said to be intuitionistic fuzzy ζ open mapping (IFζ open mapping, in short) if $f(A) \in IF\zeta OS(X)$ for every IFOS A in X.

Theorem 4.8: Let $f : (X, \tau) \to (Y, \kappa)$ be a bijective mapping. If f is an IF continuous mapping, then the following are equivalent.

(i) f is an IFζ closed mapping

(ii) f is an IFζ open mapping

(iii) f is an IFζ homeomorphism

Proof: $(i) \Rightarrow (ii)$. Let $f : (X, \tau) \to (Y, \kappa)$ be a bijective mapping and let f is an IFζ closed mapping. This implies $f^{-1} : (Y, \kappa) \to (X, \tau)$ is an IFζ continuous mapping. That is every IFOS in X is an IFζ OS in Y. Hence f is an IFζ open mapping.

$(ii) \Rightarrow (iii)$. Let $f : (X, \tau) \to (Y, \kappa)$ be a bijective mapping and let f is an IFζ open mapping. This implies $f^{-1} : (Y, \kappa) \to (X, \tau)$ is an IFζ continuous mapping. Hence f and f^{-1} are IFζ continuous mappings. That is f is an IFζ homeomorphism.

$(iii) \Rightarrow (i)$ Let f is an IF ζ homeomorphism. That is f and f^{-1} are IF ζ continuous mappings. Since every IFCS in X is an IF ζ CS in Y, f is an IF ζ closed mapping.

Remark 4.9: The composition of two IF ζ homeomorphisms need not be an IF ζ homeomorphism in general.

Example 4.10: Let $X = \{a,b\}, Y = \{c,d\}\ Z = \{u,v\}$,
$G_1 = \{\langle x,(0.8,0.6),(0.2,0.4)\rangle\}$,
$G_2 = \{\langle y,(0.6,0.1),(0.4,0.3)\rangle\}$,
$G_3 = \{\langle z,(0.4,0.4),(0.6,0.2)\rangle\}$
Then $\tau = \{0_\sim, 1_\sim, G_1\}$, $\kappa = \{0_\sim, 1_\sim, G_2\}$ and χ : $\{0_\sim, 1_\sim, G_3\}$ are IFT on X,Y and Z respectively.
Define a bijective mapping $f:(X,\tau) \to (Y,\kappa)$ by $f(a) = c$ and $f(b) = d$ and $g^{\cdot}:(Y,\kappa) \to (Z,\chi)$ by $f(c) = u\ f(d) = v$. Then f and f^{-1} are IF ζ continuous mappings. Also g and g^{-1} are IF ζ continuous mappings. Hence f and g are IF ζ homeomorphisms**.** But the composition $gof : X \to Z$ is not an IF ζ homeomorphism since gof is not an IF ζ continuous mapping.

2.4. Intuitionistic Fuzzy Z- ζ Homeomorphisms

Definition 5.1: Let f be a mapping from IFTS (X,τ) into an IFTS (Y,κ). Then f is said to be intuitionistic fuzzy ζ irresolute (IF ζ irresolute, in short) if $f^{-1}(B) \in IF\zeta O(X)$ for every IF ζ OS B in Y.

Definition 5.2: A bijection mapping $f:(X,\tau) \to (Y,\kappa)$ is called an intuitionistic fuzzy Z- ζ homeomorphism (IFZ ζ homeomorphism, in short) if IF ζ irresolute mappings.

Theorem 5.3: Every IFZ ζ homeomorphism is an IF ζ homeomorphism but not conversely.

Proof: Let $f:(X,\tau) \to (Y,\kappa)$ be an IFZ ζ homeomorphism. Let B be IFOS in Y. This implies B is an IF ζ OS in Y. By hypothesis $f^{-1}(B)$ is an IF ζ OS f and f^{-1} are in X. Hence f is an IF ζ continuous mapping. Similarly we can prove f^{-1} is an IF ζ continuous mapping. Hence f and f^{-1} are IF ζ continuous mappings. This implies the mapping f is an IF ζ homeomorphism.

Example 5.4: Let $X = \{a,b\}$, $Y = \{u,v\}$
$G_1 = \{\langle x,(0.4,0.3),(0.6,0.7)\rangle\}$,
$G_2 = \{\langle y,(0.2,0.1),(0.4,0.5)\rangle\}$
Then $\tau = \{0_\sim, 1_\sim, G_1\}$ and $\kappa = \{0_\sim, 1_\sim, G_2\}$ are IFT on X and Y respectively.
Define a bijection mapping $f:(X,\tau) \to (Y,\kappa)$ by $f(a) = u$ and $f(b) = v$. Then f is an IF ζ homeomorphism. Let us consider an IFS $H = \langle y,(0.3,0.2),(0.7,0.7)\rangle$ in Y. Clearly H is an IF ζ OS in Y. But $f^{-1}(H)$ is not an IF ζ OS in X. That is f is not an IF ζ irresolute mapping. Hence f is not an IFZ ζ homeomorphism.

Theorem 5.5: If the mapping $f : X \to Y$ is an IFZ ζ homeomorphism, then $\zeta \operatorname{int}(f^{-1}(B)) = f^{-1}(\zeta \operatorname{int}(B))$ for every IFS B in Y.

Proof: Since f is an IFZ ζ homeomorphism, f is an IF ζ irresolute mapping. Consider an IFS B in Y. Clearly $\zeta \operatorname{int}(B)$ is an IF ζ OS in Y. By hypothesis $f^{-1}(\zeta \operatorname{int}(B))$ is an IF ζ OS in X. Since $f^{-1}(\zeta \operatorname{int}(B)) \subseteq f^{-1}(B)$,
$\zeta \operatorname{int}(f^{-1}(\zeta \operatorname{int}(B))) \subseteq \zeta \operatorname{int}(f^{-1}(B))$. This implies $f^{-1}(\zeta \operatorname{int}(B)) \subseteq \zeta \operatorname{int}(f^{-1}(B))$.
Since f is an IFZ ζ homeomorphism, $f^{-1} : Y \to X$ is an IF ζ irresolute mapping. Consider an IFS $f^{-1}(B)$ in X. Clearly $\zeta \operatorname{int}(f^{-1}(B))$ is an IF ζ OS in X. This implies $(f^{-1})^{-1}(\zeta \operatorname{int}(f^{-1}(B))) = f(\zeta \operatorname{int}(f^{-1}(B)))$ is an IF ζ OS in Y. Clearly $B = (f^{-1})^{-1}(B) \supseteq (f^{-1})^{-1}(\zeta \operatorname{int}(f^{-1}(B))) = f(\zeta \operatorname{int}(f^{-1}(B))$
Therefore
$\zeta \operatorname{int}(B) \supseteq \zeta \operatorname{int}(f\ \ (\zeta \operatorname{int}(f^{-1}(B)))) = \zeta \operatorname{int}((f^{-1}(B))$. Since f^{-1} is an IF ζ irresolute mapping. Hence $f^{-1}(\zeta \operatorname{int}(B)) \supseteq f^{-1}(f(\zeta \operatorname{int}(f^{-1}(B)))) = \zeta \operatorname{int}(f^{-1}(B))$
That is $f^{-1}(\zeta \operatorname{int}(B)) \supseteq \zeta \operatorname{int}(f^{-1}(B))$.

Theorem 5.6: If $f : X \to Y$ is an IFZ ζ homeomorphism, then $\zeta \operatorname{int}(f(B)) = f(\zeta \operatorname{int}(B))$ for every IFS B in X.

Proof: Since f is an IFZ ζ homeomorphism, f^{-1} is IF ζ homeomorphism. Let us consider an IFS B in X. By theorem 5.4 $\zeta \operatorname{int}(f(B)) = f(\zeta \operatorname{int}(B))$ for every IFS B in X.

Remark: 5.7: The composition of two IFZζ homeomorphisms is IFZζ homeomorphism in general.

Proof: Let $f: X \to Y$ and $g: Y \to Z$ be any two IFZζ homeomorphisms. Let A be an IFζ OS in Z. Then by hypothesis, $g^{-1}(A)$ is an IFζ OS in Y. Then by hypothesis, $f^{-1}(g^{-1}(A))$ is an IFζ OS in X. Hence $(gof)^{-1}$ is an IFζ irresolute mapping. Now let B be an IF ζ OS in X. Then by hypothesis, $f(B)$ is an IFζ OS in Y. Then by hypothesis $g(f(B))$ is an IFζ OS in Z. This implies gof is an IFζ irresolute mapping. Hence gof is an IFZζ homeomorphism. That is the composition of two IFZζ homeomorphisms is an IFZζ homeomorphism in general.

3. References

1. Arockiarani I, Jude H. Immaculate, A Note on intuitionistic Fuzzy d-Continuous mappings, {Proceeding of National Seminar on Recent Development in Topology}, 2015, 236-244.
2. Atanassov K. Intuitionistic fuzzy sets, Fuzzy Sets and Systems 1986; 20(1):87-96.
3. Azad KK. On fuzzy Semi Ccontinuity, Fuzzy almost continuity and Fuzzy weakly continuity. J Math. Annal. Appl. 1981; 82:14-32.
4. Biljana Krsteska, Erdal Ekici. Intuitionistic fuzzy Contra Strong Pre-continuity, Faculty of Sciences and Mathematics, 2007; 21:273-284.
5. Chang CL. Fuzzy topological spaces, J Math., Anal and Appl. 1968; 24(1):182-190.
6. Coker D. An introduction to Intuitionistic fuzzy topological spaces, Fuzzy Sets and Systems. 1997; 88(1):81-89.
7. Gurcay HD, Coker AD. Es, on fuzzy continuity in intuitionistic fuzzy topological spaces, J Fuzzy Math. 1997: 2(5):365- 378.
8. Hanafy IM. Intuitionistic Fuzzy γ-continuity Canad. Math, Bull. 2009; 52(8):544-554.
9. Joen JK, Jun YB, Park JH. Intuitionistic fuzzy α-continuity and Intuitionistic fuzzy Precontinuity, IJMMS. 2005; 19:3091-3101.
10. Lee SJ, Lee EP. The category of intuitionistic fuzzy topological spaces, Bull. Korean Math. Soc. 2000; 37(1):63-76.
11. Mukherjee MN, Ghosh B. Some stronger forms of fuzzy continuous mappings on fuzzy topological spaces, Fuzzy sets and Systems 1990; 38:375-387.
12. Neelamegrajan Rajesh. *et al.* On $\tilde{g}$ semi homeomorphism in topological spaces. Annals of University of cralovamath. comp. sci. ser 2006; 33:208-215.
13. Sharmila S, Arockiarani I. Intuitionistic fuzzy ζ Open sets, (Communicated).
14. Young Bae Jun, Seok-Zun Song. Intuitionistic fuzzy semipre open sets and intuitionistic fuzzy Semipre continuous mappings, Jour. Of Appl. Math and Computing, 2005; 19:467-474.
15. Zadeh IA. Fuzzy sets, Information and Control 1965; 8:338-353.