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Exploration of Calculus Teaching and Assessment through Artwork

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Abstract
The authors are interested in conducting research of teaching pedagogy as well as assessment strategies by incorporating art in mathematics education. In particular, authors are focusing on Calculus teaching as well as assessment through artwork. During the pilot 2013 Summer Calculus course at Borough of Manhattan Community College-The City University of New York, the first author incorporated art in teaching and assessment. One purpose is to guide students to learn Calculus concepts by using and identifying certain artwork that are correlated with Calculus. The other purpose is to use artwork as a tool to assess students' application ability in Calculus.

Keywords: art-related contexts, assessment, Calculus, cognitive understanding, math anxiety, problem-solving skills

1. Introduction

1.1 Background and Problem Statement

In the traditional setting, student's math ability is evaluated by taking multiple-choice exams or written exams. Mathematics exam questions are formulated by numbers, variables, formulas, and equations in math terminology. It turns out that this traditional testing system has weakness regarding the following 3 aspects:

The first aspect is the math anxiety caused by the traditional testing format. Student will feel very stressful, develop math anxiety, and cope with math anxiety whenever tests taken. The outcome of student's math anxiety can weaken student's self-efficacy and increase student's failing test rate. For the lower-level mathematics course, such as Algebra, the data provided from CUNY Institution Testing Center showed that 28% of students who registered algebra class in fall 2012 were not confident to take CUNY CEAFE (CUNY Common Elementary Algebra Final Exam). Moreover, 62% of students who registered algebra class in fall 2012 failed. For the advanced mathematics course, such as Calculus, the situation was worse.

The second aspect is the inaccuracy of the testing system. The outcome of multiple-choice test system will not be accurate to indicate student's math ability levels when student guesses the correct answers by luck. Student who is lucky to achieve the passing scores by guessing might not be really knowledgeable of the tested subjects.

The third aspect is the lack of evaluating student's application skill in the assessment system. Some student who is able to pass tests successfully might not be able to make use of mathematical skill to cope with real-life situations. Passing the test only proves student's strong memory of math book knowledge that he/she has learned in class. Student who has successfully passed the exams may not be able to use mathematics regularly in context outside classroom. In the real-life situations, student totally forgets math skills learned in class. Student is not able to apply math skills whenever needed. Educator is frustrated with student's incompetency and disappointed with the current assessment.

How to design mathematics curriculum and teach student to use math as a basic life skill? What is the effective and efficient methodology of teaching? What is the accurate assessment method to evaluate student's ability of conceptual understanding and application skills? This is an especially acute problem in the school system. It is imperative for math educators to find a solution.

1.2 Literature Review

Math Anxiety is well-known phenomenon among students all over the world. Research of studying math anxiety, math self-efficacy and math self-concept among 41 countries was reported in 2009 [6]. Math anxiety decreases student's self-efficacy and attributes for failure on tests with test anxiety. Reciprocal relation between math anxiety and math self-concept was reported in 2012 [2]. The impact of math anxiety on student's self-efficacy, math self-concept, math attitudes was studied in 2011 [1]. The relationship of trait and test anxiety with mathematics anxiety was studied in 2000 [13].

Research regarding mathematics and art in teaching [3, 10, 12] has been studied. The majority studies of this art-based teaching in mathematics curriculum are for elementary, middle and high schools' students, such as teaching math through the visual arts for K-5 [5], improving math scores by integrating art in math class for public high school students [7], infusing the arts into math in elementary school [4]. Syamala [11] used Indian art to teach basic graph theory concepts of vertices, edges, degrees of the vertices, path, circuit and Euler path. Reily and Pagnucci's [9] claims their project of integrating art in mathematics helped students contextualize their learning.

Art expression is an integral part of human nature and has been a way of communication throughout history [8]. In past literatures, the research of connecting art and math in higher education is limited. A case study that analyzed the process of transference between art and math in a California community college by Rachford [8], the results indicated that interdisciplinary approach with art is beneficial to motivation, collaboration, understanding, and application of knowledge in the educational environment. Due to limited research study of art and math in higher education, the assessment of evaluating students' learning through arts can be a new area for exploration for future study.

2. Methodology

Authors believe that math subjects taught in art-related context will be meaningful and memorable. Math concepts such as numbers, shapes, patterns, groups, and symbols can be easily found in artwork. The cause of student's math anxiety comes from the abstraction of math subjects. In order to moderate math anxiety, authors are interested in conducting research on the pedagogy that uses images, such as artwork, as a pathway to facilitate math teaching and learning. Compared with the traditional teaching methodology, the advantage of integrating art into education is to incorporate visual images into math teaching.

In the use of mathematics knowledge, authors believe that the assessment should evaluate student's ability of recognizing underlying math knowledge in a variety of contexts and making decisions about which math techniques to apply and how to apply them. Key to the success of such an assessment is to include student's practice of math technical skills engaged with contexts. Student's ability of understanding math, applying math skills, and expressing math ideas with clarity, completeness, and accuracy will be evaluated comprehensively in the assessment.

During the pilot 2013 summer Calculus course at Borough of Manhattan Community College, the first author incorporated art in teaching and assessing. Visualization through artwork has enhanced students' intuitive understanding of Calculus concepts and Calculus theories. One purpose is to guide student to learn Calculus concepts by identifying correlation of artwork with Calculus. The other purpose is to assess students' ability by exploring Calculus knowledge through artwork. In this article, authors describe examples of the selected artwork as teaching materials or as assessment models to evaluate student's ability of cognitive understanding, interpreting math knowledge with an art-related context, and applying math principles to create art diagram design.

2.1 Calculus Teaching through Artwork

Project 1: Introducing Calculus Concepts of "Doubling Growth to Infinity" through Artwork of "Thousand-Armed Avalokiteshvara"

Fig 1: Thousand-Armed Avalokiteshvara
(F1997.17.8, HAR324 at Rubin Museum of Art in NYC).

Fig 2: Video of Dance of Thousand-Armed Avalokiteshvara at www.youtube.com

Instructor's Explanation

"Infinity" is one of the most important concepts in Calculus.

The first selected artwork was the "Thousand-Armed Avalokiteshvara". This artwork depicts the Buddha with thousand arms. Students were encouraged to relate the meaning of the math word "Infinity" to the Buddhist's supernatural power by the unbounded growth. A 4-minute video of "Dance of Avalokiteshvara with Thousand-Arms" was presented in teaching to ask student to frame a mathematical model of a function with doubling growth. By creating a function $y = f(n) = 2n$ where n represents number of dancers on stage and y represents number of Avalokiteshvara's arms indicating the power's strength, students can obtain an intuitive understanding of the growth pattern by doubling and the unbounded growth. From this lesson, students learned the mathematics definition of "Infinity" as "Unbounded Growth".

Project 2: Studying Calculus Concepts of "Vector Fields and Vector Analysis of Rotating Wheel" by Artwork of "Wheel of Life"

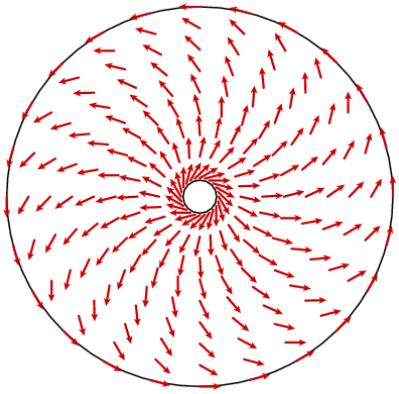

Fig 3: Vector Fields and Vector Analysis of Rotating Wheel

Fig 4: Wheel of Existence
(C2004.21.1, HAR65356 at Rubin Museum of Art in NYC)

Instructor's Explanation

Vector field and vector analysis are one of the most significant concepts in Calculus. One of the most common physical examples of vector fields is velocity field. The velocity field describes the motion of a system of particles in space. The diagram of line segments with arrows shows the velocity vectors determined by a wheel rotating on an axle.

This rotating wheel reminds students of the Wheel of Life. The Wheel of Life illustrates the essence of the Buddhist teaching. Everyone's life is different. However, no matter how different each person's life is, the wheel of life describes that everyone's life starts from the same initial point (that is in the cradle stage) and ends at the same final point (that is in the grave). In Buddhist thoughts, death will be followed by re-birth of life. Therefore, life is illustrated as a loop and zero.

Compared with the Buddhist thoughts, students were asked to find the similarity in Calculus. By applying "Fundamental Theorem of Line Integral", students can see that the integral of conservative vector fields along all possible different loops is always zero. This is true no matter how different each loop is.

The discovery of the common ground of Calculus and art will help students to understand and remember mathematical knowledge.

2.2 Calculus Assessment through Artwork

Project 3: Assessing Student's Application Ability of Using Polar Graph Equations to Create Art Diagram - "Polar Art Diagram Design"

In this pilot Calculus course, the first author designed the following project to assess students' Calculus knowledge of the polar coordinate system and to evaluate students' application skills of polar graphic equations.

a. Project Objective and Description

The objective of this polar art project was to encourage students to use their Calculus knowledge of the polar coordinate system and polar graph equations to create or design art diagrams by using Maple Software. Students were divided into groups. Students in each group worked together to complete the assignment.

Creating art diagrams was a process to compose different polar graph equations to produce different polar curves in different shapes, such as Cardioids' heart-shaped, rose flowers with multiple petals, circles with inner loops and others. The list of the fundamental shapes of polar graph equations can be found on Page 705 in the textbook of Calculus by Larson, Hostetler, and Edwards in the alternate sixth edition.

b. Learning Outcome

Working on this project, students will fully understand the graphic properties of various polar graph equations in Calculus. By completing this project, student will learn how to use Maple Software as an educational tool to create computer-generated sketch of graphic design.

c. Requirements for Students' Art Diagram

1. Each group will be required to create 3 pieces of art diagrams.
2. Each art diagram is composed by using no less than 3 different polar equations.
3. Each art diagram is required to be in at least 2 different colors.

4. Each group can create art diagrams with either repeated patterns or combination of different patterns.
5. Each group will be required to complete the project in the Math Lab by using Maple Software.
6. Each group will be required to submit a list of all polar equations that have been used in creating the artwork
7. Each group will be required to finish a writing assignment including:

1) What was the purpose of the design? (For example: We design a birthday card for our mothers because they love flowers. We pick this color because….)
2) Describe your thoughts about the project. What did you learn from this? Do you think that this is a better method to practice Calculus than the traditional way?

d. Timeline

This was a one-month project. The project started on June 5, 2013 and ended on July 5, 2013.

e. Reference

1. Textbook Resource: a list of special polar graphs on page 705 in the textbook of Calculus by Larson, Hostetler, and Edwards in the alternate sixth edition.
2. Website Resource: http://www.maplesoft.com/applications/view.aspx?SID=4087&view=html
3. Or you can use Google or other public website as reference to search the information you need.

Here let us present samples of students' works.

Students' Performance of Art Diagram and Their Report:

Spiraling Graphs and Rose Curves
-Independent Holiday Card

Fig 5: Students' Art Diagram of Holiday Greeting Card Created in 2013 Summer Pilot Calculus Course at Borough of Manhattan Community College-The City University of New York

Students also expressed their strong enthusiasm for this challenging project in their report as follows:

"It was a challenging and stimulating project as a very interesting Calculus learning experience. By integrating the concepts of polar coordinates in maple we as group were able to create a unique way to show off our artistic side. This project has been a very suitable exercise to learn about polar coordinates. This project has given us the opportunity to learn about the concepts of polar coordinates and its many uses that can be applied to real world applications that involve circular and orbital motions."

Instructor's Commends

This is one of the best student's creations. Students created a bundle of flowers from fireworks by using trigonometric functions in polar coordinates. By reading students' writing report of the creation of this Independent Holiday card, we understood that student were able to manipulate the combination between polar equations of rose curves (i.e. $r = a\cos(n\theta)$ and $r = a\sin(n\theta)$) and polar equations of spiraling graphs (i.e. $r = \theta$) as well as the product of θ and a trigonometric function (sine or cosine). By completing this polar art project, students will have many opportunities to practice polar coordinates and apply polar equations to draw curve sketching. In order to design an impressive art diagram, students will be required to try different shapes of polar graphs with appropriate diameters, angles, and patterns by trying different polar functions and testing different values of variables in the polar equations. Aesthetic development in Polar Art Diagram Design project will enhance student's critical thinking by using math skills. Learning Calculus in an enjoyable way will definitely decrease students' math anxiety.

3. Students' Feedback and Survey

In the pilot course study, students have showed great interest in learning Calculus and have better understanding of Calculus when they can see it through artwork. Students have expressed that they never thought math could be found in artwork. By incorporating artwork in math teaching, learning math becomes interesting and inspiring. This type of teaching and assessing inspired students' strong desire to learn more.

Students' Commends in This Pilot Course
Examples of students' positive feedback are quoted

"*Now, through this art project, our eyes have been opened on multiple applications of Mathematics. That's definitely one the positive outcome of the art project; it opened our eyes to applications of Mathematics that we have never heard of. Ultimately, the example, established by the art project, pushes us to look for new ways to use efficiently our knowledge in Mathematics. It is more profitable to learn Mathematics by doing projects than the traditional way which would have taught us just the math concepts and leave us with the idea that we can apply math only in the scientific field. We appreciate this new way of teaching Calculus and we hope that the experiment will be renewed and extended to all Calculus classes."*

Students' Survey in This Pilot Course

Table 1: Optimizing Student's Learning by Interpreting Math Subjects in Art-Related Concepts

Item	*Agree*	*Neutral*	*Disagree*
Increasing Conceptual Understanding	*57%*	*36%*	*7%*
Decreasing Mathematics Anxiety	*36%*	*36%*	*28%*
Strengthening Mathematics Confidence	*43%*	*29%*	*28%*
Increasing Motivation and Engagement	*50%*	*36%*	*14%*
Promoting Long-Term Memory	*43%*	*50%*	*7%*
Connecting Abstract Concepts with Concrete Objects	*71%*	*29%*	*0%*

Table 2: Enhancing Student's Self –Assessment by Using Art-Based Activities

Item	*Agree*	*Neutral*	*Disagree*
Improving Students' Application Ability of Mathematics Skills in Various Real-World Contexts	*64%*	*29%*	*7%*
Encouraging Individual Problem-Solving Skills	*57%*	*36%*	*7%*
Developing Cooperative Learning in Team Work	*79%*	*14%*	*7%*
Enhancing Students' Oral Communication Skill by Presenting Group Progress	*64%*	*36%*	*0%*
Enhancing Students' Written Communication Skills by Writing Reports of Group Achievements	*79%*	*21%*	*0%*
Helping Students Become Aware of their Strength and Weakness in Group Settings	*57%*	*29%*	*14%*

4. Conclusion and Future Research

This pilot program was designed to teach mathematical concepts relating to artwork contextual environments and to assess students' ability in applying mathematical knowledge when designing certain artwork. It is apparent that integrating art into Calculus in this pilot program showed a very important shot-term impact on promoting students' ability of conceptual understanding by connecting abstract Calculus concepts with concrete art objects, as shown in Table 1. Most notably, this pilot program marks the substantial improvement on students' ability of problems-solving skills, cooperative skills in team work, oral communication skills, and written communication skills, as indicated in Table 2.

The overall conclusion from the short-term impact of incorporating art in Calculus is positive by students' feedback. Authors believe that it will lead to great benefits and long-term academic gains for both teachers and students. Research on art-based education into college-level mathematics will be challenging with respect to the following:

1. Selecting appropriate artwork as teaching objects to facilitate students' conceptual understanding ability;
2. Designing art-based activities in the curriculum design as assessment tools to evaluate students' mathematical skills in practice.

Information gathered in this paper could lead to the future research on mathematics and art in college education.

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6. Reference

1. Ahmed A. The relationships between math anxiety, math attitudes, and self-efficacy: A structural equation model. Studia Psychologica. 2011; 53(3):263-273.
2. Ahmed W, Minnaert A, Kuyper H, van der Werf G. Reciprocal relationships between math self-concept and math anxiety. Learning and Individual Differences. 2012; 22(3): 385-389. DOI: 10.1016/j.lindif.2011.12.004
3. Brown T. The art of mathematics: Bedding down for a new era. Educational Philosophy and Theory. 2007; 39(7):755-765. DOI: 10.1111/j.1469-5812.2007.00318.x
4. Granger T. Math is art. Teaching Children Mathematics. 2000; 7(1):10-13.
5. Holtzman C, Susholtz L. Object lessons: Teaching Math through the visual arts, *K-5*. Stenhouse Publisher, Portland, 2011, 217.
6. Lee J. Universals and specifics of math self-concept, math self-efficacy, and math anxiety across 41 PISA 2003 participating countries. Learning and Individual Differences. 2009; 19(3):355-365. DOI: 10.1016/j.lindif.2008.10.009
7. McIntosh RD. Teaching Art +Math. School Arts: The Art Education Magazine for Teachers 2008; 107(8):64.
8. Rachford MK. An investigation into process of transference, through the integration of art with Science and Math curricula, in a California community college: A case study. Dissertation, School of Education, Azusa Pacific University. 2011. 175.
9. Reily EM, Pagnucci GS. Mathematics, art research, collaboration, and storytelling: The High M.A.R.C.S. Project. Mathematics Teaching in the Middle School 2007; 12(9):497-502.
10. Sharp J. Mathematics and Art. *Mathematics Teaching*. 2012; 226:20-23.
11. Syamala C. Teaching mathematics through the art of kolam. Mathematics Teaching in the Middle School 2007; 12(8):422-428.
12. Wynn T. Math into visual art into math into poetry: Rhythm of structure at the Bowery Poetry Club, New York. International Review of African American Art 2011; 23(3):62-64.
13. Zettle RD, Raines SJ. The relationship of trait and test anxiety with mathematics anxiety. College Student Journal. 2000; 34(2):246.