ISSN Print: 2394-7500
ISSN Online: 2394-5869
Impact Factor: 5.2
IJAR 2015; 1(9): 04-07
www.allresearchjournal.com
Received: 03-06-2015
Accepted: 05-07-2015

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On Nano Generalized Star Closed Sets in Nano Topological Spaces

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Abstract
The purpose of this paper is to define and study some nano-closed sets namely, nano g*-closed sets, nano g*s-closed sets, nano g*p-closed sets and nano g*r-closed sets are analysed.

Keywords: Nano g*-closed sets, nano g*s-closed sets, nano g*p-closed sets and nano g*r-closed sets.

1. Introduction
Levine [5] introduced the class of g-closed sets in 1970. Lellis Thivagar [4] introduced Nano topological space with respect to a subset X of a universe which is defined interms of lower and upper approximations of X. He has also defined Nano closed sets Nano-interior and Nano-closure of a set. Bhuvaneswari (1, 2, 3 & 7) introduced Nano g-closed, Nano gs-closed, Nano αg-closed, Nano gα-closed and Nano gr and Nano rg-closed sets.

Definition 1.1 [6]
A subset A of a topological space (X, τ) is called a generalized star closed set if cl(A) ⊆ U whenever A ⊆ U and U is g-open in (X, τ).

Definition 1.2 [4]
Let U be a non-empty finite set of objects called the universe and R be an equivalence relation. Elements belongings to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let X ⊆ U
(i) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and its denoted by $L_R(X)$.
That is $L_R(X) = U_{X\in U}\{R(X): R(X) \subseteq X\}$ where R(X) denotes the equivalence class determined by X.
(ii) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$.
That is $U_R(X) = U_{X\in U}\{R(X): R(X) \cap X \neq \Phi\}$
(iii) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not X with respect to R and it is denoted by $B_R(X)$.
That is $B_R(X) = U_R(X) – L_R(X)$.

Property 1.3 [4]
If (U, R) is an approximation space and X, Y ⊆ U, then
- $L_R(X) \subseteq X \subseteq U_R(X)$
- $L_R(\Phi) = U_R(\Phi) = \Phi$ and $L_R(U) = U_R(U) = U$
- $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
- $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$
- $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$
- $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
- $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever X ⊆ Y
- $U_R(X^C) = [L_R(X)]^C$ and $L_R(X^C) = [U_R(X)]^C$
- $U_R[U_R(X)] = L_R[U_R(X)] = U_R(X)$
- $L_R[L_R(X)] = U_R[L_R(X)] = L_R(X)$

Definition 1.4 [4]
Let U be the universe, R be an equivalence relation on U and $\tau_R(X)$ = {U, Φ, $U_R(X)$, $L_R(X)$, $B_R(X)$} where X ⊆ U. Then by Property 1.3, $\tau_R(X)$ satisfies the following axioms.

- U and Φ ∈ τ_R(X).
- The union of the elements of any sub collection of τ_R(X) is in τ_R(X).
- The intersection of the elements of any finite sub collection of τ_R(X) is in τ_R(X).

That is, τ_R(X) is a topology on U called the Nano topology on U with respect to X. $(U,\tau_R$(X)) is called the Nano topological space. Elements of the Nano topology are known as Nano open sets in U. Elements of $[\tau_R(X)]^C$ are called Nano closed sets with $[\tau_R(X)]^C$ being called dual Nano topology of τ_R(X).

Remark 1.5 [4]
If $\tau_R(X)$ is the Nano topology on U with respect to X. Then the set
B = {U, L_R(X), B_R(X)} is the basis for $\tau_R(X)$.

Definition 1.6 [4]
If $(U,\tau_R(X))$ is a nano topological space with respect to X where X ⊆ U and if A ⊆ U, then

- The Nano interior of the set A is defined as the union of all nano open subsets contained in A and is denoted by Nint(A). Nint(A) is the largest nano open subset of A.
- The Nano closure of the set A is defined as the intersection of all nano closed sets containing A and is denoted by Ncl(A). Ncl(A) is the smallest nano closed set containing A.

Definition 1.7
Let $(U,\tau_R(X))$ be a nano topological space and A ⊆ U. Then A is said to be

(1) Ng-closed [1] if Ncl(A) ⊆ V whenever A ⊆ V and V is Nano-open in U.
(2) Ngr-closed [7] if Nrcl(A) ⊆ V whenever A ⊆ V and V is Nano-open in U.
(3) Ngs-closed [2] if Nscl(A) ⊆ V whenever A ⊆ V and V is Nano-open in U.
(4) Ngα-closed [3] if Nαcl(A) ⊆ V whenever A ⊆ V and V is Nano-open in U.

1. Nano Generalized Star Closed Set

Throughout this paper (U,$\tau_R(X)$) is a Nano topological space with respect to X where X ⊆ U, R is an equivalence relation on U, U/R denotes the family of equivalence classes of U by R.

Definition 2.1
Let (U,$\tau_R(X)$) be a nano topological space. A subset A of (U,$\tau_R(X)$) is called

(i) Nano generalized star closed set (briefly Ng* closed), if Ncl(A) ⊆ V whenever A ⊆ V and V is nano g-open.
(ii) Nano generalized star semi closed set (briefly Ng*s-closed) if Nscl(A) ⊆ V whenever A ⊆ V and V is nano g-open.
(iii) Nano generalized star pre closed set (briefly Ng*p-closed) if Npcl(A) ⊆ V whenever A ⊆ V and V is nano g-open.
(iv) Nano generalized star regular closed set (briefly Ng*r-closed) if Nrcl(A) ⊆ V whenever A ⊆ V and V is nano g-open.

Example 2.2
Let U = {a, b, c, d} with U/R = {a, c, {b,d}} and X = {a, b}. Then $\tau_R(X)$ = {U, Φ, {a}, {b,d}, {a,b,d}} which are nano open sets.

- ✓ The nano closed sets = {U, Φ, {c}, {a,c}, {b,c,d}}
- ✓ The nano semi closed sets = {U, Φ, {a}, {c}, {a,c}, {b,d}, {b,c,d}}
- ✓ The nano pre closed sets = {U, Φ, {a}, {b}, {c], {d}, {a,c}, {b,c}, {c,d}, {a,b,c}, {a,c,d}, {b,c,d}}
- ✓ The nano regular closed sets = {U, Φ, {b}, {c}, {d}, {a,c}, {b,c}, {c,d}, {a,b,c}, {a,c,d}, {b,c,d}}
- ✓ The nano generalized star semi closed sets = {U, Φ, {a}, {c}, {a,c}, {b,d}, {a,b,d}, {b,c,d}}
- ✓ The nano generalized star pre closed sets = {U, Φ, {a},{b}, {c}, {d}, {a,b}, {a,c}, {a,d}, {b,c}, {b,d}, {c,d}, {a,b,c}, {a,b,d}, {a,c,d}, {b,c,d}}
- ✓ The nano generalized star regular closed sets = {U, Φ, {a},{b}, {c}, {d}, {a,b}, {a,c}, {a,d}, {b,c}, {b,d}, {c,d}, {a,b,c}, {a,b,d}, {a,c,d}, {b,c,d}}
- ✓ The nano generalized star closed sets are {U, Φ, {c}, {a,c}, {b,c}, {c,d}, {a,b,c}, {a,c,d}, {b,c,d}}

Theorem 2.3: A subset A of (U,τ_R(X)) is nano g* -closed if nano cl(A) – A contains no nonempty nano g*-closed set.

Proof: Suppose if A is nano g*-closed. Then Ncl(A) ⊆ V where A ⊆ V and V is nano g-open. Let Y be a nano g-closed subset of Ncl(A) – A. Then A ⊆ Y^C and Y^C is nano g-open. Since A is Ng*-closed, Ncl(A) ⊆ Y^C implies Y ⊆ $[Ncl(A)]^C$. That is Y ⊆ Ncl(A) and Y ⊆ $[Ncl(A)]^C$ implies Y ⊆ Φ. So Y is nonempty.

Theorem 2.4: If A and B be nano g*-closed, then A ∪ B is nano g*-closed.

Proof: Let A and B be nano g*-closed sets in (U, $\tau_R(X)$).V is nano g-open sets in U and Ncl(B) ⊆ W where B ⊆ W and W is nano g-open. Since A and B are subsets of V and W A ∪ B is a subset of VUW and V&W is nano g-open. Then Ncl(A ∪ B) = Ncl(A) ∪ Ncl(B) ⊆ VUW, which implies that A ∪ B is nano g*-closed.

Remark 2.5: The intersection of any two nano g*-closed sets in (U, $\tau_R(X)$) is nano g*-closed in(U, $\tau_R(X)$).

Proof: Let A and B are any two nano g*-closed sets. A ⊆ V, V is an nano g-open and B ⊆V, V is nano g-open.Then Ncl(A) ⊆ V, Ncl(B) ⊆ V.Therefore Ncl(A ∩ B) ⊆ V,V is nano g-open.Since A and B are nano g-closeds. Hence A ∩ B is a nano g*-closed set.

Example 2.6: Let U = {a, b, c, d, e} with U/R = {{a, b} {c, e}, {d}}. Let X = {a, d} then $\tau_R(x)$ = { U, Φ, {d},{a, b, d},{a, b}} and the nano closed sets are { U, Φ, {c, e},{a, b, c, e},{c, d, e}}. The nano g*-closed sets are {U, Φ, {c, e},{a, c, e}, {b, c, e}, {c, d, e}, {a, b, c, e},{a, c, d, e}, {b, c, d, e}}. Let A = {c, e} and B = {c, d, e} and A ∩ B = {c, e} is also nano g*-closed set.

Theorem 2.7: Every nano closed set is nano g*-closed.
Proof: Let A be a nano closed subset of U and A ⊆ V, V is nano g-open in U. since A is nano g-closed, Ncl(A) = A ⊆ V, where V is nano g-open in U. Therefore, A is nano g*-closed set.

Remark: 2.8 Converse of the above Theorem need not be true, which can be seen from the following example.

Example 2.9: In the example 2.6, the sets {a, c, e}, {a, c, d, e} are nano g*-closed set but not nano closed set.

Theorem 2.10: Every nano g*-closed set is nano g-closed.

Proof: Let A be nano g*-closed set in V. Let V be open set such that A ⊆ V.Since every nano open set is nano g-open. We have Ncl(A) ⊆ V. Therefore A is nano g-closed.

Remark 2.11: Converse of the above theorem need not be true, which can be seen from the following example.

Example 2.12: In the example 2.6, the set {a, c, d} is nano g-closed but not nano g*-closed set.

Theorem 2.13: Every nano g*-closed set is nano gr-closed.

Proof: Let A be nano g*-closed in U.Let V be nano open set such that A ⊆ V.Since every nano open set is nano g-open.We haveNrcl(A) ⊆ Ncl(A) ⊆ V implies Nrcl(A) ⊆ V.Therefore A is nano gr-closed.

Example 2.14: Let U = {a, b, c, d} with U/R = {a, c, {b,d}} and X = {a, b}. Then the nano topology $\tau_R(X)$ = {U, Φ, {a}, {b,d}, {a,b,d}}. The set {b} and {b, c} are nano gr-closed set but not nano g*-closed set.

Theorem 2.15: Every nano g*-closed set is nano gα-closed set.

Proof: Let A be nano g*-closed in U.Let V be nano open set such that A ⊆ V.Since every nano open set is nano g-open. We haveNαcl(A) ⊆ Ncl(A) ⊆ V implies Nαcl(A) ⊆ V.Therefore A is nano gα-closed.

Example 2.16: Let U = {a, b, c} with U/R = {{a}, {b, c}} and X = {a,b}. Then the nano topology $\tau_R(X)$ = {U, Φ, {a}, {b,c}}. The set {b} is nano gα-closed set but not nano g*-closed set.

Theorem 2.17: Every nano g*-closed set is nano gs-closed set.

Proof: Let A be nano g*-closed in U.Let V be nano open set such that A ⊆ V.Since every nano open set is nano g-open. We haveNscl(A) ⊆ Ncl(A) ⊆ V implies Nscl(A) ⊆ V.Therefore A is nano gs-closed.

Example 2.18: Let U = {a, b, c, d} with U/R = {a, {d}, {b,c}} and X = {a,c}. Then the nano topology $\tau_R(X)$ = {U, Φ, {a},{b,c},{a,b,c}}. The set {b, c} is nano gs-closed set but not nano g*-closed set.

Theorem 2.19: If A is nano g*-closed and A ⊆ B ⊆ Ncl(A), then B is nano g*-closed.

Proof: Let B ⊆ V where V is nano g-open in τ_R(X). Then A ⊆ B implies A ⊆ V. Since A is nano g*-closed, Ncl(A) ⊆ V. Also B ⊆ Ncl(A) implies Ncl(B) ⊆ Ncl(A). This Ncl(B) ⊆ V and so B is nano g*-closed.

Theorem 2.20: For each a ∈ U, either {a} is nano g-closed or $\{a\}^C$ is nano g*-closed in τ_R(X).

Proof: Suppose {a} is not nano g-closed in U. Then $\{a\}^C$ is not nano g-open and the only Nano g-open set containing $\{a\}^C$ is V ⊆ U. That is $\{a\}^C$ ⊆ U. Therefore Ncl$\{a\}^C$ ⊆ U which implies $\{a\}^C$ is nano g*-closed set in τ_R(X).

Theorem 2.21: Every nano g*-closed set is nano g*s-closed set.

Proof: Let A be a nano g*-closed set of U and A ⊆ V, V is nano g-open in U. Since A is nano g*-closed, Ncl(A) = A ⊆ V. That Ncl(A) ⊆ V, also Nscl(A) ⊆ Ncl(A) ⊆ V, where V is nano g-open in U. Hence Nscl(A) ⊆ V.Therefore A is nano g*s-closed set.

Example 2.22: Let U = {a, b, c, d, e} with U/R = {{a, b}, {c, e}, {d}} and X = {a, d}. Then τ_R(X) = {U, Φ, {d}, {a, b, d}, {a, b}}. The set {a, b} is nano g*s-closed set but not nano g*-closed set.

Theorem 2.23: Every nano g*-closed set is nano g*p-closed set.

Proof: Let A be a nano g*-closed set of U and A ⊆ V, V is nano g-open in U. Since A is nano g*-closed, Ncl(A) = A ⊆ V. That is Ncl(A) ⊆ V. Also Npcl(A) ⊆ Ncl(A) ⊆ V, where V is nano g-open in U. Hence Npcl(A) ⊆ V therefore A is nano g*p-closed set.

Example 2.24: In Example 2.2, the set {a} is nano g*p-closed set but not nano g*-closed set.

Theorem 2.25: Every nano g*-closed set is nano g*r-closed set.

Proof: Let A be a nano g*-closed set of V and A ⊆ V, V is nano g-open in U. Since A is nano g*-closed, Ncl(A) = A ⊆ V. That is Ncl(A) ⊆ V. Also Nrcl(A) ⊆ Ncl(A) ⊆ V, where V is nano g-open in U. Hence Nrcl(A) ⊆ V therefore A is nano g*r-closed.

Example 2.26: In Example 2.2, the set {a, b, d} is nano g*r-closed set but not nano g*-closed set.

3. Nano Generalized Star – Open Sets

Definition 3.1: A subset A of a nano topological space (U, τ_R(X)) is called nano generalized star-open (briefly nano g*-open) if A^C is nano g*-closed.

Theorem 3.2: **(i)** Every nano open set is nano g*-open.
(ii) Every nano g*-open set is nano g-open.
Proof: Proof follows from the Theorems 2.7 & 2.10.

Remark 3.3: For subsets A, B of a nano topological space (U, τ_R(X)).
(i) ∪ Ng*(int(A)) = Ng*cl(U – A)
(ii) ∪ Ng*(cl(A)) = Ng*int(U – A)

Theorem 3.4: A subset A ⊆ U is nano g*-open iff F ⊆ Nint(A) whenever F is nano g-closed set and F ⊆ A.

Proof: Let A be nano g*-open set and suppose F ⊆ A where F is nano g-closed. Then U – A is nano g*-closed set contained in the nano g-open set U – F. Hence Ncl(U – A) ⊆ U – F and U – Nint(A) ⊆ U – F. Thus F ⊆ Nint (A).
Conversely, if F is nano g-closed set with F ⊆ Nint(A) and F ⊆ A, then U – Nint(A) ⊆ U – F. Thus Ncl(U – A) ⊆ U – F. Hence U – A is nano g*-closed set and A is nano g*-open set.

Theorem 3.5: If Nint(A) ⊆ B ⊆ A and if A is nano g*-open, then B is nano g*-open.

Proof: Let Nint(A) ⊆ B ⊆ A, then $A^C \subset B^C \subset Ncl(A^C)$, where A^C is nano g*-closed and hence B^C is also nano g*-closed by Theorem 2.19. Therefore, B is nano g*-open.

Remark 3.6: If A is nano g*-closed, then Ncl(A) – A is nano g*-open.

Proof: Let A be nano g*-closed. Let F be nano g-closed such that F ⊆ Ncl(A) – A. Then F = Φ. Since Ncl(A) – A cannot have any non-empty nano g-closed set. Therefore, F ⊆ Nint(Ncl(A) – A). Hence Ncl(A) – A is nano g*-open.

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