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On nano generalized star semi closed sets in nano topological spaces

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Abstract
The purpose of this paper is to define and study nano g*s-closed sets and nano g*s-open sets in Nano topological spaces.

Keywords: Nano g*-closed sets, nano g*s-closed sets, nano gs-closed sets and nano sg-closed sets.

1. Introduction And Preliminaries

Levine [5] introduced the class of g-closed sets in 1970. Lellis Thivagar [4] introduced Nano topological space with respect to a subset X of a universe which is defined interms of lower and upper approximations of X. He has also defined Nano closed sets Nano-interior and Nano-closure of a set. Bhuvaneswari (1, 2, 3& 8) introduced Nano g-closed, Nano gs-closed, Nano αg-closed, Nano gα-closed and Nano gr and Nano rg-closed sets.

Definition 1.1 [6]
A subset A of a topological space (X,τ) is called a generalized star closed set if cl (A) ⊆ U whenever A ⊆ U and U is g-open in (X,τ).

Definition 1.2 [4]
Let U be a non-empty finite set of objects called the universe and R be an equivalence relation. Elements belongings to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let X ⊆ U

1. The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and its denoted by $L_R(X)$.
   That is $L_R(X) = \cup_{X\in U}\{R(X): R(X) \subseteq X\}$ where R(X) denotes the equivalence class determined by X.
2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$.
   That is $U_R(X) = U_{X\in U}\{R(X): R(X) \cap X \neq \Phi\}$
3. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not X with respect to R and it is denoted by $B_R(X)$.
   That is $B_R(X) = U_R(X) – L_R(X)$.

Property 1.3 [4]
If (U, R) is an approximation space and X, Y ⊆ U, then

- $L_R(X) \subseteq X \subseteq U_R(X)$
- $L_R(\Phi) = U_R(\Phi) = \Phi$ and $L_R(U) = U_R(U) = U$
- $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
- $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$
- $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$
- $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
- $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever X ⊆ Y
- $U_R(X^C) = [L_R(X)]^C$ and $L_R(X^C) =[U_R(X)]^C$
- $U_R[U_R(X)] = L_R[U_R(X)] = U_R(X)$
- $L_R[L_R(X)] = U_R[L_R(X)] = L_R(X)$

Definition 1.4 [4]
Let U be the universe, R be an equivalence relation on U and $\tau_R(X)$ = {U, Φ, U_R(X), L_R(X), B_R(X)} where X ⊆ U. Then by Property 1.3, $\tau_R(X)$ satisfies the following axioms.

- U andΦ ∈ τ_R(X).
- The union of the elements of any sub collection of τ_R(X) is inτ_R(X).
- The intersection of the elements of any finite sub collection of τ_R(X) is inτ_R(X).

That is, τ_R(X) is a topology on U called the Nano topology on U with respect to X. $(U, \tau_R$(X)) is called the Nano topological space. Elements of the Nano topology are known as Nano open sets in U. Elements of $[\tau_R(X)]^C$ are called Nano closed sets with $[\tau_R(X)]^C$ being called dual Nano topology of τ_R(X).

Remark 1.5 [4]
If $\tau_R(X)$ is the Nano topology on U with respect to X. Then the set
B = {U,L_R(X), B_R(X)} is the basis for $\tau_R(X)$.

Definition 1.6 [4]
If $(U, \tau_R(X))$ is a nano topological space with respect to X where X ⊆ U and if A ⊆ U, then

- The Nano interior of the set A is defined as the union of all nano open subsets contained in A and is denoted by Nint (A). Nint (A) is the largest nano open subset of A.
- The Nano closure of the set A is defined as the intersection of all nano closed sets containing Aand is denoted by Ncl (A). Ncl (A) is the smallest nano closed set containing A.

Definition 1.7
Let $(U, \tau_R(X))$ be a nano topological space and A ⊆ U. Then A is said to be

(1) Ng-closed [1] if Ncl (A) ⊆ V whenever A ⊆ V and V is Nano-open in U.
(2) Ngr-closed [8] if Nrcl (A) ⊆ V whenever A ⊆ V and V is Nano-open in U.
(3) Ngs-closed [2] if Nscl (A) ⊆ V whenever A ⊆ V and V is Nano-open in U.
(4) Ngα-closed [3] if Nαcl (A) ⊆ V whenever A ⊆ V and V is Nano-open in U.
(5) Ng*-closed [7] if Ncl (A) ⊆ V whenever A ⊆ V and V is Nano g-open in U.

2. Nano Generalized Star Semi Closed Set
Throughout this paper (U, $\tau_R(X)$) is a Nano topological space with respect to X where X ⊆ U, R is an equivalence relation on U, U/R denotes the family of equivalence classes of U by R.

Definition 2.1
Let (U,τ_R(x)) be a nono topological space. A subset A of (U,τ_R(x)) is called nano generalized star semi closed sets (briefly Ng*s-closed) if Nscl (A) ⊆ V whenever A ⊆ V and V is nano g-open in U.

Example 2.2
Let U = {a, b, c, d} with U/R = {{a}, {c}, {b,d}} and X = {a, b}. Then τ_R(x) = {U,Φ, {a}, {b, d}, {a, b, d}} which are nano open sets.

- ✓ The nano closed sets = {U, Φ, {c}, {a,c}, {b,c,d}}
- ✓ The nano semi closed sets = {U, Φ, {a}, {c}, {a,c}, {b,d}, {b,c,d}}
- ✓ The nano generalized star closed sets are {U, Φ, {c}, {a,c}, {b,c}, {c,d}, {a,b,c}, {a,c,d}, {b,c,d}}
- ✓ The nano generalized star semi closed sets are {U, Φ, {a}, {c}, {a,c}, {b,d}, {a,b,d}, {b,c,d}}

Theorem 2.3
Every nano closed set is nano g*s-closed set.
Proof. Let A be a nano closed set of U and A ⊆ V, Vis Nano g-open in U. Since A is Nano closed, Ncl (A) = A. So A ⊆ V implies Nscl (A) ⊆ V. But Nscl (A) ⊆ Ncl (A) implies Nscl (A) ⊆ V, A ⊆ V, V is nano g-open in U. Therefore A is nano g*s-closed set.

Example 2.4.
In example 2.2, the sets {a}, {b,d}, {a,b,d} are nano g*s-closed but not nano closed sets.

Theorem 2.5.
Every nano g*-closed set is nano g*s-closed set.
Proof. Let A be a nano g*-closed set of U and A ⊆ V, V is nano g-open in U. Since A is nano g*-closed, Ncl (A) = A. So A ⊆ V implies Ncl (A) ⊆ U. But Nscl (A) ⊆ Ncl (A) implies Nscl (A) ⊆ V, A ⊆ V, V is nano g-open in U. Therefore A is nano g*s-closed set.

Example 2.6.
Let U = {a, b, c, d, e} with U/R = {{a,b}, {c,e}, {d}} and X = {a,d}. Then τ_R(x) = {U,Φ, {d}, {a,b,d}, {a,b}}. The set {a, b} is nano g*s-closed set but not nano g*-closed set.

Theorem 2.7
Every nano g*s-closed set is nano gs-closed set.
Proof. Let A be a nano g*s-closed set of U and let V be nano open set such that A ⊆ V. Since every nano open set is nano g-open, and A is nano g*s-closed, we have Nscl (A) ⊆ V. Therefore A is nano gs-closed set.

Example 2.8
In example 2.2, the set {b} is nano gs-closed set but not in nano g*s-closed set.

Example 2.9
The notion nano g*s-closed sets and nano g-closed sets are independent.

Example 2.10
Let U = {a, b, c, d, e} with U/R = {{a, b}, {c, e}, {d}} and X = {a, d}. Then τ_R(x) = {U,Φ, {d}, {a,b,d}, {a,b}}. The set {d} is nano g*s-closed set but not nano g-closed set. B = {a, c} is nano g-closed but not nano g*s-closed set.

Theorem 2.11
The union of two nano g*s-closed sets is (U,τ_R(x)) is also a nano g*s-closed sets in (U,τ_R(x)).
Proof. Let A and B be two nano g*s-closed sets in (U, τ_R(x)). Let V be a nano g-open set in U such that A ⊆ V and B ⊆ V. Then we have A ∪ B ⊆ V. As A and B are nano g*s-closed sets in (U,τ_R(x)). Nscl (A) ⊆ V and Nscl (B) ⊆ V. Now Nscl (A ∪ B) = Nscl (A) ∪ Nscl (B) ⊆ V. Thus we have Nscl(A∪ B) ⊆ V whenever A ∪ B ⊆ V, V is nano g-open set in (U, τ_R(x)) which implies A ∪ B is a nano g*s-closed set in (U, τ_R(x)).

Theorem 2.12

Let A be a nano g*s-closed subset of $(U,\tau_R(x))$. If A ⊆ B ⊆ Nscl (A), then B is also nano g*s-closed suset of $(U,\tau_R(x))$.

Proof. Let B ⊆ V where V is nano g-open in$\tau_R(x)$. Then A ⊆ B implies A ⊆ V. Since A is nano g*s-closed, Nscl (A) ⊆ V. Also B ⊆ Nscl (A) implies Nscl (B) ⊆ Nscl (A). Thus Nscl (B) ⊆ V and so B is nano g*s-closed.

Theorem 2.13.

Let A be nano g*s-closed in $(U,\tau_R(x))$. Then Nscl (A) – A has no non-empty nano g-closed set.

Proof. Let A be nano g*s-closed sets in $(U,\tau_R(x))$, and F be nano g-closed subset of Nscl (A) – A. That is F ⊆ Nscl (A) – A, which implies that F ⊆ Nscl (A)∩ A^C. That is F ⊆ Nscl (A) and F⊆ A^C. F ⊆ A^C implies that a ⊆ F^C where F^C is a nano g-open set. Since A is nano g*s-closed, Nscl (A)⊆ F^C. That is F⊆ $[Nscl(A)]^C$. Thus F ⊆ Nscl (A)∩ $[Nscl(A)]^C$. Therefore F =Φ.

Theorem 2.14.

The intersection of any two nano g*s-closed sets is $(U,\tau_R(x))$ is also a nano g*s-closed sets in $(U,\tau_R(x))$.

Proof. Let A and B be two nano g*s-closed sets in $(U,\tau_R(x))$. Let V be a nano g-open set in U such that A ⊆ V and B ⊆ V. Then Nscl (A) ⊆ V and Nscl (B) ⊆ V. Therefore Nscl (A∩ B) ⊆ V, V is nano g-open set in $(U,\tau_R(x))$. Since A and B are nano g*s-closed sets. Hence A ∩ B is a nano g*s-closed set in $(U,\tau_R(x))$.

Example 2.15.

In example 2.2, the nano g*s-closed sets are $\tau_R(x)$ = {U,Φ, {a}, {c}, {a,c}, {b,d}, {a,b,d}, {b,c,d}}. Let A = {a, b, d} and B = {b, c, d} and A ∩ B = {b, d} is also nano g*s-closed set.

Theorem 2.16

Every nano g*s-closed set is nano sg-closed set.

Proof. Let A be a nano g*s-closed set of U and let V be nano open set such that A ⊆ V. Since every nano semi-open set such that A ⊆ V. Since every nano semi-open set is nano g-open set and A is nano g*s-closed, we have Nscl (A) ⊆ V. Therefore A is nano sg-closed set.

Example 2.17

Let U = {x,y,z} with U/R = {{x}, {y,z}}. Let X = {x,z}. Then $\tau_R(x)$ = {U,Φ, {x}, {y,z}}. The set A = {x, y} is nano sg-closed set but not nano g*s-closed set.

Remark 2.18

From the above discussions we have the following implications.

Nano-closed → Nano g*-closed → Nano
g*s-closed → Nano sg-closed → Nano
gsclosed

3. Nano Generalized Nano Star Semi-Open Sets

Definition 3.1

A subset A of a nano topological space (U, $\tau_R(x)$) is called nano generalized star semi-open (briefly, nano g*s-open), if A^C is also g*s-closed.

Theorem 3.2.

(i) Every nano open set is nano g*s-open.
(ii) Every nano g*-open set is nano g*s-open.

Proof. Proof follows from the Theorem 2.5 & 2.7.

Remark 3.3. For a subset a of a nano topological space $(U,\tau_R(x))$.

(i) ∪ Ng*s int (A) = Ng*s cl (U – A)
(ii) ∩ Ng*s int (A) = Ng*s cl (U – A).

Theorem 3.4.

A subset A ⊆ U is nano g*s-open iff F ⊆ Nsint (A) whenever F is nano g-closed set and F ⊆ A.

Proof. Let A be nano g*s-open set and suppose F ⊆ A where F is nano g-closed. Then U – A is nano g*s-closed set contained in the nano g-open set U – F. Hence Nscl (U – A) ⊆ U – F and U – Nsint (A) ⊆ U – F. Thus F ⊆ Nsint (A).
Conversely, if F is nano g-closed set with F ⊆ Nsint (A) and F ⊆ A. Then U – Nsint (A) ⊆ U – F. Thus Nscl (U – A) ⊆ U – F. Hence U – A is nano g*s-closed set and A is nano g*s-open set.

Theorem 3.5.

If Nsint (A) ⊆ B ⊆ A and if A is nano g*s-open, then B is nano g*s-open.

Proof. Let Nsint (A) ⊆ B ⊆ A, then A^C ⊆ B^C ⊆ Nscl (A^C), where A^C is nano g*s-closed and hence B^C is also nano g*s-closed by Theorem 2.12. Therefore, B is nano g*s-open.

Remark 3.6.

If Aand b are nano g*s-open subsets of a nano topological space U, then A ∪ B is also nano g*s-open in U, as seen from the following example.

Example 3.7. In example 2.2, the sets {a} and {c} are nano g*s-open sets {a} ∪ {c} is also nano g*s-open sets.

Theorem 3.6.

If A is nano g*s-closed, then Ncl (A) – A is nano g*s-open.

Proof. Let A be nano g*s-closed. Let F be nano g-closed such that F ⊆ Nscl (A) – A. Then F = Φ. Since Nscl (A) – A cannot have any non-empty nano g-closed set. Therefore, F ⊆ Nsint (Nscl (A) – A). Hence Nscl (A) – A is nano g*s-open.

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