International Journal of Applied Research

ISSN Print: 2394-7500
ISSN Online: 2394-5869
Impact Factor: 5.2
IJAR 2015; 1(9): 417-423
www.allresearchjournal.com
Received: 20-06-2015
Accepted: 21-07-2015

Monem AM
University of Sulamani – Iraq

Correspondence:
Monem AM
University of Sulamani – Iraq

Study the problem of congestion in Emkaser port by using queuing theory

Monem AM

Abstract
The Queuing theory plays an important role in solving the basic problems in every vital fields as production system, transportation systems with all its kinds: airway, naval, land and also all economics systems.
This study contains some important methods in queuing theory, and to reach a solution that treated the problem, the researcher exposed some economic indication related to the problem under study, which show that the average period of service is high. For this some basic systems and relations related to the queuing theory such as (M\M\1), and multiple (M\M\C) channels systems and the general system (G\G\1) and (G\G\C) and variety of distributions are used.
In application this contains very important conclusions to reduce the cost of waiting in queue systems or congestion in some centers of the port and improving the service level for the general interest to insure the fire distribution of ships served in that centers.

The aim of study
The importation aim is to using queuing theory to reach a solution that treated the problem under study and to find the average period of service for deceasing that costs. Applied some basic systems and relations related to the queuing theory such as (M\M\1), multiple (M\M\C) channels systems, general system (G\G\1) and (G\G\C) by using variety distributions.

Keywords: (G\G\1), (G/G/C), (M\M\1), (M\M\C) queues; parameter estimation; waiting times; Service time

1. Introduction
The cost of the waiting times of customers or units for single server queues has been studied by (A.K. Erlang – 1909), various methods of estimation Waiting time including the system (M\M\C) has been studied by (Kendall - 1930), (Dempster- *et al.* (1977) [5] studied the estimating parameters in GI/G/1 queues.
(W.G. Marchal- 1978) has been studied the system (GI\G\C) with inter-arrival and service times, See also (Bhat, and Rao-1987) [3], (J.D. Griffits- 1995) which has been studied some theoretical formula for deceasing the waiting time of units by using linear programming and queuing theory.
Basawa - *et al.* (1996) [1] presented a maximum likelihood (ML) method for estimating the parameters of the arrival and service time distributions using only the information on the waiting times of customers in a (GI/G/1) queue with "first come -first served" queue discipline. In this paper we will consider some queuing Systems which using naval units of transportation such as (M\M\1), (M\M\C), (M\G\1), and (M\G\C) where represent, Poisson distribution, Exponential distribution, Gamma or Erling distribution but (G) represent the general distribution of service time with some sufficient indications such as:
L_s: represent the expected number of units in the system.
L_q: represent the expected number of units in the queue. w_s: represent the expected number of waiting units in the system w_q: represent the expected number of waiting units in the queue.
So we have the follows:

$$L_s = \sum_{n=0}^{\infty} n\, P_n \qquad \dots (1)$$
$$L_q = \sum_{n=c}^{\infty} (n-c)\, P_n \qquad \dots (2)$$

Where (c) represent the number of service canals.

But there are a relationship between (L_s) and (w_s) also between (L_q) and (w_q) and we can find any indicator from another, now let the arrival rate equals (λ), then:

$$L_s = (\lambda w_s) \quad \dots (3)$$

This formula called (Little's formula) which can applied for any queuing system, and also $L_q = (\lambda w_q)$ … (4)
Let,)ρ(is the traffic intensity, so the probability the service canal is idle is:

$$p_0 = 1 - \rho \quad \dots (5)$$

Let,)μ(is the average of service time, so the expected of service time is ($\frac{1}{\mu}$) and we can find the following relationship:

$$w_s = (w_q + \frac{1}{\mu}) \quad \dots (6)$$

Multiply (6) with (λ(, we get :

$$L_s = (L_q + \frac{\lambda}{\mu}($$

Or $L_s = (L_q + \rho)$ … (7)

2. Some (infinite) waiting –queues system

A. Waiting –queue (G\G\C) for (infinite) arrival units

The waiting –queue (G\G\C) have more elastically with respect to others system of arrival and service distributions because we can using and applied the general system for any types of arrival and service units under study and the properties of this system have arrival distribution (G_a) with mean (t) and variance (σ^2_a) also the service of general distribution is (G_b) with mean (x) and variance (σ^2_b) with (C) the number of service with (C≥1) and the number of units in the general system is infinite.

I: let we have one service time in the general system i.e.($G_a \backslash G_b \backslash 1$) and let,

C_n: The customer of (n^{th}) number which arrive in the system.

t_n: The customer between two arrivals (n-1) and (n) in the system. x_n: The service time of (n^{th}) customer which arrive in the system

$\bar{t}$: The mean time of two arrivals.

$\bar{x}$: The mean of the service time.

w_n: The waiting time of (n^{th}) customer in the queue line. w =

w_q : The mean of waiting time in the queue line.

$\overline{U_n}$: The service time of associated with customer (C_n).

First: when arrival (C_{n+1}) units to the system before departure (C_n), we have:

$t_{n+1} + w_{n+1} = w_n + x_n$

Or $w_{n+1} = w_n + x_n - t_{n+1}$, if ($w_n + x_n - t_{n+1} \geq 0$) … (8)

Second: when arrive (C_{n+1}) units and find the system idle we have:

$w_{n+1} = 0$, if ($w_n + x_n - t_{n+1} < 0$) …(9)

Let $U_n = x_n - t_{n+1}$ …(10)

Substituting (10) in (8) and (9) we get:

$$w_{n+1} = \begin{cases} w_n + U_n & \text{if } w_n + U_n \geq 0 \\ 0 & \text{if } w_n + U_n < 0 \end{cases} \quad \dots (11)$$

3. The expected of waiting time for service in queue

Let, the system in stability, so we have (ρ = <1) where (ρ= Traffic intensity) then:

$$E(U_n) = t(\rho - 1) \quad \dots (12)$$

So the condition of stability system ($G_a \backslash G_b \backslash 1$) is:

$$E(U_n) < 0 \quad \dots (13)$$

Let, Y_n is the idle time of the service canal, so

$$Y_n = -\text{Min}[0, w_n + U_n] \quad \dots (14)$$

But whenY_n, is positive that mean the waiting time of customer (C_{n+1}) equal to zero
Therefore ($w_{n+1} = 0$) or (Y_n) equal to zero,

$$\therefore w_{n+1} * Y_n = 0 \quad \dots (15)$$

Take the difference between (11) and (14) we get:

$w_{n+1} - Y_n = w_n + U_n$ … (16)

1- $\lim_{n\to\infty} E(Y_n) = -\lim_{n\to\infty} E(U_n)$ … (17)

Put, $\lim_{n\to\infty} E(Y_n) = E(\tilde{Y}_n)$

But $\lim_{n\to\infty} E(U_n) = E(\tilde{u}_n)$

$\therefore E(\tilde{Y}) = -E(\tilde{u})$ … (18)

Also squared (16) and take (E) we get:

$E(\tilde{Y}^2) = E(\tilde{u}^2) + 2E(\tilde{u}\,\tilde{w})$ … (19)

But (U_n) and (w_n) are independent, then:

$E(\tilde{Y}^2) = E(\tilde{u}^2) + 2E(\tilde{u})E(\tilde{w})$

Or $E[\tilde{w}] = \frac{E(\tilde{u}^2)}{2E(\tilde{Y})} - \frac{E(\tilde{Y}^2)}{2E(\tilde{Y})}$ … (20)

To estimate the moment of (20) we get:

$$E[\overline{w}] = w_q = \frac{(\overline{u^2})}{2(\overline{u})} - \frac{(\overline{Y^2})}{2(\overline{u})} \quad \dots (21)$$

But from (12) we get:

1. $\bar{u} = E(U_n) = t(\rho - \bar{1})$
2. $u^2 = (\tilde{x} - \tilde{t})^2 = \sigma^2_a + \sigma_b^2 + (t^2)(1-\rho)^2$
   2
   $\overline{Y}$
3. To find $\bar{}$ (Y) and ( ),

Let: $[a_0] = P[\tilde{Y} > 0]$ = Pr[arrival and find the system is idel]
Then;

Pr [idle period ≤ Y] = $P[\tilde{Y} \leq Y / \tilde{Y} > 0]$

$\therefore \overline{Y} = E[\tilde{Y}/\tilde{Y} = 0] * \Pr[\tilde{Y} = 0] + E[\tilde{Y}/\tilde{Y} > 0] * \Pr[\tilde{Y} > 0]$

$\therefore \overline{Y} = a_0 * E[\tilde{Y}/\tilde{Y} > 0]$

Let, $\overline{I} = E[\tilde{Y}/\tilde{Y} > 0]$, so $\overline{Y} = a_0 * \overline{I}$ and $\overline{Y}^2 = a_0 * \overline{I}^2$

By substituting in moment equation (21) we get the follows:

$$w_q = \frac{\sigma_a^2 + \sigma_b^2 + (\bar{t}^2)(1-\rho)^2}{2\bar{t}(1-\rho)} - \frac{a_0 * \overline{I}^2}{2\overline{I}} \quad \dots (22)$$

Therefore the equation (22) gives the expected time of waiting of customer in queue according to (G\G\1) system, also we can find (L_q), (L_s) and (w_s) by using the equations (3), (4) and (7).

4. Goodness of Fit Test

The aim of these tests show how much comparing sampling distribution with theoretical distribution, by using some hypothesis, so (X^2) test is the important one of these goodness of tests.

4.1 X^2 − Test: Let, (x), the random variable which divided in to ($N_1, N_2, \dots, N_k$) classes with random sample (N), so $\sum_{i=1}^{k} N_i = N$, also with theoretical frequency $(\sum_{i=1}^{k} F_i = N P_i)$ where ($P_1, P_2, \dots, P_k$) is the probability for each class (i= 1, 2, 3,… k), with following hypotheses:
$H_0: F = F_0$
$H_1: F \neq F_0$
Where (F) and (F_0) are the sampling and theoretical distribution respectively.

Then; $X^2 = \sum_{i=1}^{k} \frac{(N_i - F_i)^2}{F_i}$, with (k − 1)d.f. … (23)

4.2 Kolmogorov- Simrnov Test: let, we have random sample with probability continuous function ($x_1, x_2, \dots, x_n$) with observation in order statistics ($x_1 < x_2 < \dots < x_n$).
Let we have the following Empirical cumulative distribution function

$$F_X(x) = \begin{cases} 0 \text{ for } x \leq x_1 \\ \frac{j}{n} \text{ for } x_j < x \leq x_{j+1} \\ 1 \text{ for } x > x_n \end{cases}$$

Where (n) is sample size with following hypotheses:
$H_0: F = F_0$ Vies $H_1: F \neq F_0$ Therefore:
$D = \sup_x |F(x) - F_0(x)|$, where $F_0(x) = P[X \leq x]$
We reject H_0 with (α= 0.05), if (D) $\geq D_{table}$ of Kolmogorov-Simrnov.

5. Application approach

In this application we have a sample of one center of Emkaser port which contains (22) get centers as in following table:

Table 1:

Specific service	No. of service canals	Get No.
Cargo of goods	1	1
Cargo of goods	1	2
Cargo of goods	1	3
Cargo of goods	1	4
Cargo of goods	1	5
Containers	1	6
Cargo of goods	1	7
Cargo of wheat	2	8
Cargo of goods	2	9
Cargo of goods	1	10
Cargo of goods	1	11
Containers	2	12
Cargo of goods	1	13
Cargo of goods	1	14
Containers and Cargo of goods	2	15
Cargo of goods	1	16
Cargo of goods	2	17
Cargo of goods	1	18
Cargo of goods	1	19
Containers	1	20
Cargo of wheat	1	21
passengers	1	22
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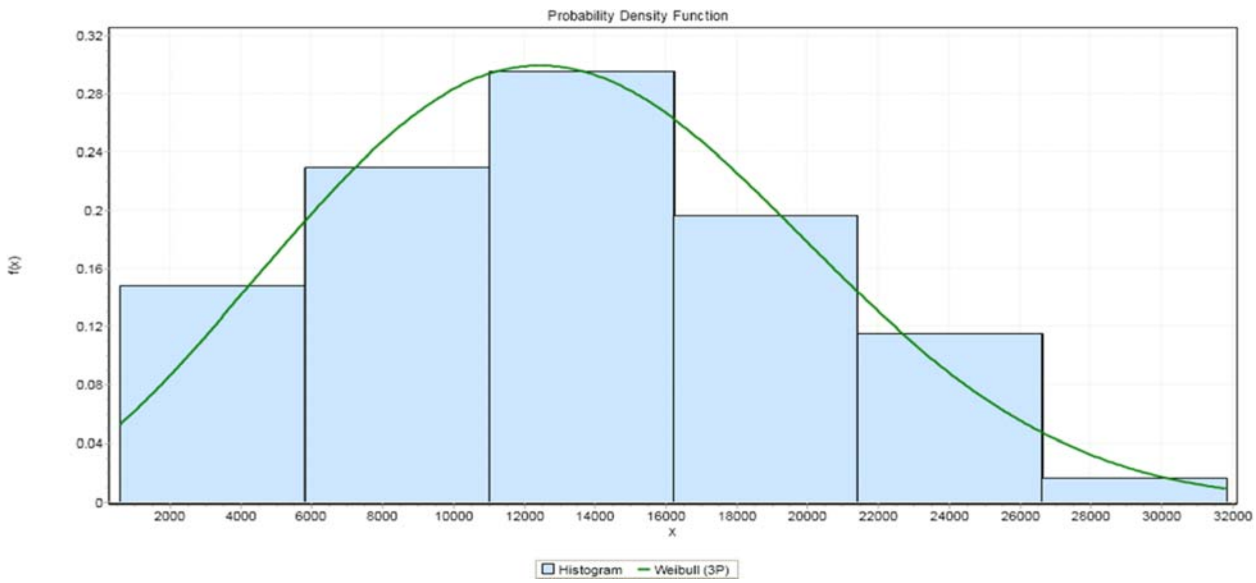
We choose the get center No. (15) Which arrival to the get many different cargos.
The application sample data taken from port for period (1/1/2014 to 30/ 06/ 2014)

5.1 Arrival data analysis: To analysis the arrival data let first checking the distribution (the time between two arrivals) of ships in this center we have:
$t_{(i,j)}$ = [Day Arrival of (j) – day arrival of (i)] ×60×24+ [hour arrival of (j) – hour arrival of (i)]×60 +[minute arrival of (j) – minute arrival of (i)] … (24)
Following table (2) represent the time between two arrival in minutes according to the formula (24) for sample size (n = 61) ships.

Table 2:

6480	11225	20400	12360	12385	11400	31820
6541	4805	19180	12900	23100	16155	21545
585	8635	10440	12410	10065	18285	18950
10070	15580	1795	7755	19285	18780	23095
20620	5390	6865	2120	15610	17670	22550
10095	14700	13260	15845	13915	19050	16455
8400	13290	13515	12990	1345	21210	8505
3380	10980	21860	7130	4730	18810	
9920	12900	1315	23220	14145	22710	



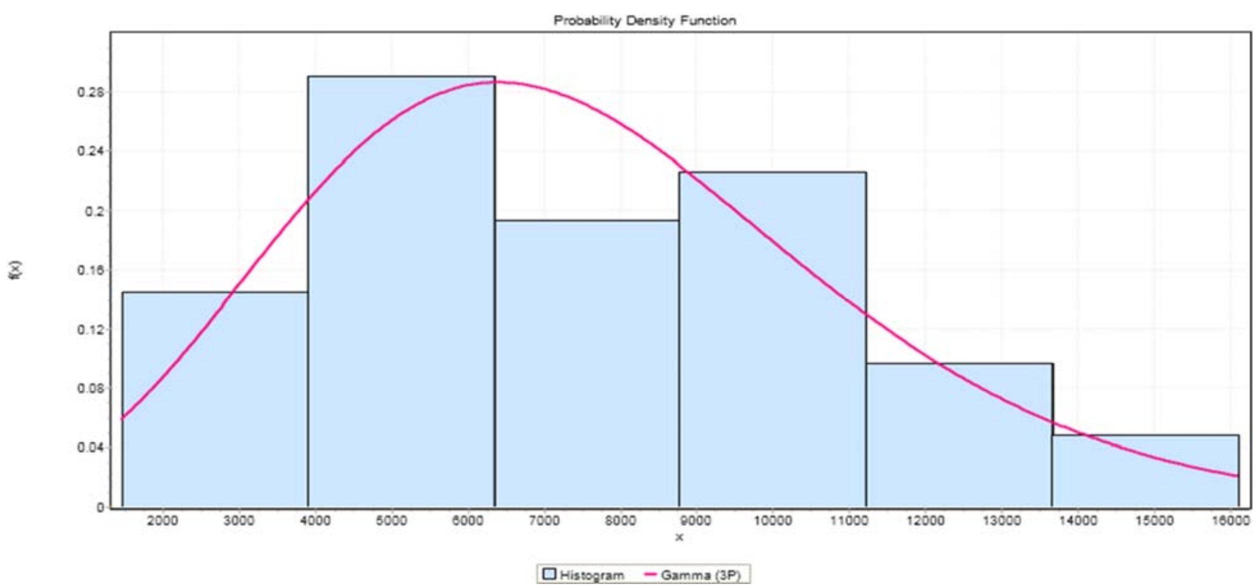
5.2 Service data analysis: For analysis the service data, we can define the service time of ship which arrive is the different between departure and interarrival time, so that the distribution of service time of ships in this center we have:

$X_{(i)}$ = [Departed day of (i) – Arrival day of (i)] $\times 60 \times 24 +$ [Departed hour of (i) – Arrival hour of (i)]$\times 60$

+[Departed minute of (i) – Arrival minute of (i)] … (25)

Where (X_i): is the time for service (i) ship, so the table (3) represent the service time which arrive in minutes according to the formula (25) for sample size (n = 62) ships. Table (3)

9665	5820	10350	5740	10715	9915	4385
1460	2105	8600	4295	12885	11730	12945
5335	14570	5630	6145	10035	8320	10175
7345	1450	4370	5650	3240	9055	11055
5505	6928	6035	12115	6545	3050	11810
10520	16110	4375	9780	2885	8250	11285
7950	10200	8290	5520	4135	8530	5415
1760	8995	4080	14515	2780	9050	6905
7865	6855	5145	9445	2600	4240	



Properties sample times "between two arrivals" for some service centers:

service centers / sample	8\1	8\2	9\1	9\2	10	12\1
N	157	193	167	78	52	63
The mean ($\bar{t}$)	4637.3	4252.9	4818.2	9660.4	15186.7	10266.4
S.d. : (σ_a)	2924.8	2807.02	3521.01	6023.9	8078.4	7259.4
(C.V.)	0.63	0.66	0.73	0.62	0.53	0.71
Min (t_i)	5	95	30	350	1790	155
Max (t_i)	13685	12785	21980	23235	33110	25985

sample \ service centers	12\2	13	15	16	17
N	65	61	86	108	229
The mean ($\bar{t}$)	11956.6	13287.3	8873.8	6699.1	3657.4
S.d. : (σ_a)	6245.5	6762.15	5723.13	4641.6	2706.7
(C.V.)	0.52	0.51	0.64	0.69	0.74
Min (t_i)	265	585	260	110	10
Max (t_i)	27070	31820	23030	28845	26510

Some properties of sample "service times" for some service centers:

sample \ service centers	8\1	8\2	9\1	9\2	10	12\1
N	176	194	168	79	53	64
The mean ($\bar{x}$)	3164.3	3273.12	3090.24	4677.6	5435	8362.6
S.d. : (σ_b)	1479.3	1725.15	1780.6	3451.5	3641.8	6176.4
(C.V.)	0.46	0.52	0.57	0.74	0.67	0.74
Min (x_i)	95	390	200	420	240	1115
Max (x_i)	8220	10980	8705	17990	15360	22945

sample \ service centers	12\2	13	15	16	17
N	66	62	87	109	230
The mean ($\bar{x}$)	8239.7	7455.6	15726.7	3529.9	4126.13
S.d. : (σ_b)	5017.4	3510.6	5862.06	1903.4	2064.98
(C.V.)	0.61	0.46	0.37	0.54	0.5
Min (x_i)	850	1450	2825	520	1110
Max (x_i)	19840	16110	29935	10630	13340

The results of (M\M\C) system comparing with the general system (G\G\C) for some service centers:

1- The service center (8/1) with system (M/M/C):

Lq	LS	Wq	WS	% P_O	% ρ	C
1.465	2.147	4.719	6.917	31.772	68.228	1
0.518	0.772	0.289	2.487	49.127	34.114	2
0.0102	0.692	0.0327	2.231	50.414	22.743	3

With double work service time in center (8/1) with system (M/M/C) we have:

Lq	LS	Wq	WS	% P_O	% ρ	C
0.176	0.518	0.569	1.667	65.886	34.114	1
0.0102	0.351	0.033	1.132	70.857	17.057	2
0.0007	0.342	0.002	1.101	71.084	11.371	3

Service center (8/1) with system (G/G/C):

Lq	LS	Wq	WS	% P_O	% ρ	C
0.45	1.13	1.45	3.65	31.772	68.228	1
0.159	0.5	0.513	1.612	49.127	34.114	2
0.003	0.23	0.009	0.742	50.414	22.743	3

With double work service time in center (8/1) with system (G/G/C) we have:

Lq	LS	Wq	WS	% P_O	% ρ	C
0.054	0.395	0.175	1.273	65.886	34.114	1
0.003	0.174	0.0099	0.559	70.857	17.057	2
0.0002	0.114	0.0006	0.366	71.084	11.371	3

2- Service center (9/1) with system (M/M/C)

Lq	LS	Wq	WS	% P_O	% ρ	C
1.154	1.796	3.863	6.012	35.759	64.241	1
0.073	0.716	0.247	2.397	51.377	32.121	2
0.008	0.651	0.0269	2.177	52.494	21.414	3

With double work service time in center (9/1) with System (M/M/C) we have:

Lq	LS	Wq	WS	% P_O	% ρ	C
0.152	0.473	0.508	1.584	67.88	32.121	1
0.008	0.329	0.028	1.103	72.324	16.06	2
0.0005	0.322	0.002	1.077	72.52	10.707	3

Service center (9/1) with system (G/G/C):

Lq	LS	Wq	WS	% P_O	% ρ	C
0.498	1.413	1.669	3.819	35.759	64.241	1
0.032	0.353	0.1067	3.819	51.377	32.121	2
0.0034	0.2176	0.0114	0.728	52.494	21.414	3

With double work service time in center (9/1) with system (G/G/C) we have:

Lq	LS	Wq	WS	% P_O	% ρ	C
0.065	0.386	0.219	1.295	67.88	32.121	1
0.004	0.164	0.0123	0.549	72.324	16.06	2
0.0002	0.107	0.0007	0.358	72.52	10.707	3

3- Service center (12/1) with system (M/M/C)

Lq	LS	Wq	WS	% P_O	% ρ	C
3.594	4.409	25.729	31.565	18.487	81.513	1
0.162	0.977	1.162	6.998	42.08	40.75	2
0.020	0.835	0.146	5.98	44.028	27.171	3

With double work service time in center (12/1) with system (M/M/C) we have:

Lq	LS	Wq	WS	% P_O	% ρ	C
0.281	0.687	2.007	4.925	59.244	40.756	1
0.017	0.425	0.126	3.044	66.14	20.378	2
0.002	0.408	0.009	2.927	66.504	13.585	3

Service center (12/1) with system (G/G/C):

Lq	LS	Wq	WS	% Po	% ρ	C
1.86	2.677	13.33	19.165	18.487	81.513	1
0.084	0.492	0.602	3.52	42.08	40.75	2
0.0105	0.282	0.075	2.021	44.028	27.171	3

With double work service time in center (12/1) with system (G/G/C) we have:

Lq	LS	Wq	WS	% Po	% ρ	C
0.1452	0.553	1.039	3.957	59.244	40.756	1
0.009	0.213	0.0659	1.524	66.14	20.378	2
0.0007	0.137	0.005	0.978	66.504	13.585	3

4- Service center (15/1) with system (M/M/C):

Lq	LS	Wq	WS	% Po	% ρ	C
2.02	2.75	25.5	34.72	26.66	73.33	1
0.114	0.85	1.44	10.69	46.34	36.66	2
0.013	0.75	0.17	9.43	47.86	24.44	3

With double work service time in center (15/1) with system (M/M/C) we have:

Lq	LS	Wq	WS	% Po	% ρ	C
0.212	0.578	2.68	7.31	63.33	36.66	1
0.013	0.379	0.161	4.79	69.01	18.33	2
0.001	0.367	0.011	4.64	69.28	12.22	3

Service center (15/1) with system (G/G/C):

Lq	LS	Wq	WS	% Po	% ρ	C
0.345	1.078	4.35	13.6	26.66	73.33	1
0.0195	0.386	0.245	4.875	46.34	36.66	2
0.0023	0.246	0.03	3.115	47.86	24.44	3

With double work service time in center (15/1) with system (G/G/C) we have:

Lq	LS	Wq	WS	% Po	% ρ	C
0.036	0.403	0.458	5.087	63.33	36.66	1
0.002	0.186	0.028	2.342	69.01	18.33	2
0.0001	0.1224	0.0013	1.545	69.28	12.22	3

5.3 The results of X^2 – Test and (Kolmogorov- Simrnov) -Test for distribution times between two arrivals:

	Statistical test			Parameter values		Arrival distribution	Service center
D0.95	D_c	X02.95	X_C2	β	k		
0.102	0.0799	9.48	8.32	0.00065	3	Erlang	8\1
0.097	0.0368	11.07	1.45	0.00047	2	Erlang	8\2
0.105	0.044	5.84	3.81	0.00042	2	Erlang	9\1
0.151	0.113	7.81	6.14	0.00031	3	Erlang	9\2
0.184	0.123	5.99	5.79	0.00026	4	Erlang	10
0.168	0.075	7.815	1.648	0.00019	2	Erlang	12\1
0.165	0.116	3.81	2.756	0.00033	4	Erlang	12\2
0.210	0.147	5.99	3.652	0.00278	5	Erlang	15\1
0.198	0.137	5.99	4.53	0.00024	4	Erlang	15\2
0.130	0.0632	5.99	2.42	0.00029	2	Erlang	16
0.125	0.0785	5.99	2.037	0.00028	2	Erlang	17\1
0.129	0.0797	7.815	4.707	0.00052	4	Erlang	17\2

5.4 The results of X^2 – Test and (Kolmogorov- Simrnov) -Test for distribution service times:

	The statistical test			Parameters value		Service dist.	Service center
D0.95	D_c	X02.95	X_C2	β	k		
0.1023	0.0348	7.815	3.19	0.00158	5	Erlang	8\1
0.0974	0.033	7.815	1.179	0.00122	4	Erlang	8\2
0.1047	0.0779	11.07	9.156	0.00097	3	Erlang	9\1
0.1505	0.0637	5.99	1.465	0.00042	2	Erlang	9\2
0.1831	0.0654	5.99	0.304	0.00036	2	Erlang	10
0.1669	0.1087	7.815	7.122	0.00023	2	Erlang	12\1
0.1644	0.0831	5.99	2.448	0.00036	3	Erlang	12\2
0.2076	0.1384	3.84	1.619	0.00052	7	Erlang	15\1
0.1962	0.1371	3.84	1.228	0.00056	10	Erlang	15\2
0.1300	0.0984	5.99	4.866	0.00084	3	Erlang	16
0.1250	0.1235	5.99	4.052	0.0024	8	Erlang	17\1
0.1288	0.1178	7.815	6.677	0.00079	4	Erlang	17\2

6. Conclusion and Recommendations

1- From the study it was discovered that the problem of port congestion in emkacer port is not caused by only inadequate berthing space but majorly by the operational inefficiency of the Port Managers and Operators coupled with long years of infrastructural development neglect and there is not good organization procedure or there is no capacity for service all arrival units in this center.

2- The most distributions of time between two arrivals it's so far than Exponential according to application results, because most of them (Erlang) or (Gamma) dist. The mean arrival rate is (1.15) (ship/ weekly) for each service center. 3- the most distributions of service time according to application are (Erlang) or (Gamma) dist. According to all statistical tests, the mean service rate is (2.2) (Ship/ weekly) for each service center

3- The expected No. of units in service center (L_q) = (0.205) unit in center (9), (1.8216) unit in center (15) and this to decreasing with adding another service center.
4- The expected No. of units arrival to the system (L_S = 0.74) unit in center (9), (2.7) unit in center (15) and this to decreasing with adding another service center.
5- The minimum expected time for wait in queue (W_q = 0.314) day in center (9) but the maximum expected time for wait in queue (W_q = 11.2) days in center (15).
6- Queue theory is a viable tool for solving congestion problems and its application in this study has helped to identify the cause of congestion in the ports and has also provided the Port Managers with useful set of decision making formulas with algorithm for designing Port systems and services.
7- Increased competition, booming in international maritime trade and the associated port congestion is crucial in ports rethinking, how to bolster capacity and improve service quality, to maintain current and attract new business.

7. References

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