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Characterizations of mildly nano gb-normal spaces

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Abstract
The purpose of this paper is to introduce a new class of nano-normal spaces in a nano topological space. We obtain the relationships of such normal spaces, and present some properties and establish various preservation theorems.
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Keywords: nano b-normal space, nano gb-normal space, nano gb-closed map, quasi nano gb-closed map, almost nano gb-normal space, mildly nano gb-normal space.

1. Introduction
Normality is an important topological property which was initially taken by Vigilino [20] who defined semi normal spaces in the year 1971. Then Signal and Arya [19] introduced the class of almost normal spaces and proved that a space is normal if and only if it is both a semi-normal space and an almost normal space. In recent years, many others have studied several forms of normality [9, 10, 15, 17]. On the other hand, the notions of p-normal spaces, s-normal spaces were introduced by Paul and Bhattacharya [18]. Levine [11] initiated the investigation of g-closed sets in topological spaces, since then many modifications of g-closed sets were defined and investigated by number of topologists [4-7]. Ahmad Al-Omari and Mohd. Salmi Md. Noorani [1] introduced the concepts of gb-closed sets. In 2013, the concept of nano topology was introduced by Lellis Thivagar [12], which was defined in terms of approximations and boundary region of a subset of a universe using an equivalence relation on it.
The aim of this paper is to introduce a new class of different normal spaces, namely nano gb-normal spaces, mildly nano gb-normal spaces and almost nano gb-normal spaces in nano topological spaces. The relations among nano b-normal spaces, nano p-normal spaces, nano s-normal spaces and also properties of nano gb-normal spaces are investigated. Moreover, we study new forms of nano gb-closed functions and obtain properties of these new forms of nano gb-closed functions and preservation theorems.

2. Preliminaries
Throughout this paper $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ mean nano topological spaces on which no separation axioms are assumed unless explicitly stated. The nano closure of A and the nano interior of A are denoted by Ncl(A) and Nint(A) respectively.

Definition 2.1 [21]: Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let X $\subseteq$ U

1. The lower approximation of X with respect to R is the set of all objects, which can be for certainly classified as X with respect to R and it is denoted by $L_R(X)$. That is

$L_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x$\in$U.

2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is
$U_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \phi\}$
3. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not-X with respect to R and it is denoted by $B_R(X)$. That is
$B_R(X) = U_R(X) – L_R(X)$.

Definition 2.2 [12]**:** Let U be non-empty, finite universe of objects and R be an equivalence relation on U. Let $X \subseteq U$. Let $\tau_R(X)$ = {U, ϕ, $L_R(X)$, $U_R(X)$, $B_R(X)$}. Then $\tau_R(X)$ is a topology on U, called as the nano topology with respect to X. Elements of the nano topology are known as the nano-open sets in U and $(U, \tau_R(X))$ is called the nano topological space. $[\tau_R(X)]^c$ is called as the dual nano topology of $\tau_R(X)$. Elements of $[\tau_R(X)]^c$ are called as nano closed sets.

Definition 2.3 [12]: Let $(U, \tau_R(X))$ be a nano topological space and $A \subseteq U$. Then A is said to be
(i) nano semi-open if $A \subseteq$ Ncl(Nint(A))
(ii) nano pre-open if $A \subseteq$ Nint(Ncl(A))
(iii) nano α -open if $A \subseteq$ Nint(Ncl(Nint(A)))
(iv) nano semi pre-open if $A \subseteq$ Ncl(Nint(Ncl(A)))
(v) nano b-open if $A \subseteq$ Ncl(NintA) $\cup$ Nint(NclA).
NSO(U, X), NPO(U, X), NαO(U, X), NSPO(U, X) and NBO(U, X) respectively denote the families of all nano semi-open, nano pre-open, nano α-open, nano semi pre-open and nano b- open subsets of U.
Let $(U, \tau_R(X))$ be a nano topological space and $A \subseteq U$. A is said to be nano semi closed, nano pre-closed, nano α-closed, nano semi pre closed and nano b-closed if its complement is respectively nano semi-open, nano pre-open, nano α-open, nano semi pre open and nano regular open.

Definition 2.4 [6]**:** A subset A of a nano topological space $(U, \tau_R(X))$ is called nano generalized b-closed (briefly, nano gb-closed), if Nbcl(A) $\subseteq$ G whenever $A \subseteq G$ and G is nano open in U.

Definition 2.5 [13]**:** A function
$f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is called a nano continuous if the inverse image of every nano closed set in $(V, \tau_{R'}(X))$ is nano closed in $(U, \tau_R(X))$.

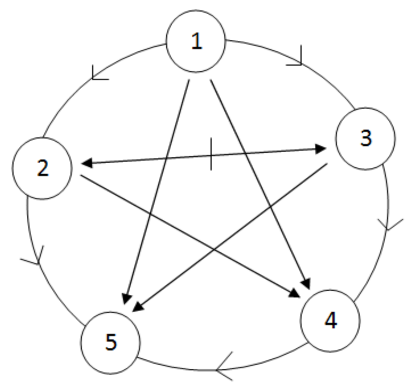
Definition 2.6 [7]**:** A function $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is called a nano gb-irresolute if the inverse image of every nano gb-closed set in $(V, \tau_{R'}(X))$ is nano gb-closed in $(U, \tau_R(X))$.

Definition 2.7: A space X is said to be p-normal [17] (resp.s-normal [14]) if for any pair of disjoint closed sets A and B, there exist disjoint preopen (resp. semi open) sets U and V such that $A \subset U$ and $B \subset V$.

Definition 2.8: A nano topological space $(U, \tau_R(X))$ is said to nano b-normal if for any pair of disjoint nano closed sets A and B, there exist disjoint nano b-open sets M and N such that $A \subset M$ and $B \subset N$.

3. Nano gb-normal spaces
Definition 3.1: A nano topological space $(U, \tau_R(X))$ is said to be nano gb-normal if for any pair of disjoint nano closed sets A and B, there exist disjoint nano gb-open sets M and N such that $A \subset M$ and $B \subset N$.

Remark 3.2: The following diagram holds for a nano topological space $(U, \tau_R(X))$.
1. nano normal, 2. nano semi normal, 3. nano pre-normal, 4. nano b-normal, 5. nano gb-normal



None of these implications is reversible as shown by the following examples.

Example 3.3: Let U = {a, b, c, d} with U/R = {{d}, {a, b, c}, {b, c, d}} and X = {b, d}. The nano topology is defined as $\tau_R(X)$ = {U, ϕ, {b, d}, {a, b, d}, {b, c, d}}. Then $\tau_R(X)$ is nano b-normal but not nano s-normal.

Example 3.4: Let U = {a, b, c, d} with U/R = {{a, b}, {c}, {a, b, d}} and X = {a, b}. The nano topology is defined as $\tau_R(X)$ = {U, ϕ, {a}, {b}, {a, b}, {a, b, c}, {a, b, d}}. Then $\tau_R(X)$ is nano b-normal but not nano p-normal.

Example 3.5: Let U = {a, b, c, d} with U/R = {{b, d}, {a, b}, {a, c, d}} and X = {b}. The nano topology is defined as $\tau_R(X)$ = {U, ϕ, {b, d}, {a, b, d}, {b, c, d}}. Then $\tau_R(X)$ is nano p-normal but not nano normal.

Theorem 3.6: For a nano topological space $(U, \tau_R(X))$ the following are equivalent:
1) U is nano gb-normal,
2) For every pair of nano open sets M and N whose union is U, there exist nano gb-closed sets A and B such that $A \subset M$ and $B \subset N$ and $A \cup B = U$,
3) For every nano closed set H and every nano open set K containing H, there exists a nano gb-open set M such that $H \subset M \subset$ Ngb-cl(M) $\subset K$.

Proof: (1) $\Rightarrow$ (2): Let M and N be a pair of nano open sets in a nano gb-normal space U such that U = M $\cup$ N. Then, U \ M, U \ N are disjoint nano closed sets. Since U is nano gb-normal there exist nano gb-open sets M_1 and such N_1 such that U \ M $\subset M_1$ and U \ N $\subset N_1$.
Let A = U \ M_1, B = U \ N_1. Then A and B are nano gb-closed sets such that A $\subset$ M and B $\subset$ N and A $\cup$ B = U.
(2) $\Rightarrow$ (3) Let H be a nano closed set and K be a nano open set containing H. Then U \ H and K are nano open sets whose union is U. Then by (2), there exist nano gb-closed sets P_1 and P_2 such that P_1 $\subset$ U \ H and P_2 $\subset$ K and $P_1 \cup P_2$ = U. Then H $\subset$ U \ P_1 and U \ K $\subset$ U \ P_2 and
(U \ P_1) $\cap$ (U \ P_2) = ϕ. Let M = U \ P_1 and N = U \ P_2.
Then M and N are disjoint nano gb-open sets such that H $\subset$ M $\subset$ U \ N $\subset$ K.
As U \ N is nano gb-closed set, we have Ngb-cl(M) $\subset$ U \ N and H $\subset$ M $\subset$ Ngb-cl(M) $\subset$ K.
(3) $\Rightarrow$ (1) : Let H_1 and H_2 be any two disjoint nano closed sets of U. Put K = U \ H_2, then
H_2 $\cap$ K = ϕ. H_1 $\subset$ K where K is an nano open set. Then by (3), there exists a nano gb-open set M of U such that
H_1 $\subset$ M $\subset$ Ngb-cl(M) $\subset$ K.
It follows that H_2 $\subset$ U \ Ngb- cl(M) = N, say, then N is nano gb-open and M $\cap$ N = ϕ.
Hence H_1 and H_2 are separated by nano gb-open sets M and N. Therefore U is nano gb-normal.

Definition 3.7: A function $f:(U,\tau_R(X))\rightarrow(V,\tau_{R'}(Y))$ is called strongly nano gb-open if f(M) $\in$ NgbO(V) for each M $\in$ NgbO(U).

Definition 3.8: A function $f:(U,\tau_R(X))\rightarrow(V,\tau_{R'}(Y))$ is called strongly nano gb-closed if f(M) $\in$ NgbC(V) for each M $\in$ NgbC(U).

Theorem 3.9: A function $f:(U,\tau_R(X))\rightarrow(V,\tau_{R'}(Y))$ is strongly nano gb-closed if and only if for each subset B in V and for each nano gb-open set M in U containing $f^{-1}(B)$, there exists a nano gb-open set N containing B such that (i) $f^{-1}(N)\subset$ M.

Proof: Suppose that f is strongly nano gb-closed. Let B be a subset of V and M $\in$ NgbO(U) containing $f^{-1}(B)$. Put N = V \ f(U \ M), then N is a nano gb-open set of V such that B $\subset$ N and $f^{-1}(N)\subset$ M.
Conversely, let K be any nano gb-closed set of U. Then $f^{-1}(V\setminus f(K))$ $\subset$ U \ K and
U \ K $\in$ NgbO(U). There exists a nano gb-open set N of V such that V \ f(K) $\subset$ N and $f^{-1}(N)\subset$ U \ K. Therefore, we have f(K) $\supset$ V \ N and K $\subset f^{-1}(V\setminus N)$. Hence, we obtain f(K) = V \ N and f(K) is nano gb-closed in V. This shows that f is strongly nano gb-closed.

Theorem 3.10: If $f:(U,\tau_R(X))\rightarrow(V,\tau_{R'}(Y))$ is a strongly nano gb-closed continuous function from a nano gb-normal space U onto a space V, then V is nano gb-normal.

Proof: Let K_1 and K_2 be disjoint nano closed sets in V. Then $f^{-1}(K_1)$ and $f^{-1}(K_2)$ are nano closed sets. Since U is nano gb-normal, then there exist disjoint nano gb-open sets M and N such that $f^{-1}(K_1)\subset$ M and $f^{-1}(K_2)\subset$ N. By Theorem 3.6, there exist nano gb-open sets A and B such that $K_1 \subset$ A, $K_2 \subset$ B, $f^{-1}(A)\subset$ M and $f^{-1}(B)\subset$ N. Also, A and B are disjoint. Thus, V is nano gb-normal.

Definition 3.11: A function $f:(U,\tau_R(X))\rightarrow(V,\tau_{R'}(Y))$ is a nano α - closed if for each nano closed set M in U, f(M) is nano α -closed in V.

Theorem 3.12: If $f:(U,\tau_R(X))\rightarrow(V,\tau_{R'}(Y))$ is an nano α -closed continuous surjection and U is nano normal, then V is nano gb-normal.

Proof: Let A and B be disjoint nano closed sets of V. Then $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint nano closed sets of U by continuity of f. Since U is nano normal, there exist disjoint nano open sets M and N such that $f^{-1}(A)\subset$ M and $f^{-1}(B)\subset$ N. By Theorem 6 [17], there are disjoint nano α - open sets G and H in V such that A $\subset$ G and B $\subset$ H. Since every nano α -open set is nano gb-open, G and H are disjoint nano gb-open sets containing A and B respectively. Therefore, V is nano gb-normal.

Definition 3.13: A function $f:(U,\tau_R(X))\rightarrow(V,\tau_{R'}(Y))$ is called almost nano gb-irresolute if for each u in U and each nano gb-neighbourhood N of f(u), N$gb-cl(f^{-1}(N))$ is a nano gb-neighbourhood of u.

Lemma 3.14: For a function $f:(U,\tau_R(X))\rightarrow(V,\tau_{R'}(Y))$, the following are equivalent:

(i) f is almost nano gb-irresolute,

(ii) $f^{-1}(N)\subset$ Ngb-int(Ngb-cl($f^{-1}(N)$)) for every N $\in$ NgbO(V).

Theorem 3.15: A function $f:(U,\tau_R(X))\rightarrow(V,\tau_{R'}(Y))$ is almost nano gb-irresolute if and only if f(Ngb-cl(M)) $\subset$ Ngb-cl(f(M)) for every M $\in$ NgbO(U).

Proof: Let M $\in$ NgbO(U). Suppose v $\notin$ Ngb-cl(f(M)). Then there exists N $\in$ NgbO(V, v) such that N $\cap$ f(M) = ϕ. Hence, $f^{-1}(N)\cap$ M= ϕ. Since M $\in$ NgbO(U), we have
Ngb-int(Ngb-cl($f^{-1}(N)$)) $\cap$ Ngb-cl(M) = ϕ.
Then by the previous lemma, $f^{-1}(N)\cap$ Ngb-cl(M) = ϕ and hence N $\cap$ f(Ngb-cl(M)) = ϕ.
This implies that v $\notin$ f(Ngb-cl(M)).
Conversely if N $\in$ NgbO(V), then P = U \ Ngb-cl($f^{-1}(N)$) $\in$ NgbO(U). By assumption, f(Ngb-cl(P)) $\subset$ Ngb-cl(f(P)) and hence
U \ Ngb-int(Ngb-cl($f^{-1}(N)$)) = Ngb-cl(P)

$$\subset f^{-1}(Ngb-cl(f(P)))$$

$\subset f^{-1}(Ngb-cl(f(U \setminus f^{-1}(N))))$
$\subset f^{-1}(Ngb-cl(V \setminus N))$
$\subset f^{-1}(V \setminus N)$ = U \ $f^{-1}(N)$.

Therefore, $f^{-1}(N) \subset$ Ngb-int(Ngb-cl($f^{-1}(N)$)). By Lemma 3.14, f is almost nano gb-irresolute.

Theorem 3.16: If $f:(U,\tau_R(X)) \rightarrow (V,\tau_{R'}(Y))$ is a strongly nano gb-open continuous almost nano gb-irresolute function from a nano gb-normal space U onto a space V, then V is nano gb-normal.

Proof: Let A be a nano closed subset of V and B be a nano open set containing A. Then by continuity of f, $f^{-1}(A)$ is nano closed and $f^{-1}(B)$ is a nano open set of U such that $f^{-1}(A) \subset f^{-1}(B)$. As U is nano gb-normal, there exists a nano gb-open set M in U such that $f^{-1}(A) \subset$M$\subset$Ngb-cl(M) $\subset f^{-1}(B)$ by Theorem 3.6. Then, $f(f^{-1}(A)) \subset$ f(M) $\subset$f(Ngb-cl(M)) $\subset f(f^{-1}(B))$. Since f is strongly nano gb-open almost nano gb-irresolute surjection, we obtain A $\subset$f(M) $\subset$Ngb-cl(f(M)) $\subset$B. Then again by Theorem 3.6 the space V is nano gb-normal.

4. Almost and mildly nano gb-normal Spaces

Definition 4.1: A nano topological space $(U,\tau_R(X))$ is said to almost nano gb-normal if for each nano closed set A and nano regular closed set B such that A $\cap$B = ϕ, there exist disjoint nano gb-open sets M and N such that A$\subset$M and B$\subset$N.

Theorem 4.2: For a nano topological space $(U,\tau_R(X))$ the following are equivalent:
1) U is almost nano gb-normal,
2) For every pair of nano sets M and N, one of which is nano open and the other is nano regular open whose union is U, there exist nano gb-closed sets A and B such that A$\subset$M and B$\subset$N and A $\cup$B = U,
3) For every nano closed set A and every nano regular open set B containing A, there exists a nano gb-open set N such that A $\subset$N $\subset$ Ngb-cl(N)$\subset$B.

Proof: Similar to Theorem 3.6.

Definition 4.3: A function $f:(U,\tau_R(X)) \rightarrow (V,\tau_{R'}(Y))$ is called

(i) nano R-map if $f^{-1}(N)$ is nano regular open in U for every nano regular open set N of V,
(ii) completely nano continuous if $f^{-1}(N)$ is nano regular open in U for every nano open set N of V,
(iii) quasi nano gb-closed if f(A) is nano closed in V for each A$\in$NgbC(U)

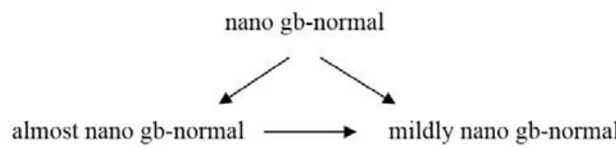
Theorem 4.4: If $f:(U,\tau_R(X)) \rightarrow (V,\tau_{R'}(Y))$ is a nano continuous strongly nano gb-open nano R-map and almost nano gb-irresolute surjection from an almost nano gb-normal space U onto a space V, then V is almost nano gb-normal.

Proof: Similar to Theorem 3.16

Corollary 4.5: If $f:(U,\tau_R(X)) \rightarrow (V,\tau_{R'}(Y))$ is a completely nano continuous strongly nano gb-open and almost nano gb-irresolute surjection from an almost nano gb-normal space U onto a space V, then V is almost nano gb-normal.

Definition 4.6: A nano topological space $(U,\tau_R(X))$ is said to mildly nano gb-normal if for every pair of disjoint nano regular closed sets A and B of U, there exist disjoint nano gb-open sets M and N such that A$\subset$M and B$\subset$N.

Remark 4.7: The following diagram holds for a nano topological space $(U,\tau_R(X))$:



None of the above implications are reversible.

Theorem 4.8: For a nano topological space $(U,\tau_R(X))$ the following are equivalent:
1) U is mildly nano gb-normal,
2) For every pair of nano regular open sets M and N, whose union is U, there exist nano gb-closed sets A and B such that A$\subset$M and B$\subset$N and A $\cup$B = U,
3) For any nano regular closed set A and every nano regular open set B containing A, there exists a nano gb-open set M such that A $\subset$M $\subset$ Ngb-cl(M)$\subset$B.
4) For ever pair of disjoint nano regular closed sets A and B, there exists a nano gb-open sets M and N such that A $\subset$M and B$\subset$N and Ngb-cl(M) $\cap$Ngb-(N) = ϕ.

Proof: Similar to Theorem 3.6.

Theorem 4.9: If $f:(U,\tau_R(X)) \rightarrow (V,\tau_{R'}(Y))$ is an strongly nano gb-open nano R-map and almost nano gb-irresolute function from a mildly nano gb-normal space U onto a space V, then V is mildly nano gb-normal.

Proof: Let A be nano regular closed set and B be a nano regular open set containing A. Then by nano R-map of f, $f^{-1}(A)$ is a nano regular closed set contained in the nano regular open set $f^{-1}(B)$. Since U is mildly nano gb-normal, there exists a nano gb-open set N such that

$f^{-1}(A) \subset$ N$\subset$ Ngb-cl(N) $\subset f^{-1}(B)$.

by Theorem 4.8. As f is strongly nano gb-open and an almost nano gb-irresolute surjection, it follows that f(N) $\in$ NgbO(V) and A$\subset$f(N) $\subset$Ngb-cl(f(N)) $\subset$B. Hence V is mildly nano gb-normal.

Theorem 4.10: If $f:(U,\tau_R(X)) \rightarrow (V,\tau_{R'}(Y))$ is nano R-map, strongly nano gb-closed function from a mildly nano gb-normal space U onto a space V, then V is mildly nano gb-normal.

Proof: Similar to Theorem 3.10.

Theorem 4.11: Let $f:\left(U,\tau_R(X)\right)\rightarrow\left(V,\tau_{R'}(Y)\right)$ is a nano continuous quasi nano gb-closed surjection and U is nano gb-normal, then V is normal.

Proof: Let P_1 and P_2 be any disjoint nano closed sets of V. Since f is nano continuous, $f^{-1}(P_1)$ and $f^{-1}(P_2)$ are disjoint nano closed sets of U. Since U is nano gb-normal, there exist disjoint $M_1, M_2 \in$ NgbO(U) such that $f^{-1}(P_i) \subset M_i$ for i = 1, 2. Put N_i = V- f(U-M_i), then N_i is nano open in V, $P_i \subset N_i$ and $f^{-1}(N_i) \subset M_i$ for i = 1, 2. Since $M_1 \cap M_2 = \phi$ and f is surjective; we have $N_1 \cap N_2 = \phi$. This shows that V is nano normal.

Theorem 4.12: Let $f:\left(U,\tau_R(X)\right)\rightarrow\left(V,\tau_{R'}(Y)\right)$ be a nano closed nano gb-irresolute injection and V is nano gb-normal, then U is nano gb-normal.

Proof: Let Q_1 and Q_2 be any disjoint nano closed sets of U. Since f is nano closed injection, f(Q_1)and f(Q_2) are disjoint nano closed sets of V. Since V is nano gb-normal, there exist disjoint $N_1, N_2 \in$ NgbO(V) such that f(Q_i)$\subset N_i$ for i = 1, 2. Since f is nano gb-irresolute $f^{-1}(N_1)$ and $f^{-1}(N_2)$ are nano gb-open sets of U and $Q_i \subset f^{-1}(N_i)$ for i = 1, 2. Now put M_i = Ngb- $\text{int}(f^{-1}(N_i))$ for i = 1, 2. Then $M_i \in$ NgbO (U), $Q_i \subset M_i$ and $M_1 \cap M_2 = \phi$. This shows that U is nano gb-normal.

Lemma 4.13: A function $f:\left(U,\tau_R(X)\right)\rightarrow\left(V,\tau_{R'}(Y)\right)$ is almost nano gb-closed if and only if for each subset B of V and each $M \in$ NRO(U) containing $f^{-1}(B)$, there exists a nano gb-open set N of V such that $B \subset N$ and $f^{-1}(N) \subset M$.

Lemma 4.14: A function $f:\left(U,\tau_R(X)\right)\rightarrow\left(V,\tau_{R'}(Y)\right)$ is almost nano gb-closed, then for each nano closed set B of V and each $M \in$ NRO(U) containing $f^{-1}(B)$, there exists $N \in$ NbO(V) such that $B \subset N$ and $f^{-1}(N) \subset M$.

Theorem 4.15: Let $f:\left(U,\tau_R(X)\right)\rightarrow\left(V,\tau_{R'}(Y)\right)$ is a nano continuous almost nano gb-closed surjection. If U is nano normal, then V is nano gb-normal.

Proof: Let P_1 and P_2 be any disjoint, nano closed sets of V. Since f is nano continuous, $f^{-1}(P_1)$ and $f^{-1}(P_2)$ are disjoint nano closed sets of U. Since U is nano normal, there exist disjoint nano open sets M_1 and M_2 such that $f^{-1}(P_i) \subset M_i$ for i = 1, 2. Now put G_i = Nint(Ncl(M_i)) for i = 1, 2, then $G_i \in$ NRO(U), $f^{-1}(P_i) \subset M_i \subset G_i$ and $G_1 \cap G_2 = \phi$. By Lemma 4.13, there exists a nano gb-open set N_i of V_i such that $P_i \subset N_i$ and $f^{-1}(N_i) \subset M_i \subset G_i$ where i = 1, 2. Since $G_1 \cap G_2 = \phi$ and f is surjective, we have $N_1 \cap N_2 = \phi$ This shows that V is nano gb-normal.

Corollary 4.16: Let $f:\left(U,\tau_R(X)\right)\rightarrow\left(V,\tau_{R'}(Y)\right)$ is a nano continuous nano b-closed surjection. If U is nano normal, then V is nano gb-normal.

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