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A new class of connected spaces in intuitionistic topological spaces

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Abstract
In this paper we present anew class of connected spaces namely intuitionistic fuzzy d-connected and intuitionistic fuzzy d-extremally disconnected spaces in an intuitionistic fuzzy topological space. We obtain several properties and some characterization concerning connectedness in these spaces.

Keywords: intuitionistic fuzzy sets, intuitionistic fuzzy d-open sets, intuitionistic fuzzy d-irresolute mapping, intuitionistic fuzzy d-connectedness.

1. Introduction
Ever since the introduction of fuzzy sets by Zadeh [16], the fuzzy concept has invaded almost all branches of Mathematics. Atanassov [3] introduced the notion of intuitionistic fuzzy sets. Using this notion Coker [6] initiated the concept of intuitionistic fuzzy topological space. Connectedness is one of the basic idea in topology. Turanli *et al.* [15] introduced several types of fuzzy connectedness in intuitionistic fuzzy topological spaces and investigated some interrelations between them together with the preservation properties under fuzzy continuous functions. The notion of intuitionistic fuzzy d-continuous mappings was introduced by I. Arockiarani *et al.* [1]. In this paper intuitionistic fuzzy d-connected spaces along with some interesting properties and their characterization are studied.

2. Preliminaries
Definition 2.1 [3]: Let X be a nonempty fixed set. An *intuitionistic fuzzy set* (IFS, for short) A is an object having the form $A=\{\langle x,\mu_A(x),\nu_A(x)\rangle : x\in X\}$ where the function $\mu_A : X\to I$ and $\nu_A : X\to I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\nu_A(x)$) of each element $x\in X$ to the set A, respectively, and $0\le \mu_A(x)+\nu_A(x)\le 1$ for each $x\in X$.
Obviously, every fuzzy set A on a nonempty set X is an IFS having the form $A\{\langle x,\mu_A(x),1-\mu_A(x)\rangle : x\in X\}$.

Definition 2.2 [3]: Let X be a nonempty set and the IFS's A and B be in the form $A=\{\langle x,\mu_A(x),\nu_A(x)\rangle : x\in X\}$, $B=\{\langle x,\mu_B(x),\nu_B(x)\rangle : x\in X\}$, and let $A=\{A_j : j\in J\}$ be an arbitrary family of IFS's in X. then we define

1. $A\subseteq B$ if and only if $\forall x\in X, \mu_A(x)\le\mu_B(x)$ and $\nu_A(x)\ge\nu_B(x)$;
2. $\overline{A}=\{\langle x,\nu_A(x),\mu_A(x)\rangle : x\in X\}$;
3. $\cap A_j=\{\{\langle x,\wedge\mu_{A_j}(x),\vee\nu_{A_j}(x)\rangle : x\in X\}$;
4. $\cup A_j=\{\{\langle x,\vee\mu_{A_j}(x),\wedge\nu_{A_j}(x)\rangle : x\in X\}$;
5. $1_{\sim}=\{\langle x,1,0\rangle : x\in X\}$ and $0_{\sim}=\{\langle x,0,1\rangle : x\in X\}$;

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Definition 2.4 [5]: The intuitionistic fuzzy set $c(\alpha,\beta)=\langle x, c_{\alpha}, c_{1-\beta}\rangle$ where $\alpha\in(0,1]$ and $\beta\in[0,1)$ and $\alpha+\beta\leq 1$ is called an intuitionistic fuzzy point (IFP for short) in X.

Definition 2.5 [6]: Two IFSs are said to be q-coincident (AqB in short) if and only if there exists an element $x\in X$ such that $\mu_A(x)>\nu_B(x)$ or $\nu_A(x)<\mu_B(x)$.

Definition 2.7 [5]: An intuitionistic fuzzy topology (IFT, for short) on a nonempty set X is a family τ of IFS's in X satisfying the following axioms:

(i) $0_{\sim}, 1_{\sim}\in\tau$.

(ii) $A_1\cap A_2\in\tau$ for any $A_1, A_2\in\tau$.

(iii) $\cup A_j\in\tau$ for any $\{A_j : j\in J\}\subseteq\tau$.

Definition 2.8 [6]: The complement $\overline{A}$ of IFOS A in IFTS (X, τ) is called an *intuitionistic fuzzy closed set* (IFCS, for short).

Definition 2.9 [5]: Let(X,τ) be an IFTS and A= $\langle x,\mu_A(x),\nu_A(x)\rangle\}$ be an IFS in X. Then the fuzzy interior and fuzzy closure of A is are denoted and defined by

cl(A)= $\cap$ {K:K is an IFCS in X and $A\subseteq K$ }

int(A)= $\cup$ {G:G is an IFOS in X and $G\subseteq A$}

Note that, for any IFS A in (X,τ), we have cl($\bar{A}$)=$\overline{int(A)}$and$int(\bar{A})$=$\overline{cl(A)}$.

Definition 2.11 [1]: Let A be an IFS in an IFTS (X,τ), then A is an *intuitionistic fuzzy d- open set* (IFDOS) if $A\subseteq scl(b\operatorname{int}(A))\cup cl(\operatorname{int}(A))$.

Definition 2.12 [4]: Let f be a mapping from an IFTS (X,τ) into IFTS (Y,κ). Then f is said to be intuitionistic fuzzy continuous if $f^{-1}(B)\in IFO(X)$ for every $B\in\kappa$.

Definition 2.13 [5]: An IFS A is said to be intuitionistic fuzzy dense (IFD for short) in another IFS B in an IFT (X, τ), if cl(A) = B.

Definition 2.14[3]: Let f be a mapping from an IFTS (X,τ) into IFTS (Y,κ). Then f is said to be intuitionistic fuzzy d-continuous mapping if $f^{-1}(B)\in IFDO(X)$ for every $B\in\kappa$.

Definition 2.13 [2] Let $f:X\rightarrow Y$ be a mapping from an IFTS X into an IFTS Y. The mapping f is called an *intuitionistic fuzzy d - irresolute*, if $f^{-1}(B)$ is an $IFdOS$ in X, for each $IFdOS$ B in Y

Definition 2.16 [12]: Let $c(\alpha,\beta)$ be an IFP of an IFTS ((X,τ) An IFS A of X is called an intuitionistic fuzzy neighborhood (IFN for short) of $c(\alpha,\beta)$ if there exists an IFOS B in X such that $c(\alpha,\beta)\in B\subseteq A$.

Definition 2.17 [6]: Two intuitionistic fuzzy sets A and B are said to be q-quasi coincident (AqB) if there exists an element $x\in X$ such that $\mu_A(x)>\nu_B(x)$ or $\nu_A(x)<\mu_B(x)$.

Definition 2.18 [6]: Two intuitionistic fuzzy sets A and B are said to be non q-coincident ($A\tilde{q}B$)if and only if $A\subseteq\overline{B}$.

Definition 2.19 [13]: An IFTS (X,τ) is called intuitionistic fuzzy C_5 -connected between two intuitionistic fuzzy sets A and B if there is no IFOS E in (X,τ) such that $A\subseteq E$ and $E\tilde{q}B$.

3. Intuitionistic Fuzzy d-Connectedness In Intuitionistic Fuzzy Topological Spaces

Definition 3.1: An intuitionistic fuzzy topological space (X,τ) is intuitionistic fuzzy d-disconnected if there exists an intuitionistic fuzzy d-open sets A, B in X, $A\neq 0\sim$, $B\neq 0\sim$ such that $A\cup B=1\sim$ and $A\cap B=0\sim$. If X is not IFd-disconnected then it is said to be intuitionistic fuzzy d-connected.

Example 3.2: Let $X=\{a,b\}$, $\tau=\{0\sim,1\sim,A\}$ where $A=\{\langle x,(0.4,0.2),(0.3,0.5)\rangle; x\in X\}$, $B=\{\langle x,(0.4,0.4),(0.2,0.5)\rangle; x\in X\}$ A and B are intuitionistic fuzzy d-open sets in X, $A\neq 0\sim$, $B\neq 0\sim$ and $A\cup B=B=1\sim$, $A\cap B=A=0\sim$. Hence X is intuitionistic fuzzy d-connected.

Example 3.3: Let $X=\{a,b\}$, $\tau=\{0\sim,1\sim,A\}$ where $A=\{\langle x,(0.4,0.2),(0.3,0.5)\rangle; x\in X\}$, $B=\{\langle x,(0.1,0.0),(0,1)\rangle; x\in X\}$, $C=\{\langle x,(0.0,0.1),(0.1,0.0)\rangle; x\in X\}$ B and C are intuitionistic fuzzy d-open sets in X, $B\neq 0\sim$, $C\neq 0\sim$ and $B\cup C=1\sim$, $B\cap C=0\sim$. Hence X is intuitionistic fuzzy d-connected.

Definition 3.4: An IFTS (X,τ) is $IFdC_5$ -disconnected if there exists IFS A in X, which is both IFdOS and IFdCS such that $A\neq 0\sim$ and $A\neq 1\sim$. If X is not $IFdC_5$ -disconnected then it is said to be an $IFdC_5$ -connected.

Example 3.5: Let $X=\{a,b\}$, $\tau=\{0\sim,1\sim,A\}$ where $A=\{\langle x,(0.4,0.2),(0.3,0.5)\rangle; x\in X\}$, A is an intuitionistic fuzzy d-open sets in X. But A is not an IFdCS since $\sin t(bcl(A))\cup\operatorname{int}(cl(A))\not\subseteq A$ and $1\sim\neq A\neq 0\sim$. Hence X is $IFdC_5$ -connected.

Example 3.6: Let $X=\{a,b\}$, $\tau=\{0\sim,1\sim,A\}$ where $A=\{\langle x,(0.4,0.2),(0.3,0.5)\rangle; x\in X\}$,

$B=\{\langle x,(0.1,0.0),(0.0,0.1)\rangle; x\in X\}$ B is an intuitionistic fuzzy d-open sets in X. Also B is an IFdCS since $\sin t(bcl(A))\cup \text{int}(cl(A))=0\sim\subseteq B$. Hence there exists an intuitionistic fuzzy set B in X such that $1\sim\neq B\neq 0\sim$ which is both intuitionistic fuzzy d-open and intuitionistic fuzzy d-closed set in X. Thus X is $IFdC_5$-disconnected.

Proposition 3.7: Every $IFdC_5$-connected space is IFd-connected.

Proof: Suppose that there exists non-empty intuitionistic fuzzy d-open sets A and B such that $A\cup B=1\sim$ and $A\cap B=0\sim$ (IFd-disconnected) then $\mu_A\vee\mu_B=1, \nu_A\wedge\nu_B=0$ and then $\mu_A\wedge\mu_B=0, \nu_A\vee\nu_B=1$. In other words then $\overline{B}=A$. Hence A is an IFd-clopen which implies X is $IFdC_5$-disconnected.

Remark 3.8: The converse of the above proposition need not be true.

Example 3.9: Let $X=\{a,b\}$, $\tau=\{0\sim,1\sim,A\}$ where, $A=\{\langle x,(0.5,0.5),(0.5,0.5)\rangle; x\in X\}$, $B=\{\langle x,(0.4,0.2),(0.3,0.5)\rangle; x\in X\}$ A is anintuitionistic fuzzy d-open sets in X. And B is an IFdOS in X. Also $1\sim\neq A\cup B=\{\langle x,(0.5,0.5),(0.3,0.5)\rangle; x\in X\}$, $0\sim\neq A\cap B=\{\langle x,(0.4,0.2),(0.5,0.5)\rangle; x\in X\}$, Hence X is intuitionistic fuzzy d-connected. Since IFS A is both intuitionistic fuzzy d-open and intuitionistic fuzzy d-closed set in X, X is $IFdC_5$-disconnected.

Proposition 3.10: Let $f:(X,\tau)\to(Y,\sigma)$ be an intuitionistic fuzzy d-irresolute surjection, If (X,τ) is intuitionistic fuzzy d-connected, then (Y,σ) is intuitionistic fuzzy d-connected.

Proof: Assume that (Y,σ) is not intuitionistic fuzzy d-connected then there exists non-empty intuitionistic fuzzy d-open sets A and B in (Y,σ) such that $A\cup B=1\sim$ and $A\cap B=0\sim$. Since f is an intuitionistic fuzzy d-irresolute mapping $C=f^{-1}(A)\neq 0\sim$, $D=f^{-1}(B)\neq 0\sim$ which are intuitionistic fuzzy d-open sets in X. And $f^{-1}(A)\cup f^{-1}(B)=f^{-1}(1\sim)=1\sim$ which implies $C\cup D=1\sim$. $f^{-1}(A)\cap f^{-1}(B)=f^{-1}(0\sim)=0\sim$ $=f^{-1}(0\sim)=0\sim$ which implies $C\cap D=0\sim$. Thus X is IFd-disconnected, which is a contradiction to our assumption. Hence Y is intuitionistic fuzzy d-connected.

Proposition 3.11: (X,τ) is $IFdC_5$-connected iff there exists no-nonempty intuitionistic fuzzy d-open sets A and B in X such that $A=\overline{B}$.

Proof: Suppose that A and B are intuitionistic fuzzy d-open sets in X such that $A\neq 0\sim\neq B$ and $A=\overline{B}$. Since $A=\overline{B}$, $\overline{B}$ is an IFdOS and B is an IFdCS. And $A\neq 0\sim\Rightarrow B\neq 1\sim$. But this is a contradiction to the fact that X is $IFdC_5$-connected.

Conversely, let A be both intuitionistic fuzzy d-open set and intuitionistic fuzzy d-closed set in X such that $0\sim\neq A\neq 1\sim$. Now take $B=\overline{A}$. B is an intuitionistic fuzzy d-open set and $A\neq 1\sim$ which implies $B=\overline{A}\neq 0\sim$ which is a contradiction

Definition 3.12: An IFTS (X,τ) is intuitionistic fuzzy d-strongly connected if there exists no non-empty IFdCS A and B in X such that $\mu_A+\mu_B\subseteq 1, \nu_A+\nu_B\supseteq 1$. In other words an IFTS (X,τ) is intuitionistic fuzzy d-strongly connected if there exists no nonempty intuitionistic fuzzy d-closed sets A and B such that $A\cap B=0\sim$.

Proposition 3.13: An IFTS (X,τ) is intuitionistic fuzzy d-strongly connected if there exists no non-empty IFdOS A and B in X, $A\neq 1\sim\neq B$ such that $\mu_A+\mu_B\supseteq 1, \nu_A+\nu_B\subseteq 1$.

Example 3.14: Let $X=\{a,b\}$, $\tau=\{0\sim,1\sim,A\}$ where $A=\{\langle x,(0.5,0.5),(0.5,0.5)\rangle; x\in X\}$, $B=\{\langle x,(0.4,0.2),(0.6,0.8)\rangle; x\in X\}$ A is an IFdOS in X. And B is an IFdOS in X since $B\subseteq scl(\text{bint}(A))\cup cl(\text{int}(A))$. Also $\mu_A+\mu_B\subseteq 1, \nu_A+\nu_B\supseteq 1$. Hence X is intuitionistic fuzzy d-strongly connected.

Proposition 3.15: Let $f:(X,\tau)\to(Y,\sigma)$ be a intuitionistic fuzzy d-irresolute surjection. If X is an IFd-strongly connected, then so is Y.

Proof: Suppose that Y is not IFd-strongly connected then there exists intuitionistic fuzzy d-closed set C and D in Y such that $C\neq 0\sim, D\neq 0\sim$, $C\cap D\neq 0\sim$. Since f is intuitionistic fuzzy d-irresolute, $f^{-1}(C)$, $f^{-1}(D)$ are intuitionistic fuzzy d-closed sets in X and $f^{-1}(C)\cap f^{-1}(D)=0$, $f^{-1}(C)\neq 0\sim$, $f^{-1}(D)\neq 0\sim$. If $f^{-1}(C)=0\sim$ then $f(f^{-1}(C))=C$ which implies $f(0\sim)=C$. So $C=0\sim$ a contradiction). Hence X is intuitionistic fuzzy d-strongly disconnected, a contradiction. Thus (Y,σ) is intuitionistic fuzzy d-strongly connected.

Remark 3.16: Intuitionistic fuzzy d-strongly connected and $IFdC_5$-connected spaces are independent.

Example 3.17: Let $X=\{a,b\}$, $\tau=\{0\sim,1\sim,A\}$ where $A=\{\langle x,(0.5,0.5),(0.5,0.5)\rangle; x\in X\}$,

$B = \{\langle x,(0.4,0.2),(0.6,0.8)\rangle; x \in X\}$ A is an IFdOS in X. And B is an IFdOS in X since $B \subseteq scl(bint(A)) \cup cl(int(A))$ .Also $\mu_A + \mu_B \subseteq 1, \nu_A + \nu_B \supseteq 1$. Hence X is intuitionistic fuzzy d-strongly connected. But X is not $IFdC_5$-connected, since A is both intuitionistic fuzzy d-open and intuitionistic fuzzy d-closed set in X.

Example 3.18: Let $X = \{a,b\}$, $\tau = \{0\sim, 1\sim, A, B, A\cup B, A\cap B\}$ where $A = \{\langle x,(0.5,0.6),(0.4,0.3)\rangle; x \in X\}$, $B = \{\langle x,(0.5,0.4),(0.2,0.4)\rangle; x \in X\}$ X is not $IFdC_5$-connected. But X is not is intuitionistic fuzzy d-strongly connected since A and B are IFdOS in X such that $\mu_A + \mu_B \supseteq 1, \nu_A + \nu_B \subseteq 1$.

Lemma: (i) $A \cap B = 0\sim \Rightarrow A \subseteq \overline{B}$ (ii) $A \not\subseteq \overline{B} \Rightarrow A \cap B \neq 0\sim$

Definition 3.19: A and B are non-zero intuitionistic fuzzy sets in (X,τ) . Then A and B are said to be

i) Intuitionistic fuzzy d-weakly separated if $dcl(A) \subseteq \overline{B}$ and $dcl(B) \subseteq \overline{A}$.
ii) Intuitionistic fuzzy d-q-separated if $dcl(A) \cap B = 0\sim = A \cap dcl(B)$.

Definition 3.20: An IFTS (X,τ) is said to be $IFdC_5$-disconnected if there exists intuitionistic fuzzy d-weakly separated non-zero intuitionistic fuzzy sets A and B in (X,τ) such that $A \cup B = 1\sim$.

Example 3.21: Let $X = \{a,b\}$, $\tau = \{0\sim, 1\sim, A\}$ where $A = \{\langle x,(0.4,0.2),(0.3,0.5)\rangle; x \in X\}$, $B = \{\langle x,(1,0),(0,1)\rangle; x \in X\}$, $C = \{\langle x,(0,1),(1,0)\rangle; x \in X\}$ B and C are IFdOS in X, $dcl(B) \subseteq \overline{C}$ and $dcl(C) \subseteq \overline{B}$. Hence B and C are intuitionistic fuzzy d-weakly separated and $B \cup C = 1\sim$ So X is $IFdC_5$-disconnected.

Definition 3.22: An IFTS (X,τ) is said to be $IFdC_M$-disconnected if there exists intuitionistic fuzzy d-q separated non-zero intuitionistic fuzzy sets A and B in (X,τ) such that $A \cup B = 1\sim$.

Example 3.23: Let $X = \{a,b\}$, $\tau = \{0\sim, 1\sim, A\}$ where $A = \{\langle x,(0.4,0.2),(0.3,0.5)\rangle; x \in X\}$, $B = \{\langle x,(1,0),(0,1)\rangle; x \in X\}$, $C = \{\langle x,(0,1),(1,0)\rangle; x \in X\}$ B and C are IFdOS in X, $dcl(B) \cap C = 0\sim$ and $B \cap dcl(C) = 0\sim$ which implies B and C are intuitionistic fuzzy d-q-separated and $B \cup C = 1\sim$ .So X is $IFdC_M$-disconnected.

Remark 3.24: An IFTS (X,τ) is said to be $IFdC_5$-connected if and only if (X,τ) is $IFdC_M$-connected.

Definition 3.25: An IFS A in (X,τ) is said to IFd-regular open set if dint(dcl(A))=A and intuitionistic fuzzy d–regular closed set if dcl(dint(A))=A.

Definition 3.26: An IFTS (X,τ) is said to be IFd-super disconnected if there exists an intuitionistic fuzzy d-regular open set A in X such that $0\sim \neq A \neq 1\sim$. X is called intuitionistic fuzzy d-super connected if X is not intuitionistic fuzzy d-super disconnected.

Example 3.27: Let $X = \{a,b\}$, $\tau = \{0\sim, 1\sim, A\}$ where $A = \{\langle x,(0.4,0.2),(0.3,0.5)\rangle; x \in X\}$, $B = \{\langle x,(1,0),(0,1)\rangle; x \in X\}$, $C = \{\langle x,(0,1),(1,0)\rangle; x \in X\}$ B and C are IFdOS in X, and $dcl(d\,int(B)) = B$ which implies B is anintuitionistic fuzzy d-regular open set in X. Hence X is an intuitionistic fuzzy d-super disconnected.

Proposition 3.28: Let (X,τ) be an IFTS. Then the following are equivalent:

i) X is intuitionistic fuzzy d-super connected.
ii) For each IFdOS $A \neq 0\sim$ in X, we have $dcl(A) = 1\sim$.
iii) For each IFdCS $A \neq 1\sim$ in X, we have $d\,int(A) = 0\sim$.
iv) There exists no IFdOSs A and B in X such that $A \neq 0\sim \neq B$ and $A \subseteq \overline{B}$.
v) There exists no IFdOSs A and B in X such that $A \neq 0\sim \neq B$, $B = \overline{dcl(A)}$ and $A = \overline{dcl(B)}$.
vi) There exists no IFdCSs A and B in X such that $A \neq 1\sim \neq B$, $B = \overline{d\,int(A)}$ and $A = \overline{d\,int(B)}$.

Proof: $(i) \Rightarrow (ii)$ Assume that there exists an $A \neq 0\sim$ such that $dcl(A) \neq 1\sim$. Take A=dint(dcl(A)). Then A is a proper d-regular open set in X which contradicts that X is intuitionistic fuzzy d-super connectedness.

$(ii) \Rightarrow (iii)$ Let $A \neq 1\sim$ be an intuitionistic fuzzy d-closed set in X. if we take $B = \overline{A}$ then B is an intuitionistic fuzzy d-open set in X and $B \neq 0\sim$. Hence by (ii) $dcl(B) = 1\sim \Rightarrow \overline{dcl(B)} = 0\sim \Rightarrow d\,int(\overline{B}) = 0\sim \Rightarrow d\,int(A) = 0\sim$.

$(iii) \Rightarrow (iv)$ Let A and B are intuitionistic fuzzy d-open sets in X such that $A \neq 0\sim \neq B$ and $A \subseteq \overline{B}$. Since $\overline{B}$ is an intuitionistic fuzzy d-closed set in X, $\overline{B} \neq 1\sim$ by (iii) $d\,int(\overline{B}) = 0\sim$ .But $A \subseteq \overline{B}$ implies

$0\sim\neq A = d\,\text{int}(A) \subseteq d\,\text{int}(\overline{B}) = 0\sim$ which is a contradiction.

$(iv) \Rightarrow (i)$ Let $0\sim \neq A \neq 1\sim$ be an intuitionistic fuzzy d-regular open set in X. If we take $B = \overline{dcl(A)}$ we get $B \neq 0\sim$. (If not B=0~ implies $\overline{dcl(A)} = 0\sim$ $\Rightarrow \overline{dcl(A)} = 1\sim \Rightarrow A = d\,\text{int}(dcl(A)) = d\,\text{int}(1\sim)$ $= 1\sim \Rightarrow A = 1\sim$ a contradiction to $A \neq 1\sim$)Also we have $A \subseteq \overline{B}$ which is also a contradiction. Therefore X is intuitionistic fuzzy d-super connected.

$(i) \Rightarrow (v)$ Let A and B be two intuitionistic fuzzy d-open set in (X,τ) such that $A \neq 0\sim \neq B$, $B = \overline{dcl(A)}$ and $A = \overline{dcl(B)}$. Now we have $d\,\text{int}(dcl(A)) = d\,\text{int}(\overline{A})$ $= \overline{dcl(B)} = A$, $A \neq 0\sim$ and $A \neq 1\sim$, since if $A = 1\sim$ then $1\sim = \overline{dcl(B)} \Rightarrow dcl(B) = 0\sim \Rightarrow B = 0\sim$. But $B \neq 0\sim$ and $A \neq 1\sim$ implies A is proper intuitionistic fuzzy d-regular open set in (X,τ) which is a contraction to (i). Hence (v) is true.

$(v) \Rightarrow (i)$ Let A be an intuitionistic fuzzy d-open set in X such that A=dint(dcl(A)), $0\sim \neq A \neq 1\sim$. Now take $B = \overline{dcl(A)}$. In this case, we get $B \neq 0\sim$ and B is an intuitionistic fuzzy d-open set in X and $B = \overline{dcl(A)}$ and $\overline{dcl(B)} = \overline{dcl(\overline{dcl(A)})} = \overline{\overline{d\,\text{int}(dcl(A))}}$=dint (dcl(A))=A. But this is a contradiction to (v). Therefore (X,τ) is intuitionistic fuzzy d-super connected space.

$(v) \Rightarrow (vi)$ Let A and B be intuitionistic fuzzy d-closed sets in (X,τ) such that $A \neq 1\sim \neq B$, $B = \overline{d\,\text{int}(A)}$ and $A = \overline{d\,\text{int}(B)}$. Taking $C = \overline{A}$ and $D = \overline{B}$, C and D become intuitionistic fuzzy d-open sets in (X,τ) and $C \neq 0\sim \neq D$, $\overline{dcl(C)} = \overline{dcl(\overline{A})} = \overline{\overline{d\,\text{int}(A)}} =$ dint(A)= $= \overline{B}$=D and similarity $\overline{dcl(D)} = C$. But this is a contradiction to (v). Hence (vi) is true. $(vi) \Rightarrow (v)$ We can prove this by a similar way as in $(v) \Rightarrow (vi)$.

Proposition 3.29: Let $f:(X,\tau) \rightarrow (Y,\sigma)$ be a intuitionistic fuzzy d-irresolute surjection. If X is an IFd-super connected, then so is Y.

Proof: Suppose that Y is intuitionistic fuzzy d-super disconnected. Then there exists intuitionistic fuzzy d-open sets C and in Y such that $C \neq 0\sim \neq D$, $C \subseteq \overline{D}$. Since f is intuitionistic fuzzy d-irresolute, $f^{-1}(C)$ and $f^{-1}(D)$ are intuitionistic fuzzy d-open sets in X and $C \subseteq \overline{D}$ implies,

$f^{-1}(C) \subseteq f^{-1}(\overline{D}) = \overline{f^{-1}(D)}$. Hence

$f^{-1}(C) \neq 0\sim \neq f^{-1}(\overline{D})$ which means that X is an intuitionistic fuzzy d-super disconnected which is a contradiction.

Definition 3.30: An IFTS (X,τ) is called intuitionistic fuzzy d-connected between two intuitionistic fuzzy sets A and B if there is no IFdOS E in (X,τ) such that $A \subseteq E$ and $E \tilde{q} B$.

Example 3.31: Let $X = \{a,b\}$, $\tau = \{0\sim, 1\sim, D\}$ where $D = \{\langle x,(0.5,0.5),(0.4,0.4)\rangle; x \in X\}$, (X,τ) be intuitionistic fuzzy topological space. Consider the intuitionistic fuzzy set $A = \{\langle x,(0.2,0.4),(0.7,0.6)\rangle; x \in X\}$ $B = \{\langle x,(0.5,0.5),(0.4,0.4)\rangle; x \in X\}$, D is an IFdOS in X. Hence X is an intuitionistic fuzzy d- connected between A and B.

Theorem 3.32: If an IFTS (X,τ) is an intuitionistic fuzzy d-connected between two intuitionistic fuzzy sets Aad B, then it is intuitionistic fuzzy C_5-connected between two intuitionistic fuzzy sets A and B.

Proof: Suppose (X,τ) is not intuitionistic fuzzy C_5-connected between two intuitionistic fuzzy sets A and B then there exists an IFOS E in (X,τ) such that $A \subseteq E$ and $E \tilde{q} B$ .Since for every intuitionistic fuzzy open set and intuitionistic fuzzy d-open set, there exists an IFdOS E in (X,τ) such that $A \subseteq E$ and $E \tilde{q} B$ which implies (X,τ) is not intuitionistic fuzzy d-connected between A and B, a contradiction to our hypothesis. Therefore, (X,τ) is intuitionistic fuzzy C_5-connected between A and B.

Example 3.33: Let $X = \{a,b\}$, $\tau = \{0\sim, 1\sim, D\}$ where $D = \{\langle x,(0.5,0.5),(0.4,0.4)\rangle; x \in X\}$, (X,τ) be intuitionistic fuzzy topological space. Consider the intuitionistic fuzzy set $A = \{\langle x,(0.2,0.4),(0.7,0.6)\rangle; x \in X\}$ $B = \{\langle x,(0.5,0.5),(0.4,0.4)\rangle; x \in X\}$, D is an IFdOS in X. Hence X is an intuitionistic fuzzy dC_5- connected between A and B. Consider the intuitionistic fuzzy set $C = \{\langle x,(0.5,0.4),(0.5,0.6)\rangle; x \in X\}$ C is an intuitionistic fuzzy d-open set such that $A \subseteq C$ and $C \subseteq \overline{B}$ which implies (X,τ) is intuitionistic fuzzy d-disconnected between A and B

Theorem 3.34: Let (X,τ) be an IFTS and A and B be intuitionistic fuzzy sets in (X,τ) .If AqB then (X,τ) is intuitionistic fuzzy d-connected between A and B.'

Proof: Suppose (X,τ) is not intuitionistic fuzzy d-connected between A and B. Then there exists an intuitionistic fuzzy d-open set E in (X,τ) such that $A \subseteq E$ and $E \subseteq \overline{B}$. This implies that $A \subseteq \overline{B}$. That is $A \tilde{q} B$ which is a contradiction to our hypothesis. Therefore (X,τ) is intuitionistic fuzzy d-connected between A and B.

The converse of the above theorem need not be true.

Example 3.35: Let $X = \{a, b\}$, $\tau = \{0\sim, 1\sim, D\}$ where $D = \{\langle x, (0.5, 0.5), (0.4, 0.4)\rangle ; x \in X\}$, (X,τ) be intuitionistic fuzzy topological space. Consider the intuitionistic fuzzy set $A = \{\langle x, (0.2, 0.4), (0.7, 0.6)\rangle ; x \in X\}$ $B = \{\langle x, (0.5, 0.5), (0.5, 0.4)\rangle ; x \in X\}$, D is an IFdOS in X. Hence X is an intuitionistic fuzzy d- connected between A and B. But A is not q-coincident with B since $\mu_A(x) < \nu_A(x)$.

Definition 3.36: Let N be an IFS in IFTS (X,τ)

1. If there exists intuitionistic fuzzy d-open sets M and W in X satisfying the following properties, Then N is called $IFdC_i$ -disconnected(i=1,2,3,4)

$C_1 : N \subseteq M \cup W, M \cap W \subseteq \overline{N}, N \cap M \neq 0\sim, N \cap W \neq 0\sim,$

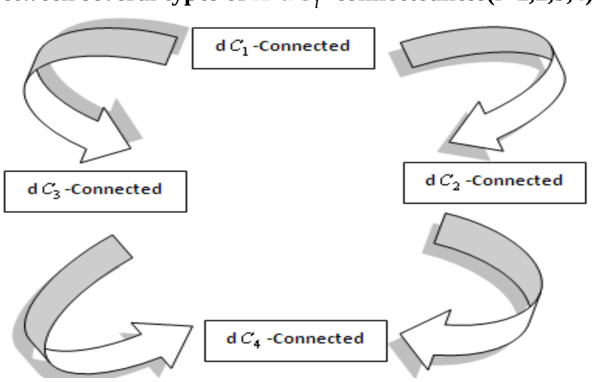
$C_2 : N \subseteq M \cup W, N \cap M \cap W = 0\sim, N \cap M \neq 0\sim, N \cap W \neq 0\sim,$

$C_3 : N \subseteq M \cup W, M \cap W \subseteq \overline{N}, M \not\subseteq \overline{N}, W \not\subseteq \overline{N},$

$C_4 : N \subseteq M \cup W, N \cap M \cap W = 0\sim,, M \not\subseteq \overline{N}, W \not\subseteq \overline{N},$

2. N is said to be $IFdC_i$ -connected(i=1,2,3,4) if N is not $IFdC_i$ -disconnected(1=1,2,3,4).

Obviously we can obtain the following implication between several types of $IFdC_i$ -connectedness(i=1,2,3,4)



Example 3.37: Let $X = \{a, b, c\}$, $\tau = \{0\sim, 1\sim, D, E\}$ where $D = \{\langle x, (0.4, 0.1, 0.3), (0.6, 0.9, 0.7)\rangle ; x \in X\}$, $E = \{\langle x, (0.5, 0.3, 0.4), (0.5, 0.7, 0.6)\rangle ; x \in X\}$ (X,τ) be intuitionistic fuzzy topological space. Consider the intuitionistic fuzzy set $A = \{\langle x, (0.3, 0.1, 0.2), (0.7, 0.9, 0.8)\rangle ; x \in X\}$ A is $IFdC_2$ -connected, $IFdC_3$ -connected, $IFdC_4$ -connected but $IFdC_1$ -disconnected.

Example 3.38: Let $X = \{a, b\}$, $\tau = \{0\sim, 1\sim, D, E, D \cup W, D \cap W\}$ where $D = \{\langle x, (0.3, 0.9), (0.7, 0.1)\rangle ; x \in X\}$, $E = \{\langle x, (0.9, 0.7), (0.1, 0.3)\rangle ; x \in X\}$ (X,τ) be intuitionistic fuzzy topological space. Consider the intuitionistic fuzzy set $A = \{\langle x, (0.2, 0.2), (0.8, 0.8)\rangle ; x \in X\}$ A is $IFdC_4$ -connected but $IFdC_3$ -disconnected.

Example 3.39: Let $X = \{a, b\}$, $\tau = \{0\sim, 1\sim, D, E, D \cup W\}$ where $D = \{\langle x, (0.0, 0.2), (1, 0.8)\rangle ; x \in X\}$, $E = \{\langle x, (0.2, 0), (0.8, 1)\rangle ; x \in X\}$ (X,τ) be intuitionistic fuzzy topological space. Consider the intuitionistic fuzzy set $A = \{\langle x, (0.1, 0.1), (0.9, 0.9)\rangle ; x \in X\}$ D and E are intuitionistic fuzzy d-open sets in X. Then N is $IFdC_4$ -connected but $IFdC_2$ -disconnected.

4. Intuitionistic Fuzzy d-Extremally Disconnectedness In Intuitionistic Fuzzy Topological Spaces

Definition 4.1: Let (X,τ) be any IFTS. X is called intuitionistic fuzzy d-extremally disconnected if the d-closure of every IFdOS in X is IFdOS.

Theorem 4.2: For an IFTS (X,τ) the following are equivalent

1. (X,τ) is an intuitionistic fuzzy d-extremally disconnected space.
2. For each intuitionistic fuzzy d-closed set A dint(A) is an intuitionistic fuzzy d-closed set.
3. For each intuitionistic fuzzy d-open set A, $dcl(A) = \overline{dcl(\overline{dcl(A)})}$.
4. For each intuitionistic fuzzy d-open set A and B with $dcl(A) = \overline{B}$, $dcl(A) = \overline{dcl(B)}$.

Proof: $(i) \Rightarrow (ii)$ Let A be an intuitionistic fuzzy d-close d. Then $\overline{A}$ is an intuitionistic fuzzy d-open set. So $dcl(\overline{A}) = \overline{d\,\text{int}(A)}$ is an intuitionistic fuzzy d-open set. Thus dint(A) is an intuitionistic fuzzy d-closed set in (X,τ)

.

$(i) \Rightarrow (ii)$ Let A be an intuitionistic fuzzy d-open set. Then $dcl(\overline{dcl(A)}) = dcl(d\,\text{int}(\overline{A}))$.

$\overline{dcl(\overline{dcl(A)})} = \overline{dcl(d\,\text{int}(\overline{A}))}$. Since A is an intuitionistic fuzzy d-open set, $\overline{A}$ is an intuitionistic fuzzy d-closed set. So by (ii) $d\,\text{int}(\overline{A})$ is an intuitionistic fuzzy d-closed set. That is $dcl(d\,\text{int}(\overline{A})) = d\,\text{int}(\overline{A})$. Hence $\overline{dcl(d\,\text{int}(\overline{A}))} = \overline{d\,\text{int}(\overline{A})}$ =dcl (A).

$(iii) \Rightarrow (iv)$ Let A and B be any two intuitionistic fuzzy d-open set in (X,τ) such that $dcl(A) = \overline{B}$ (iii) implies $dcl(A) = \overline{dcl(\overline{dcl(A)})} = \overline{dcl(\overline{B})} = \overline{dcl(B)}$.

$(iv) \Rightarrow (i)$ Let A be any intuitionistic fuzzy d-open set in (X,τ). Put $B = \overline{dcl(A)}$. Then $dcl(A) = \overline{B}$. Hence by (iv) $dcl(A) = \overline{dcl(B)}$. Therefore dcl (A) is an intuitionistic fuzzy d-open set in (X,τ). That is (X,τ) an extremally disconnected space.

5. References

1. I. Arockiarani, H. Jude Immaculate, A Note on intuitionistic fuzzy d-continuous mappings, Proceeding of National Seminar on Recent Development in Topology, 2015, 236-244.
2. I. Arockiarani, H. Jude Immaculate, A New Class of Fuzzy Irresolute Mappings on Intuitionistic Spaces, Communicated.
3. Atanassov K. Intuitionistic fuzzy sets, Fuzzy Sets and System 1986; 20:87-96.
4. Chang CL. Fuzzy topological spaces, J Math Anal Appl. 1968; 24:182-190.
5. Coker D. An introduction to intuitionistic fuzzy topological spaces, Fuzzy sets and systems 1997; 88:81-89.
6. Coker D, Demrci M. On intuitionistic fuzzy points, NIFS 1995; 1(2):79-84.
7. Gurcay H, Coker D. On fuzzy continuity in intuitionistic fuzzy topological spaces, J Fuzzy Math.1997; 5:365-378.
8. I. M. Hanafy and El-Arish, Completely continuous functions in intuitionistic fuzzy topological spaces, Czechoslovak Mathematical journal, (2003), 793-803.
9. Joung Kon Jeon, Young Bae Jun and Jin Han Park, Intuitionistic fuzzy alpha continuity and intuitionistic fuzzy pre continuity, International journal of Mathematics and Mathematical Sciences, (2005), 3091-3101.
10. R. Renuka and V. Seenivasan, On β –connectedness in intuitionistic fuzzy topological spaces, Gen. Math. Notes, Vol 19(1)(20013),16-27.
11. Seok Jong Lee and EunPyo Lee, “The category of intuitionistic fuzzy topological spaces”, Bull. Korean Math. Soc, (2000), 63-76.
12. Seok Jong Lee and EunPyo Lee, The category of intuitionistic fuzzy topological spaces, Bull. Korean Math. Soc, (2000), 63-76.
13. R. Santhi and D. Jayanthi, Intuitionistic fuzzy generalized semi-pre closed sets, Tripura Math. Soci, (2009), 61-72.
14. R. Santhi and D. Jayanthi, Generalized semi-pre connectedness in intuitionistic fuzzy topological spaces, Annals of fuzzy mathematics and informatics, 3(2)(2012), 243-253.
15. Turnali, D Coker. Fuzzy connectedness in intuitionistic fuzzy topological space, Fuzzy sets and Systems, 116(2000), 369-375
16. L. A. Zadeh. Fuzzy sets, Information and Control, 9(1965), 338-353.