International Journal of Applied Research 2016; 2(1): 121-125

ISSN Print: 2394-7500
ISSN Online: 2394-5869
Impact Factor: 5.2
IJAR 2016; 2(1): 121-125
www.allresearchjournal.com
Received: 18-11-2015
Accepted: 20-12-2015

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A collaborative vendor-buyer inventory model under budget capacity constraints

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Abstract
The objective of this work is to develop an inventory model considering budget capacity constraints and to study the effect of coordination on the performance of the system. A collaborative inventory model is created to get the optimal results of lot size, setup cost, and the total number of shipments from vendor to buyer in a production run, to minimize the joint expected total cost. The requirement rate of the end customer is estimated to be responsive to the selling cost. The inventory model is developed to get optimal values of the selling cost, item order quantity and number of consignments. Mathematical modeling concept is used to obtain the profit functions of vendor as well as buyer in joint form & iterative algorithm is used to solve the inventory model. It is found that under joint optimization, the requirement rate and the stock profit are higher than their values under independent optimization, especially for the more price sensitive requirement.

Keywords: Joint inventory, pricing, vendor, buyer, requirement and stock

1. Introduction

In couple of years, joint inventory model management with various types of decisions made by the firm like pricing, quality of product, time-to-market period has fascinated the scrutiny of countless researchers because these judgments must be compatible to each other in order to obtain maximum profit. In general, Inventory manager has to decide strategy for context prices and planning for how much inventory to hold. Most of the researchers and academicians have focused on deciding pricing strategy, which control requirements, and production inventory decisions, which express the cost of satisfying those requirements, simultaneously. The joint inventory model endures because the inventory refurbishment decisions are considered separately from the perspectives of either buyer or vendor based on economic order quantity (EOQ). To determine the effective solutions, both buyer & vendor should coordinate and collaborate parallel to accomplish acceptable inventory management.

Whitin has first presented this type of line of research in 1955. He examined EOQ inventory model with a linear function for a buyer that has a price dependent requirement. His work contribute and encouraged many researchers to interrogate joint pricing as well as ordering problems. Rosenberg in 1999, Lau in 2003 focused on demand functions inventory model. Roy, Dave, Burwell, Lin, Fitzpatrick and Ho in 2011, focused on quantity discount. Likewise, in last decade Chung and Wee introduced joint pricing and ordering problems. In a physical structure, sometimes it may happen that goods received by the buyer are not perfectly produced due to either the rudimentary production process of the vendor or damage in transit. Later, Goyal and Huang introduced joint vendor buyer inventory EOQ models for items with imperfect quality. They pretended that items of imperfect quality detected in the screening round are sold with rebate in cost. Some more researchers Hsu, Bag, Jindal, Chakraborty, Lin, Solanki and Kurdhi showed joint vendor buyer production inventory EOQ model under imperfect quality items in different conditions without any error in the inspection process.

Summarizing the previously proposed work, there is a less efforts done in the field of joint production inventory EOQ marketing models attracted on providing the consequences of coordination on the performance, especially when the requirement rate is a function of the selling price. Hence, the objective of this paper is to study a joint inventory EOQ model that grants operations as well as pricing decisions.

It is also used to investigate the effect of coordination on the overall system. Additionally, the production rate is finite and it is directly proportional to the requirement rate. To optimize the joint total profit (vendor and buyer's profit), the selling price, order quantity and number of shipments will be determined in this proposed work. This paper is organized as follows: In Section 2, pre-defined assumptions and notations for vendor buyer inventory EOQ model are provided. Section 3 provides the mathematical modelling concept for requirement function. Section 4 presents sensitivity analysis with suitable numerical example. Conclusion is summarized in Section 5.

2. Preliminaries

The mathematical modeling concept in this paper is presented based on the following assumptions and notations:

- Single vendor & single buyer inventory model
- Shortage is not considered.
- Every produced unit having cost Rs. A by the vendor is sold out to the buyer in Rs. B and again it reaches to the consumers with cost Rs. C. The relationship between them C>B>A.
- The requirement rate is decreasing function of selling cost, $F(C) = \alpha C^{-\beta}$. Where α is a scaling factor having value greater than zero and β is the index of cost elasticity having value greater than 1.
- The buyer inventory is continuously reviewed.
- Vendor sells Q quantity on buyer's demand. $Q_v = n*Q$, where n is no. of shipment to the buyer's.
- Production rate is constant & higher than the requirement rate. Ratio of requirement rate to production rate is less than or equal to unity.
- The buyer's inventory holding cost (HC_b) per item per unit time is greater than vendor's inventory holding cost (HCv) per item per unit time.
- The other parameters are:

V→ supplier setup cost, B→ buyer fixed order & transportation cost, r→ unit variable cost for handling & receiving an order, S→ buyer unit selling cost, ρ→ production rate, Q→ buyer order quantity and n→ no. of shipments

3. Mathematical Formulation

In this section, a joint inventory single supplier& buyer EOQ model and its optimization algorithm is developed. The coordination system is such that the supplier receives the buyer's requirementone produces the single product at a finite production rate PR. The supplier refills the order in an equi-sized shipments. Figure 1 represent the inventory model based on requirement.

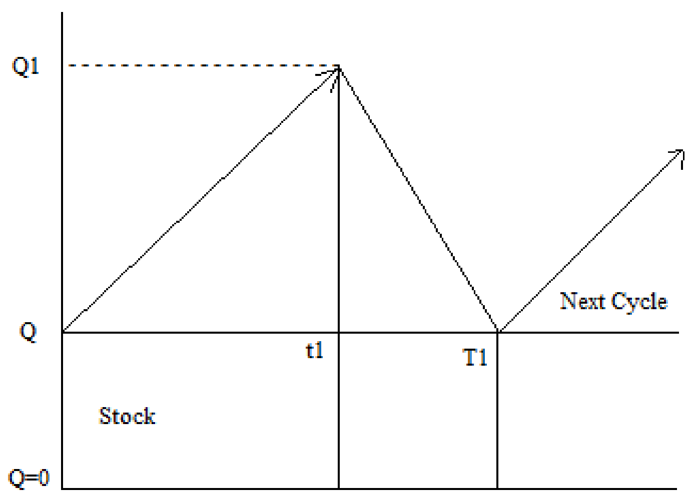

Fig 1: Inventory model with linear requirement

Joint Vendor-Buyer Profit:

If the buyer selects its selling price and ordering quantity (S, Q), and the supplierdecides its number of shipment n, then the overall system profit under independent optimization, OSP(S, Q, n), is equal to the sum of both buyer & supplier profits, i.e., OSP(S, Q, n) = SPB(S, Q) + SPV(n). The situation where supplier & buyer aim to coordinate by sharing information with each other to conclude the best solution for the joint supply system. Hence, the average joint total profit function is given by

$$\text{Max. AJTP}(S,Q,n) = (S - c - r)\alpha . S^{-\beta} - \frac{(\frac{V}{n} + B).\alpha . S^{-\beta}}{Q} - \frac{HC_b Q}{2} - \frac{HC_v Q}{2}[(2-n)\rho + n - 1] \tag{1}$$

Subject to: S> (c + r), Q>0 and n ϵ N.

To interrogate the consequences of the number of shipments on the joint total profit function, the II-order derivative of AJTP (S, Q, n) with respect to n is calculated as follow:

$$\frac{\partial^2 AJTP(S,Q,n)}{\partial n^2} = \frac{-2V\alpha S^{-\beta}}{Qn^3} < 0 \tag{2}$$

By solving eq. 1 & 2, result shows that for fixed S and Q, AJTP (S, Q, n) is a concave function ofshipment (n). First find the optimal solution, number of shipments (n*), is reduced to find a nearest integer. Second, by taking the I-order & II-order partial derivatives of AJTP (S, Q, n) with respect to Q for fixed value of n and s.

$$\frac{\partial AJTP(S,Q,n)}{\partial Q}=\frac{(V+nB)\alpha S^{-\beta}}{nQ^2}-\frac{HC_b}{2}-\frac{HC_v}{2}[(2-n)\rho+n-1] \quad (3)$$

$$\frac{\partial^2 AJTP(S,Q,n)}{\partial Q^2}=\frac{-2(V+nB)\alpha S^{-\beta}}{nQ^3} \quad (4)$$

The second derivative of AJTP is less than zero then there exist a unique solution which is denoted by:

$$Q^*=\sqrt{\frac{2\alpha S^{-\beta}(V/n+B)}{\{HC_v[(2-n)\rho+n-1]+HC_b\}}} \quad (5)$$

Substituting this Q* in eq. 1, results AJTP is a function of two variables S, n. For the constant value n find the first derivative and set it equal to zero (necessary condition for optimality). Again find the second derivative of AJTP w.r.t. S and check whether it is less than zero. This is sufficient condition for optimality.

4. Numerical Example & Analysis

Given data: B=Rs. 200 per order, V= Rs. 1200 per set up, c= Rs. 2.5 per unit, r= Rs. 1 per unit, w= Rs. 5 per unit, ρ=0.8, HC_b= Rs. 0.5 per unit per year, HC_v= Rs. 0.25 per unit per year, α=30000 and β=1.245.

Thus the requirement rate F(S) = $30000*S^{-1.245}$ and improvement I = (AJTP-OSP)*100/OSP is calculated to focus on joint optimization. AJTP & OSP shows the total vendor buyer system profit under joint and independent optimization respectively.

Find the optimal solution (S*, Q*, n) by using following steps:

1. Assume n=0, and set AJTP equal to zero.
2. Set n=1 and determine S* by putting first derivative of AJTP w.r.t. S equals to zero
3. Calculate AJTP (S^n, Q^n, n) using eq. 1. If AJTP (S^n, Q^n, n)>=AJTP (S^{n-1}, Q^{n-1}, n-1) go to step four, otherwise optimum solution is (S*, Q*,n*)=(S^{n-1}, Q^{n-1}, n-1).
4. Put n=n+1 and repeat from step 2.

The optimum value of S, Q, n & vendor buyer system profit under independent optimization are 31, 1825.3, 8 and 112976 and in case of joint optimization values are 18.6, 2188.4, 9 and 116600. So this vendor buyer coordination provide improved solution in terms of overall system profit. In order to highlight the performance of vendor buyer coordination in the overall profit, analternative of haphazardly generated problem instances are resolved and summarized in following table 1 by varying some model parameters and study their impact on the optimal solution and the overall profit.

Table 1: A haphazardly generated problem example

α	β	ρ	B	V	HC_b	HC_v	r	w	c	OSP	AJTP	I (%)
381330	1.32	0.8	2508	11247	1.78	1.32	15.3	17.8	13.2	56,567	56,686	0.2
764660	2.59	0.8	2576	2394	1.68	0.82	5.8	16.8	8.2	349	439	25.9
568210	1.62	0.85	1786	2202	6.90	1.64	10.6	69.0	16.4	15,136	20,249	33.8
833750	1.33	0.9	1640	8993	3.61	0.77	17.9	36.1	7.7	118,870	128,620	8.2
723330	1.46	0.9	1297	1375	1.04	0.54	5.6	10.4	5.4	89,635	92,705	3.4
847210	1.30	0.9	1840	3000	5.04	1.70	18.7	50.4	17.0	126,469	133,930	5.9
555060	2.08	0.85	2764	6123	1.97	0.53	4.7	19.7	5.3	4,327	6,575	52.0
723330	2.83	0.9	1297	1375	1.04	0.54	5.6	10.4	5.4	388	481	24.1
109520	1.16	0.95	2756	15136	1.32	0.53	12.7	13.2	5.3	41,425	41,689	0.6
369700	1.89	0.7	2367	2514	4.59	0.93	10.0	45.9	9.3	2,811	4,258	51.5

Sensitivity analysis for the unit inventory costs ratio HC_v /HC_b:

Table 2: Sensitivity analysis for the unit inventory costs ratio α=HC_v / HC_b

HC_b	α	Independent Optimization							Joint Optimization					I(%)
		S	Q	n	SPB	SPV	OSP	Q_v	S	Q	n	AJTP	Q_v	
0.5	0.2	31	1825.3	12	103,390	9,928	113,318	21903.6	18.4	2385.7	13	117,060	31014.1	3.30
	0.4	31	1825.3	9	103,390	9,688	113,078	16427.7	18.5	2230.1	10	116,730	22301.0	3.23
	0.6	31	1825.3	7	103,390	9,491	112,881	12777.1	18.6	2180.7	8	116,470	17445.6	3.18
	0.8	31	1825.3	6	103,390	9,317	112,707	10951.8	18.7	2116.6	7	116,240	14816.2	3.14
1.0	0.2	31	1825.3	6	103,390	9,152	112,542	10951.8	18.7	1966	7	116,040	13762.0	3.11
	0.4	31.3	1284.6	12	103,020	9,600	112,620	15415.2	18.7	1672	13	116,250	21736.0	3.22
	0.6	31.3	1284.6	9	103,020	9,262	112,282	11561.4	18.8	1560.5	10	115,800	15605.0	3.13
	0.8	31.3	1284.6	7	103,020	8,986	112,006	8992.2	19.0	1524.1	8	115,430	12192.8	3.06
1.5	0.2	31.3	1284.6	6	103,020	8,740	111,760	7707.6	19.1	1477.8	7	115,110	10344.6	3.00
	0.4	31.3	1284.6	5	103,020	8,509	111,529	6423.0	19.2	1371.3	7	114,830	9599.1	2.96
	0.6	31.5	1045.1	12	102,730	9,363	112,093	12541.2	18.9	1355.8	13	115,640	17625.4	3.16
	0.8	31.5	1045.1	9	102,730	8,951	111,681	9405.9	19.1	1264	10	115,080	12460.0	3.04

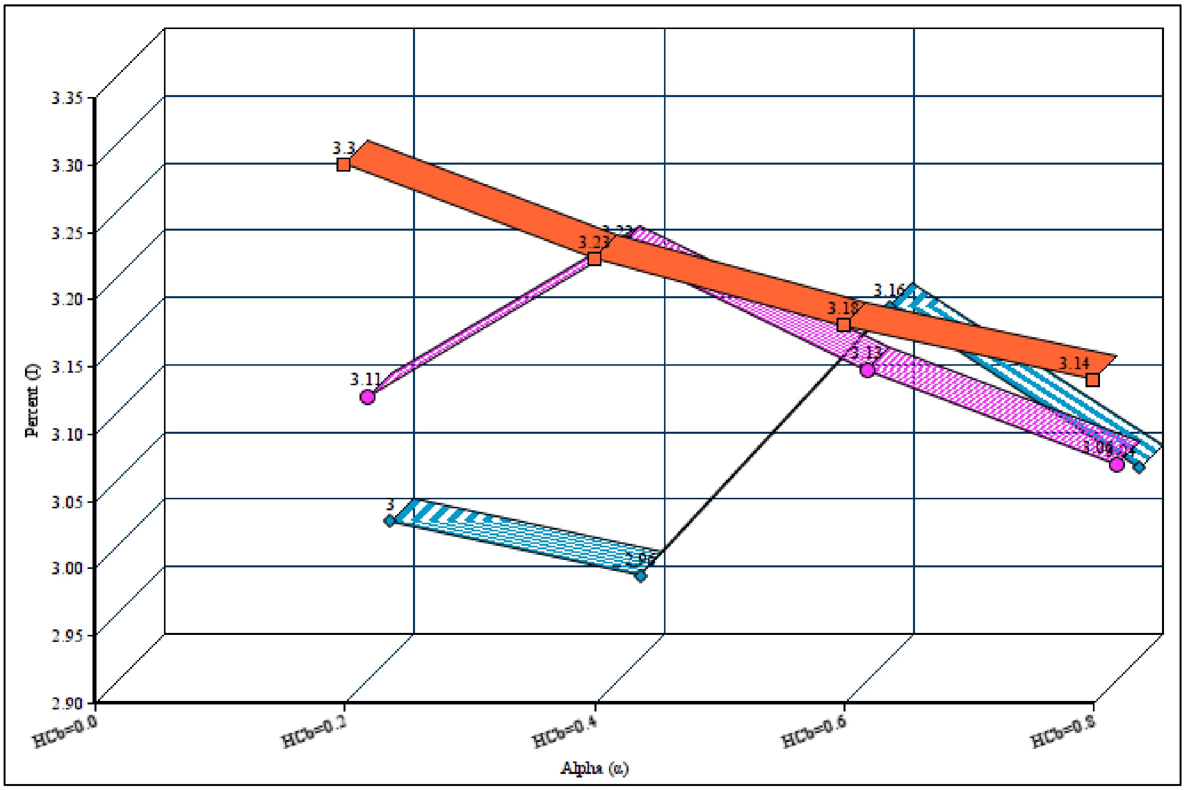

Fig 2: Sensitivity Analysis curve

Effect of $\alpha=HC_v / HC_b$ on percentage improvement (I) and no. of shipment is shown in figure 2. It is seen that, joint optimization will be less favorable when there is not a significant difference between the buyer as well as vendor holding costs. Under independent optimization, the growth of HCv does not impact the buyer selling price and the demand rate, but the vendor choose to keep less no. of stocks. So, it is lessening the number of shipments (n) to diminish its inventory level. Now, under joint optimization the selling price increases when decreasing the number of shipments (n). Therefore, the improvement of AJTP, where both the selling price and number of shipments vary, is lower than the increase of OSP, where the number of shipments only changes. Consequently, the percentage improvement reduces.

5. Conclusion

In this paper, a joint vendor buyer inventory EOQ model for a two-stage supply is proposed. It is considered that the requirement rate is a function of the selling price. Likewise, the overall cost functions are designed, and the optimum values of the selling price (S), order quantity (Q) and number of shipments (n) are obtained for independent as well asjoint optimizations. A mathematical model & sensitivity analysis are described, and the main following concluding remarks are obtained. The optimum value of selling price in case of independent optimization is higher than its value in case of joint optimization, and so coordination increases the requirement and profit of the supply. Moreover, supply members can get more and more profit from coordination in this competitive environment in which sensitivity of the requirement to cost is high. Other concluding remark is that increasing the unit purchasing cost, which is given by the buyer to the vendor leads to increase in the percentage improvement. At last, coordination of the supply is quite less appealing when the vendor setup cost is considerably more than the buyer ordering cost. This work can be proceed for multiple vendors and buyers supply system. Also, the inventory model can be developed for imperfect products as well as for deteriorating items.

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