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# Study of the various probability distributions for estimating life table scale parameters

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**Abstract**
Length of life is fundamental dimension of human prosperity. The construction of mathematical model of mortality is very important for an inferential decision. Mathematical modeling of mortality is also crucial for forecasting of mortality. There has been dramatic increase in the life expectancy of human in all industrial countries in the last century. In the first decade increase was due to decline in mortality rate of infants and in later decades was due to mortality rates of adults. For forecasting mortality certain parameters of mortality is to be estimated. In the present paper a study of the parameters of mortality estimated for forecasting mortality by various continuous probability distributions is made.

**Keywords:** Mortality forecasting, life expectancy at mode, force of mortality, age specific growth rate.

## 1. Introduction
Modeling of mortality is important for the forecasting of mortality. The relative importance of mortality forecasting is considered because increase in life expectancy beyond limit. Human mortality has improved substantially over the last century. Since life expectancy is continuously increasing beyond certain limit there is a need of forecasting of mortality. Previously in the past mortality was forecasted on the basis of subjective judgment but in the last 15 years more sophisticated tools are used and applied for forecasting mortality. Instead of deterministic point forecast nowadays most of the models use probability distribution for forecasting mortality.
However certain models that are used in mortality forecasting are as follows.

i) Extrapolative: - includes extrapolation of aggregate measures such as life expectancy and survival functions.
ii) Explanation:- make use of structural or epidemiological models of mortality [dependence of lung cancer in tobacco smoking]
iii) Expectation: - forecasts are based on subjective opinions of expert.

It is very important in the field of demography and actuarial science to find an appropriate model of mortality for forecasting. Forecasting is mainly depends upon the purpose of forecast and data availability. In this paper we studied the various parametric models of mortality for estimating scale parameters required for constructing life tables.

## 2. Terms used in mortality forecasting
**2.1. Force of mortality:** Since death can occur at any time $l_x$ is a continuous function of x. At any age x the rate of decrease in $l_x$ is given by the expression

$$\lim_{t\to 0}\frac{l_x - l_{x+t}}{t} = -\lim_{t\to 0}\frac{l_{x+t} - l_x}{t} = -\frac{d}{d_x} l_x \qquad \text{….. (2.1.1)}$$

Where $\frac{d}{d_x} l_x$ is the differential coefficient of $l_x$ with respect to x.

The force of mortality at age x is defined as the ratio of instantaneous rate of decrease in $l_x$ to the value of $l_x$ denoting $\mu_x$

$$\mu_x = -\frac{1}{l_x}\frac{d}{d_x}l_x = -\frac{d}{d_x}\log l_x \qquad \ldots\ldots (2.1.2)$$

### 2.2. Life expectancy at mode
Let the radix of the life table at time t is l (o, t). The life table density function describing complete life span expresses distribution of death at age x and time t denoted by d (x, t).
Mean age at death- life expectancy at birth

$e_0(t) = \int_0^w xd(x,t)$ where w is the terminal age at which $l_x$ vanishes also we have $\int_0^w d(x,t) = 1$

Median age at death- Age at which half of the population died

i.e. survival function is equal to $\frac{1}{2}$

$l(M_d, t) = 0.5$

Modal age at death- Age at which most of the death occurs.
M (t) = {x/max [d(x, t)]}
Here the distribution is considered to be bimodal one mode at age 0 and the later mode found at older ages in this distribution the focus is made on the latter age.
M (t) = {x/max [d(x, t)] for x>5}

### 2.3. Survival curve
The survival curve is simply proportion of lives surviving at each age. The survival curve is better known to actuaries as tPx probability that a person of age x survives till age x+t. The survival curve starts at 10 (or 100%) as everyone is alive at outset and decreases monotonically towards zero (or 0%) as most of the people dies at later age.

### 2.4. Euler mascheroni constant
The numerical value of this constant is given to 50 decimal places. It is defined as the limiting difference between the harmonic series and the natural logarithms.

$$\gamma = \lim_{n\to\infty}\left[\sum_{k=1}^{n}\frac{1}{k} - \ln(n)\right]$$

= 0.577221566……

### 2.5. Euler gompertz constant
Euler Gompertz constant is given by

$$p = \int_1^\infty \frac{\exp(1-x)}{x}dx$$

= 0.5963473623…..

### 2.6. Age specific death rate
The age specific death rate is the ratio of the number of deaths within a specified age group in a given geographic area during a certain period of time to the corresponding population at risk of the same age group in the same geographic area during a specified time period of study.

## 3. Estimation of life table parameters using different probability distribution

### 3.1. Gompertz distribution
The British actuary, Benjamin Gompertz, proposed a simple formula in 1825 for describing the mortality rates of the elderly. This famous law states that death rate increases exponentially with age
In terms of actuarial notation

$$\mu(x) = Ae^{kx} \qquad \ldots\ldots (3.1.1)$$

Where $\mu(x)$ is the force of mortality A>0, k>0, A represents general mortality level and k is the age specific growth rate of the force of mortality
Considering A = k $e^{-km}$ = $\mu(m)e^{-km}$

$\mu(x) =$ k $e^{-km}$ $e^{kx}$

= k $e^{k(x-m)}$

$$\mu(x) = \mu(m)\, e^{k(x-m)} \qquad \ldots\ldots (3.1.2)$$

The survival function is given by

$$l_x = \exp\left[\frac{A}{k} - \frac{A}{k}e^{kx}\right]$$

$$l_x = \exp\left[e^{-km} - e^{k(x-m)}\right]$$

$$l_x = \exp\left[e^{-m\times\frac{1}{1/k}}\; e^{(x-m)\times\frac{1}{1/k}}\right]$$

$$l_x = \exp\left[e^{-m/s} - e^{(x-m)/s}\right] \qquad \ldots\ldots (3.1.3)$$

Survival function is recognized by the mode m & a spread parameter s = $1/k$. Since with human population $e^{-km} = 0$ the survival function can be approximated for -∞<x<∞

$$l_x = \exp\left(-\frac{A}{k}e^{kx}\right)$$

$$l_x = \exp\left(-e^{k(x-m)}\right) \qquad \ldots\ldots (3.1.4)$$

The life expectancy at age x can be approximated by

$$e_x = \frac{\dfrac{\gamma + \ln\left(\dfrac{A}{k}\right) + k\times x\times\dfrac{A}{k}\exp(kx)}{k}}{\exp\left(\dfrac{A}{k} - \dfrac{A}{k}e^{kx}\right)}$$

Where $\gamma$ is the Euler Mascheroni constant

$$= \frac{\dfrac{\gamma + k(x-m) - \exp(k(x-m))}{k}}{\exp(e^{-km} - e^{k(x-m)})}$$

Putting x = 0 one gets the life expectancy at birth

$$e_0 = \mu = \frac{\ln\left(\dfrac{A}{k}\right) + \gamma - \dfrac{A}{k}}{k} \approx \frac{\ln\left(\dfrac{A}{k}\right) + \gamma}{k} \approx m - \frac{\gamma}{k} \quad \ldots (3.1.5)$$

Life expectancy at the modal age

$$e_m = \frac{p}{k} = \frac{0.59634736}{k} \quad \text{(c.f. Pollard 1998)}$$

Where p is the Euler Gompertz constant

The Gompertz law of mortality so popular in years still provide a good general representation of the age pattern of mortality at the age near which most of the death occurs and about spread parameter of death, the modal age at death & the force of mortality at the modal age.

The Gompertz distribution is a continuous probability distribution and is often applied to describe the distribution of adult life span by the demographers and actuaries for the analysis of survival.

### 3.2. Inverse Gompertz model

A closely associated model with the Gompertz model is the Inverse Gompertz model which is mainly reflected to the value of m. The inverse Gompertz model is skewed to the right as opposite to the Gompertz model which is skewed left. The probability density function of the inverse Gompertz distribution is given by the expression

$$f(x) = \frac{\frac{1}{s}\exp\{-\frac{x-m}{s} - e^{-(x-m)/s}\}}{1-\exp\{e^{-m/s}\}}$$

….. (3.2.1) (c.f Jacques F. Carriere 1992)

The life table parameters are find out to be as follows

The force of mortality is

$$\mu(x) = \frac{\frac{1}{s}\exp\left\{-\frac{x-m}{s}\right\}}{\left(\exp\left\{e^{-(x-m)/s} - 1\right\}\right)} \qquad \text{……. (3.2.2)}$$

The inverse Gompertz survival function is given by

$$l_x = \frac{\left[1-\exp\left\{-e^{-(x-m)/s}\right\}\right]}{\left[1-\exp\left\{-e^{m/s}\right\}\right]} \qquad \text{…… (3.2.3)}$$

### 3.3. Weibull model

This distribution is described by Waloddi Weibull who described it in the year 1951. The weibull model is the most widely used parametric model for estimating mortality parameters. It is well known that the Weibull distribution is the most popular distribution in the analysis of lifetime data. The probability density function of the weibull model is given by

$$f(x) = \frac{1}{s}\left(\frac{x}{m}\right)^{\frac{m}{s}-1} \exp\left\{-\left(\frac{x}{m}\right)^{m/s}\right\} \qquad \text{…… (3.3.1)}$$

The force of mortality is given by

$$\mu(x) = \frac{1}{s}\left(\frac{x}{m}\right)^{\frac{m}{s}-1} \qquad \text{…… (3.3.2)}$$

The survival function of the weibull model is given by the expression

$$l_x = \exp\left\{-\left(\frac{x}{m}\right)^{m/s}\right\} \qquad \text{…… (3.3.3)}$$

It is clear through the Weibull distribution is that Weibull probability density function can be decreasing or increasing or unimodal depending on the shape of the distribution parameter.

### 3.4. Inverse Weibull distribution

Due to the flexibility of Weibull PDF the inverse Weibull distribution has been extensively used. A model that is closely associated with the weibull's law is the inverse weibull model in which the probability density function is given by

$$f(x) = \frac{1}{s}\left(\frac{x}{m}\right)^{-\frac{m}{s}-1} \exp\left\{-\left(\frac{x}{m}\right)^{-m/s}\right\}$$

The life table parameters for estimating mortality are calculated using inverse weibull model is as follows

The force of mortality

$$\mu(x) = \frac{\frac{1}{s}\left(\frac{x}{m}\right)^{-\frac{m}{s}-1}}{\left(\exp\left\{\left(\frac{x}{m}\right)^{-m/s} - 1\right\}\right)}$$

…… (3.4.1) ( c.f Jacques F. Carriere 1992)

The survival function is given by

$$l_x = 1-\exp\left\{-\left(\frac{x}{m}\right)^{-m/s}\right\} \qquad \text{…… (3.4.2)}$$

### 4. Applications in forecasting mortality

The prediction of future mortality rates is a problem of fundamental importance for the insurance and pension industry. For the insurance and pension industry the pricing and reserving of annuities depends on three things: stock market return, interest rates & future mortality rates. The more recent application of weibull model is to model various phases of mortality experience. Weon model derived from Weibull distribution using age dependant shape parameter is the complementarity between survival and mortality. Mortality tables are used by particularly actuaries', demographist or people dealing with public health to make various studies on issues like migrations, fertility, and population estimation.

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