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Some results on 3-Equitable prime cordial labeling of graphs

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Abstract
A 3-equitable prime cordial labeling of a graph G with vertex set V is a bijection f from V to $\{1,2,3,\ldots..|V|\}$ such that if an edge uv is assigned the label 1 if $gcd\big(f(u),f(v)\big)=1$ and $gcd\big(f(u)+f(v),f(u)-f(v)\big)=1$ the label 2 and if $gcd\big(f(u),f(v)\big)=1$ and $gcd\big(f(u)+f(v),f(u)-f(v)\big)=2$ and 0 otherwise, then the number of edges labeled with i and the number of edges labeled with j differ by atmost 1 for $0 \le i,j \le 2$, if a graph has a 3-equitable prime cordial labeling then it is called a 3-equitable prime cordial graph. In this paper, we investigate the 3-equitable prime cordial labeling behavior of cycle with three chords and a cycle.

Keywords: 3-equitable prime cordial labeling, 3-equitable prime cordial graph

Introduction
The graphs considered here are finite, connected, undirected and simple. The vertex set and edge set of a graph G are denoted by $V(G)$ and $E(G)$ respectively. For various graph theoretic notations and terminology we follow Harary [1] and for number theory we follow Burton [2]. We will give the brief summary of definitions which are useful for the present investigations.

Definition 1: If the vertices of the graph are assigned values subject to certain conditions it is known as graph labeling.
Most interesting graph labeling problems have three important characteristics.
1. A set of numbers from which vertex labels are chosen.
2. A rule that assigns a value to each edge.
3. A condition that these values must satisfy.

For detailed survey on graph labeling one can refer Gallian [3]. Vast amount of literature is available on different types of graph labeling. According to Beineke and Hegde [4] graph labeling serves as a frontier between number theory and structure of graphs.
Labeled graphs have variety of applications in coding theory, particularly for missile guidance codes, design of good radar type codes and convolution codes with optimal autocorrelation properties. Labeled graph plays vital role in the study of X-ray crystallography, communication network and to determine optimal circuit layouts. A detailed study of variety of applications of graph labeling is given by Bloom and Golomb [5].
The present work is to aimed to discuss one such labeling known as 3-equitable prime cordial labeling.

Definition 2: Let $G=\big(V(G),E(G)\big)$ be a graph. A mapping $f:V(G)\to\{0,1\}$ is called binary vertex labeling of G and $f(v)$ is called the label of the vertex v of G under f.
For and edge $e=uv$, the induced edge labeling $f^*=E(G)\to\{0,1\}$ is given by $f^*(e)=|f(u)-f(v)|$. Let $v_f(0), v_f(1)$ be the number of vertices of G having labels of 0 and 1 respectively under f and $e_f(0)$, $e_f(1)$ be the number of edges having labels 0 and 1 respectively under f^*.

Definition 3: A binary vertex labeling of a graph G is called a cordial labeling if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is cordial if it admits cordial labeling.
The concept of cordial labeling was introduced by Cahit [6]. Many researchers have studied cordiality of graphs. Cahit [6] proved that tree is cordial and K_n is cordial if and only if $n \leq 3$. Vaidya *et al*. [7] have also discussed the cordiality of various graphs.

Definition 4: Let $G = (V, E)$ be a graph. A mapping $f: V(G) \to \{0, 1, 2\}$ is called ternary vertex labeling of G and $f(v)$ is called the label of the vertex v of G under f.
For an edge $e = uv$, the induced edge labeling is given $f^* = E(G) \to \{0, 1, 2\}$ is given by
$f^*(e) = |f(u) - f(v)|$. Let $v_f(0), v_f(1), v_f(2)$ be the number of vertices of G having labeling 0,1,2 respectively under f and $e_f(0)$,$e_f(1)$,$e_f(2)$ be the number of edges having labels 0,1,2 respectively under f^*.

Definition 5: A ternary vertex labeling of a graph G is called a 3-equitable labeling if $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$ for all $0 \leq i, j \leq 2$. A graph G is 3-equitable if it admits 3-equitable labeling.
The concept of 3-equitable labeling was introduced by Cahit [8]. Many researchers have studied 3-equitability of graphs. Cahit [8] proved that C_n is 3-equitable except $n \equiv 3(mod\ 6)$. In the same paper he proved that an Eulerian graph with number of edges congruent to $3(mod\ 6)$ is not 3-equitable. Yourssef [9] proved that W_n is 3-equitable for all $n \geq 4$.

Definition 6: A prime cordial labeling of a graph G with vertex set $V(G)$ is a bijection defined by $f: V(G) \to \{1, 2, 3, \dots . |V|\}$
defined by $f(e = uv) = 1$ if $\gcd(f(u), f\ (v)) = 1$
$= 0\ otherwise$
and $|e_f(i) - e_f(j)| \leq 1$. A graph which admits prime cordial labeling is called a prime cordial graph.
The concept of prime cordial labeling was introduced by Sundaram *et al*. [10] and in the same paper they investigate several results on prime cordial labeling. Vaidya and Vihol [11] have also discussed prime cordial labeling in the context of graph operations.
By combining the concepts of prime cordial labeling and 3-equitable labeling and by using the result in Number Theory that $\gcd(a, b) = 1$, then $\gcd(a + b, a - b) = 1$ or 2, we introduce a new concept called 3-equitable prime cordial labeling as follows.

Definition 7: A 3-equitable prime cordial labeling of a graph G with vertex set $V(G)$ is a bijection $f: V(G) \to \{1, 2, 3, \dots \dots, p\}$ defined by

$$f(e = uv) = \begin{cases} 1\ if\ gcd(f(u), f(v)) = 1\ and\ gcd(f(u) + f(v), f(u) - f(v)) = 1 \\ 2\ if\ gcd(f(u), f(v)) = 1\ and\ gcd(f(u) + f(v), f(u) - f(v)) = 2 \\ 0\ otherwise \end{cases}$$

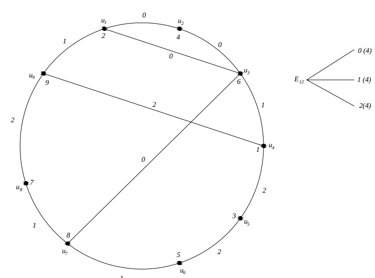
And $|e_f(i) - e_f(j)| \leq 1$ for all $0 \leq i, j \leq 2$. A graph which admits 3-equitable prime cordial labeling is called a 3-equitable prime cordial graph.

Definition 8: A chord of cycle C_n is an edge joining two non-adjacent vertices of C_n.

Theorem 1
Cycle C_n with three chord is 3-equitable prime cordial . n is odd except $n \equiv 1(mod3)(n \geq 9)$

Proof
Let G be a cycle C_n with three chords.
Let u_1, u_2, u_3 be the vertices of cylcle C_n and let $e = u_1u_3$ be one chord of cycle C_n. $e = u_4u_n$ and $e = u_3u_{n-2}$ be the other two chords of C_n.
put $f(u_1) = 2, f(u_2) = 4, f(u_3) = 6$ and $f(u_n) = n$
Label the next three vertices is u_4, u_5, u_6 by consecutive odd numbers $1, 2, 3$ respectively, again the next three vertices u_7, u_8, u_9 by consecutive even numbers $8, 10, 12$ respectively. Repeat the process until all the vertices get labeled of there does not exist three even numbers (or) odd numbers then according to existence of number give the label.

Illustration: The 3-equitable prime cordial labeling of cycle C_9 with three chords is an shown below.



Theorem 2
Cycle C_n with three chord is 3-equitable prime cordial. – n is even $n \geq 12$.

Proof
Let G be cycle C_n with three chords $u_1, u_2, \dots \dots u_n$ be the vertices of the cycle C_n and let $e = u_1u_3$ be the one chord of the cycle C_n.

Case (i)
$n \equiv 0(mod3)$ except $n \equiv 0(mod10)$
Put $f(u_1) = 1, f(u_2) = 3, f(u_3) = 5$.

Label the next three vertices u_4, u_5, u_6 by the consecutive even numbers $2, 4, 6$ respectively. The next three vertices u_7, u_8, u_9 by consecutive odd numbers $7, 9, 11$ respectively., and repeat this process until all the vertices are labeled.
In this case are chord is u_1u_3 and other two chords are u_3u_n and u_4u_{n-1}

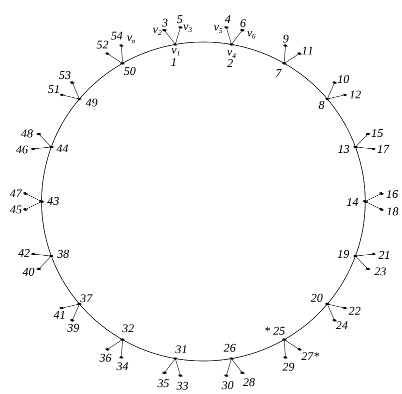
Case (ii)
$n \equiv 1(mod3)$
Here one chord is u_1u_3, and the other two chords are u_3u_n, u_4u_{n-1}

Case (iii)
$n \equiv 2(mod3)$
Here one chord is u_1u_3, and the other two chords are u_4u_n, u_4u_{n-2}

Case (iv)
$n \equiv 0(mod3)\ n \equiv 0(mod10)$
Here one chord is u_1u_3, and the other two chords are u_3u_{n-1}, u_4v_4.

Theorem 3
C_n with two pendent vertices is equitable prime cordial. n is even $n \geq 10$

Proof
Let G be a graph. Let $v_1, v_2, v_3, \dots . v_n$ be the vertices of the cycle.
Let $f(v_1) = 1, f(v_2) = 3, f(v_3) = 5$ and $f(v_n) = n$
Label the next three vertices v_4, v_5, v_6 by consecutive odd numbers $2, 4, 6$ respectively. Again the next three vertices v_7, v_8, v_9 by consecutive even numbers $7, 9, 11$ respectively. Repeat the problem until all the vertices get labeled. If there does not exist three even numbers (or) odd numbers then according to the existence of numbers give the label.
Here interchanging the labeling of the vertices.
(25+k 30) by (25+k 30) + 2, k = 0,1,2,3,……………



Conclusion
By using a property from number theory, we have introduced 3-equitable prime cordial labeling of graphs. In the present work, we investigated 3-equitable prime cordial labeling behavior of standard graphs only. To investigate analogous results for different graphs as well as in the context of various graph labeling problems is an open area of research.

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