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On soft- π gp-separation axioms

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Abstract

The aim of the paper is to study soft- π gp-separation axioms and obtain some of their characterization.

Keywords: Soft- π gp- T_i -spaces. ($i=0, 1, 2$)

1. Introduction

General Topology plays a significant role in space time geometry as well as in different branches of Mathematics. Molodtsov^[12] introduced soft set sets as a new mathematical tool for dealing with uncertainties which is free from the difficulties. A soft set is a collection of approximation descriptions of an object. The concept of π -closed sets in topological space was initiated by Zaitsav^[15] and the concept of π g-closed set was introduced by Noiri and Dontchev^[13]. Soft separation axioms for soft topological spaces were studied and introduced by Shabir *et al.*^[14]. Arokia Lancy and Arockiarani. I^[2] introduced soft- β -separation axioms and derived some of its characterization. Soft semi-open sets and its properties along with soft semi-separation axioms were introduced and studied by Bin Chen^[3]. Introduction of soft point along with the study on interior point, neighborhood system, continuity and compactness is found in due to Zorlutuna^[16]. Many Researchers extended the result of generalization of various soft closed sets in many directions. P.K. Das^[6], Kharal *et al.*^[10] developed soft function over classes of soft sets. Cigdem Gunduz Aras *et al.*,^[1] in 2013 studied and discussed the properties of soft continuous mappings. V. Christy and K.Mohana^[4] studied the characterization of soft- π gp-closed sets in soft topological spaces.

In this paper, we investigate the soft- π gp-separation axioms such as soft- π gp- T_i ($i=0, 1, 2$)-spaces.

2. Preliminaries

Let U be an initial universe set and E be a collection of all possible parameters with respect to U , where parameters are the characteristics or properties of objects in U . Let $P(U)$ denote the power set of U , and let $A \subseteq E$.

Definition: 2.1^[12] A pair (F, A) is called a soft set over U , where F is a mapping given by $F: A \rightarrow P(U)$. In other words, a soft set over U is a parameterized family of subsets of the universe U . For a particular $e \in A$, $F(e)$ may be considered the set of e -approximate elements of the soft set (F, A) .

Definition: 2.2^[12] Two soft set (F, A) and (G, B) over a common universe U is said to be soft equal if (F, A) is a soft subset (G, B) and (G, B) is a soft subset of (F, A) .

Definition: 2.3:^[11] A soft set (F, A) over X is said to be absolute soft set denoted by \bar{A} or X_A , if for all $e \in A, F(e) = X$. Clearly, we have $X'_A = \phi_A$ and $\phi'_A = X_A$

Let (F, E) be a soft set X . The soft set (F, E) is called a soft point, denoted by (X_e, E) , if for the element $e \in E$, $F(e) = \{x\}$ and $F(e') = \phi$ for all $e' \in E - \{e\}$.

Definition: 2.4^[11] For two soft sets (F, A) and (G, B) over a common universe U , Union of two soft sets of (F, A) and (G, B) is the soft set (H, C) , where $C = A \cup B$, and $\forall e \in C$,

$$H(e) = \begin{cases} F(e) & , \text{if } e \in A - B \\ G(e) & , \text{if } e \in B - A \\ F(e) \cup G(e) & , \text{if } e \in A \cap B \end{cases}$$

We write $(F,A) \cup (G,B) = (H,C)$.

Definition: 2.5 ^[14] Let τ be the collection of soft sets over X , and then τ is called a soft topology on X if τ satisfies the following axioms:

- (i) ϕ, \tilde{X} belong to τ
- (ii) The union of any number of soft sets in τ belongs to τ .
- (iii) The intersection of any two soft sets in τ belongs to τ .

The triplet (X, τ, E) is called a soft topological space over X . Let (X, τ, E) be a soft space over X , then the members of τ are said to be soft open sets in X .

Definition: 2.6 ^[6] Let (X, τ, E) be a soft topological space over X and the soft interior of (F, E) denoted by $\text{Int}(F, E)$ is the union of all soft open subsets of (F, E) . Clearly, (F, E) is the largest soft open set over X which is contained in (F, E) . The soft closure of (F, E) denoted by $\text{Cl}(F, E)$ is the intersection of all closed sets containing (F, E) . Clearly, (F, E) is smallest soft closed set containing (F, E) .

$$\text{Int}(F, E) = \cup \{ (O, E) : (O, E) \text{ is soft open and } (O, E) \subseteq (F, E) \}.$$

$$\text{Cl}(F, E) = \cap \{ (O, E) : (O, E) \text{ is soft closed and } (F, E) \subseteq (O, E) \}.$$

Definition: 2.7 ^[7] The Intersection (H, C) of two soft sets (F, A) and (G, B) over a common universe U denoted $(F, A) \cap (G, B)$ is defined as $C = A \cap B$ and $H(e) = F(e) \cap G(e)$ for all $e \in C$.

Definition: 2.8 ^[14]

- 1. a soft-semi open ^[9], if $(A, E) \subseteq \text{Cl}(\text{Int}(A, E))$
- 2. a soft-regular open ^[7], if $(A, E) = \text{Int}(\text{Cl}(A, E))$.
- 3. a soft-pre-open ^[7], if $(A, E) \subseteq \text{Int}(\text{Cl}(A, E))$.
- 4. a soft-clopen ^[9], if (A, E) is both soft open and soft closed.

The complement of the soft semi open, soft regular open, soft pre-open sets are their respective soft semi closed, soft regular closed and soft pre-closed sets.

The finite union of soft regular open sets is called soft π -open set and its complement is soft- π -closed set. The soft regular open set of X is denoted by $\text{SRO}(X)$ or $\text{SRO}(X, \tau, E)$.

Definition: 2.9 ^[8] Let (X, τ, E) and (Y, τ^*, E) be two soft topological spaces. A function $f: (X, \tau, E) \rightarrow (Y, \tau^*, E)$ is said to be Soft-continuous ^[14], if $f^{-1}(F, E)$ is soft-open in (X, τ, E) , for every soft-open set (F, E) of (Y, τ^*, E) .

Definition: 2.10 ^[4]

- 1. A soft subset (A, E) of a soft topological space X is called soft- π gp-closed set in X if $\text{spcl}(A, E) \subseteq (U, E)$ whenever $(A, E) \subseteq (U, E)$ and (U, E) is soft- π -open in X .
- 2. Let X and Y be two topological spaces and the function $f: X \rightarrow Y$. Then the function f is soft- π gp-irresolute if $f^{-1}(F, E)$ is soft- π gp-open in X , for every soft- π gp-open set (F, E) of Y .

Definition 2.11: ^[5] Let (X, τ, E) and (Y, τ^*, E) be two soft topological spaces and $f: (X, \tau, E) \rightarrow (Y, \tau^*, E)$ be a function. Then the function f is said to be soft- π gp-continuous function if $f^{-1}(G, E)$ is soft- π gp-closed(open) set in (X, τ, E) for every soft-closed(open) set (G, E) of (Y, τ^*, E) .

Definition: 2.12 ^[5]: Let (X, τ, E) and (Y, τ^*, E) be two soft topological spaces and $f: (X, \tau, E) \rightarrow (Y, \tau^*, E)$ be a function. Then the function f is soft-contra- π gp-continuous if $f^{-1}(F, E)$ is soft- π gp-closed in (X, τ, E) for every soft-open (F, E) in (Y, τ^*, E) .

Definition: 2.13: ^[5]

- 1. The Soft π gp-Closure of a soft set (G, E) is defined to be the intersection of all soft π gp-closed sets containing the soft set (G, E) and is denoted by $s\text{-}\pi\text{gp-cl}(G, E)$.
- 2. The Soft π gp-Interior of a soft set (G, E) is defined to be the union of all soft π gp-open sets contained the soft set (G, E) and is denoted by $s\text{-}\pi\text{gp-int}(G, E)$.

Definition: 2.14: ^[9] A function $f: (X, \tau, E) \rightarrow (Y, \tau^*, E)$ is called soft regular open if $f((F, A))$ is soft-regular open (soft-regular-closed) if $f(F, A)$ is soft-regular-open (soft-regular-closed) in Y , for every soft-open (soft-closed) set of X .

Definition: 2.15: ^[9]

- 1. A soft topological space X is said to be soft- T_0 -space if for two disjoint points x and y , there exist a soft open sets (F, E) and (G, E) containing one but not the other.
- 2. A soft topological space X is said to be soft- T_1 -space if for two disjoint points x and y of X , there exist a soft open sets (F, E) and (G, E) such that $x \in (F, E)$, $y \notin (F, E)$ and $y \in (G, E)$, $x \notin (G, E)$.
- 3. A soft topological space X is said to be soft- T_2 -space if for two disjoint points x and y of X , there exist a disjoint open sets (F, E) and (G, E) such that $x \in (F, E)$ and $y \in (G, E)$.

Definition: 2.16: ^[11] A space X is called soft-connected if X is not the union of two disjoint non-empty soft-open sets. Throughout this paper we denote (X, τ, E) , (Y, τ^*, E) and (Z, τ^{**}, E) as X, Y and Z .

3. Soft- π gp-separation axioms

Definition: 3.1

- 1. A soft topological space X is said to be soft- π gp- T_0 -space if for two disjoint points x and y of X , there exists a soft- π gp- open sets (F, E) and (G, E) such that $x \in (F, E)$ or $y \in (F, E)$ and $y \in (G, E)$ or $x \in (G, E)$.
- 2. A soft topological space X is said to be soft- π gp- T_1 -space if for two disjoint points x and y of X , there exists a soft- π gp- open sets (F, E) and (G, E) such that $x \in (F, E)$ and $y \notin (F, E)$ and $y \in (G, E)$ and $x \notin (G, E)$.
- 3. A soft topological space X is said to be soft- π gp- T_2 -space if for two disjoint points x and y of X , there exists a disjoint soft- π gp- open sets (F, E) and (G, E) such that $x \in (F, E)$ and $y \in (G, E)$.

Theorem: 3.2

A soft space X is soft- π gp- T_0 -space iff soft π gp-closures of distinct points are distinct.

Proof: Let x and y be distinct points of X . Since X is a soft- π gp- T_0 -space, there exists a soft π gp-open set (F, E) such

that $x \in (F,E)$ and $y \notin (F,E)$. Consequently, $X-(F,E)$ is a soft- π grp-closed set containing y but not x . But soft- π grp-cl(y) is the intersection of all soft- π grp-closed sets containing y . Hence $y \in$ soft- π grp-cl(y), but $x \notin$ soft- π grp-cl(y) as $x \notin X-(F,E)$. Therefore, soft- π grp-cl(x) \neq soft- π grp-cl(y). Conversely, let soft- π grp-cl(x) \neq soft- π grp-cl(y) for $x \neq y$. Then there exists at least one point $z \in X$ such that $z \notin$ soft- π grp-cl(y). We have to prove $x \notin$ soft- π grp-cl(y), because if $x \in$ soft- π grp-cl(y), then $\{x\} \subset$ soft- π grp-cl(y) \Rightarrow soft- π grp-cl(x) \subset soft- π grp-cl(y). So, $z \in$ soft- π grp-cl(y), which is a contradiction. Hence $x \notin$ soft- π grp-cl(y) $\Rightarrow x \in X -$ soft- π grp-cl(y), which is a soft- π grp-open set containing x but not y . Hence X is a soft- π grp- T_0 -space.

Theorem: 3.3

A soft space X is soft- π grp- T_1 -space iff each singleton is soft- π grp-closed set.

Proof: Let X be a soft- π grp- T_1 -space and $x \in X$. Let $y \in X-\{x\}$. Then for $x \neq y$, there exists soft- π grp-open set U_y such that $y \in U_y$ and $x \notin U_y$. Conversely, $y \in U_y \subset X-\{x\}$. That is $X-\{x\} = \cup\{U_y : y \in X-\{x\}\}$, which is soft- π grp-open set. Hence $\{x\}$ is soft- π grp-closed set. Conversely, suppose $\{x\}$ is soft- π grp-closed set for every $x \in X$. Let $x, y \in X$ with $x \neq y$. Now, $x \neq y \Rightarrow y \in X-\{x\}$. Hence $X-\{x\}$ is soft- π grp-open set containing y but not x . Similarly, $X-\{y\}$ is soft- π grp-open set containing x but not y . Therefore, X is a soft- π grp- T_1 -space.

Theorem: 3.4

If every point (x,E) of X is a soft- π grp-closed, then X is a soft- π grp- T_1 -space.

Proof: Let (x,E) and (y,E) be two distinct points of X and by hypothesis (x,E) and (y,E) are soft π grp-closed. Then (x,E)' and (y,E)' are soft- π grp-open such that (y,E) \in (x,E)', (y,E) \notin (x,E) and (x,E) \in (y,E)', (x,E) \notin (y,E)'. Hence the soft topological space X is a soft- π grp- T_1 -space.

Theorem: 3.5

If $f: X \rightarrow Y$ is soft- π grp -continuous injection and Y is soft- T_1 -space, then X is soft- π grp- T_1 - space.

Proof: Let $f: X \rightarrow Y$ be a soft- π grp- continuous injection and Y be soft- T_1 -space. For any two distinct points x_1, x_2 of the soft space X , there exists distinct points y_1, y_2 of the soft space Y such that $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Since Y is soft- T_1 - space, there exists soft open sets (F, E) and (G, E) in Y such that $y_1 \in (F,E)$ and $y_2 \notin (F,E)$ and $y_1 \notin (G,E)$, $y_2 \in (G,E)$. i.e. $x_1 \in f^{-1}((F,E))$, $x_1 \notin f^{-1}((G,E))$ and $x_2 \in f^{-1}((G,E))$, $x_2 \notin f^{-1}((F,E))$. Since f is soft- π grp - continuous, $f^{-1}((F,E))$, $f^{-1}((G,E))$ are soft- π grp -open sets in X . Thus for two distinct points x_1, x_2 of the soft space X , there exists soft- π grp - open sets $f^{-1}((F,E))$ and $f^{-1}((G,E))$ such that $x_1 \in f^{-1}((F,E))$, $x_2 \notin f^{-1}((F,E))$ and $x_2 \in f^{-1}((G,E))$, $x_2 \notin f^{-1}((F,E))$. Therefore, X is a soft- π grp - T_1 - space.

Theorem: 3.6

If $f : X \rightarrow Y$ be soft- π grp -irresolute function, and Y is soft- π grp - T_1 - space, then X is soft- π grp - T_1 -space.

Proof: Let x_1, x_2 be two distinct points in a soft space X . Since f is injective, there exists distinct points y_1, y_2 of a soft space Y such that $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Since Y is soft- π grp - T_1 -space, there exists soft- π grp- open sets (F,E) and (G,E) in Y such that $y_1 \in (F,E)$ and $y_2 \notin (F,E)$ and $y_1 \notin (G,E)$, $y_2 \in (G,E)$. i.e. $x_1 \in f^{-1}((F,E))$, $x_1 \notin f^{-1}((G,E))$ and $x_2 \in f^{-1}((G,E))$, $x_2 \notin f^{-1}((F,E))$. Since f is soft- π grp- irresolute, $f^{-1}((F,E))$, $f^{-1}((G,E))$ are soft- π grp - open sets in X . Thus, for two distinct points x_1, x_2 of X , there exists soft- π grp- open sets $f^{-1}((F,E))$ and $f^{-1}((G,E))$ such that $x_1 \in f^{-1}((F,E))$, $x_1 \notin f^{-1}((G,E))$ and $x_2 \in f^{-1}((G,E))$, $x_2 \notin f^{-1}((F,E))$. Hence X is a soft- π grp- T_1 - space.

Theorem: 3.7

If $f : X \rightarrow Y$ be soft- π grp -continuous injection, and Y is soft- T_2 -space, then X is soft- π grp - T_2 -space.

Proof: Let $f : X \rightarrow Y$ be soft- π grp- continuous injection and Y be soft- T_2 -space. Then for any two distinct points x_1 and x_2 of a soft space X , there exists distinct points y_1, y_2 of a soft space Y such that $y_1 = f(x_1)$, $y_2 = f(x_2)$. Since Y is soft- T_2 -space, there exists disjoint soft open sets (F,E) and (G,E) in Y such that $y_1 \in (F,E)$ and $y_2 \in (G,E)$. i.e. $x_1 \in f^{-1}((F,E))$, $x_2 \in f^{-1}((G,E))$. Since f is soft- π grp - continuous, $f^{-1}((F,E))$ & $f^{-1}((G,E))$ are soft- π grp-open sets in X . since f is soft injective, $f^{-1}((F,E)) \cap f^{-1}((G,E)) = f^{-1}((F,E) \cap (G,E)) = f^{-1}(\emptyset) = \emptyset$. Thus, for two disjoint points x_1, x_2 of X , there exists disjoint soft- π grp-open sets $f^{-1}((F,E))$ and $f^{-1}((G,E))$ such that $x_1 \in f^{-1}((F,E))$ and $x_2 \in f^{-1}((G,E))$. Hence X is soft- π grp- T_2 -space.

Theorem: 3.8

If $f: X \rightarrow Y$ be the soft- π grp-irresolute injective function and Y is soft- π grp- T_2 -space, then X is soft- π grp- T_2 -space.

Proof: Let x_1, x_2 be any two distinct points in a soft space X . Since f is soft injective, there exists distinct points y_1, y_2 of a soft space Y such that $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Since Y is soft- π grp- T_2 -space, there exist disjoint soft- π grp-open sets (F,E) and (G,E) in Y such that $y_1 \in (F,E)$ and $y_2 \in (G,E)$. i.e. $x_1 \in f^{-1}((F,E))$, $x_2 \in f^{-1}((G,E))$. Since f is soft- π grp-irresolute, $f^{-1}((F,E))$, $f^{-1}((G,E))$ are disjoint soft- π grp-open sets in X . Thus, for two distinct points x_1, x_2 of X , there exists disjoint soft- π grp-open sets $f^{-1}((F,E))$ and $f^{-1}((G,E))$ such that $x_1 \in f^{-1}((F,E))$ and $x_2 \in f^{-1}((G,E))$. Hence X is a soft- π grp- T_2 -space.

Theorem: 3.9

A soft topological space X is soft- π grp- T_2 -space iff for distinct points x, y of X , there exists a soft- π grp-open set (F,E) containing x but not y such that $y \notin$ soft- π grp-cl(F,E).

Proof

Let x and y be two distinct points in a soft- π grp- T_2 -space X . Then there exists distinct soft- π grp-open sets (G,E) and (H,E) such that $x \in (G,E)$ and $y \in (H,E)$. This implies $x \in (H,E)'$. So, $(H,E)' = (F,E)$ is soft- π grp-closed set containing x but not y and soft- π grp-cl(F,E)= (F,E) . Hence $y \notin$ soft- π grp-cl(F,E). Let x and y be two distinct points of X . Then there exists a soft- π grp-open set (F,E) containing x but not y such that $y \notin$ soft- π grp-cl(F,E) $\Rightarrow y \in$ (soft- π grp-cl(F,E))'. Hence (F,E) and (soft- π grp-cl(F,E))' are disjoint soft- π grp- open sets containing x and y respectively. Therefore the space X is a soft- π grp- T_2 -space.

Definition: 3.10

A space X is said to be soft Urysohn space, if each pair of distinct soft points and x and y of X , there exist two soft open sets (U,E) and (V,E) such that $x \in (U,E)$ and $y \in (V,E)$ and $s\text{-cl}(U,E) \cap s\text{-cl}(V,E) = \emptyset$.

Theorem: 3.11

Let X and Y are soft spaces. If for each pair of distinct soft points x and y in X , there exist a function f of X into Y such that $f(x) \neq f(y)$, Y is an soft-Urysohn space and f is soft contra- π grp-continuous at x and y , then X is $s\text{-}\pi$ grp- T_2 -space.

Proof

Let x and y be any distinct soft points in X . Then, there exists a soft-Urysohn space Y and a function $f: X \rightarrow Y$ such that $f(x) \neq f(y)$ and f is soft contra- π grp-continuous at x and y . Let $z=f(x)$ and $v=f(y)$. Then $z \neq v$. Since Y is soft-Urysohn, there exist soft open sets (A,E) and (B,E) containing z and v , respectively such that $s\text{-cl}(A,E) \cap s\text{-cl}(B,E) = \emptyset$. Since f is soft-contra- π grp-continuous at x and y , then there exists- π grp-open sets (F,E) and (G,E) containing x and y respectively such that $f(F,E) \subseteq s\text{-cl}(A,E)$ and $f(G,E) \subseteq s\text{-cl}(B,E)$. Since $s\text{-cl}(A,E) \cap s\text{-cl}(B,E) = \emptyset$. We have $(F,E) \cap (G,E) = \emptyset$. Hence X is $s\text{-}\pi$ grp- T_2 -space.

Definition: 3.12

A space X is called soft- π grp-connected, if X is not the union of two disjoint nonempty $s\text{-}\pi$ grp-open sets.

Theorem: 3.13

A space X is soft- π grp-connected, if every soft-contra- π grp-continuous function from a space X into any soft T_0 -space Y is constant.

Proof

Suppose that X is not soft- π grp-connected and that every soft-contra- π grp-continuous function from X into Y is constant. Since X is not soft- π grp-connected, there exists a proper nonempty soft- π grp-clopen subset (A,E) of X . Then f is non-constant and soft-contra- π grp-continuous such that Y is soft T_0 -space, which is a contradiction. Hence, X must be soft- π grp-connected.

Theorem: 3.14

If $f: X \rightarrow Y$ is a soft-contra- π grp-continuous surjection and X is soft- π grp-connected, then Y is soft connected.

Proof

Suppose that Y is not a soft-connected space. Then there exist nonempty disjoint open sets (F,E) and (G,E) such that $Y = (F,E) \cup (G,E)$. Therefore (F,E) and (G,E) are soft-clopen in Y . Since f is soft-contra- π grp-continuous, $f^{-1}(F,E)$ and $f^{-1}(G,E)$ are soft- π grp-open in X . Moreover, $f^{-1}(F,E)$ and $f^{-1}(G,E)$ are nonempty disjoint and $X = f^{-1}(F,E) \cup f^{-1}(G,E)$. This shows that X is not soft- π grp-connected. This contradicts that Y is not soft-connected. Hence Y is soft-connected.

Definition: 3.15

A subset (A,E) of a soft space X is said to be soft- π grp-dense in X , if $s\text{-}\pi$ grp-cl $(A,E) = X$.

Theorem: 3.16

If $f: X \rightarrow Y$ and $g: X \rightarrow Y$ are soft-contra- π grp-continuous and Y is soft-Urysohn, then $(F,E) = \{x \in X : f(x) = g(x)\}$ is soft- π grp-closed in X .

Proof

Let $x \in X \setminus (F,E)$. Then $f(x) \neq g(x)$. Since Y is soft-Urysohn, there exist soft open sets (A,E) and (B,E) such that $f(x) \in (A,E)$, $g(x) \in (B,E)$ and $s\text{-cl}(A,E) \cap s\text{-cl}(B,E) = \emptyset$. Since f and g are soft-contra- π grp-continuous, $f^{-1}(s\text{-cl}(A,E)) \in S\pi$ GPO(X) and $g^{-1}(s\text{-cl}(B,E)) \in S\pi$ GPO(X). Let $(G,E) = f^{-1}(s\text{-cl}(A,E))$ and $(K,E) = g^{-1}(s\text{-cl}(B,E))$. Then (G,E) and (K,E) contains x . Set $(V,E) = (G,E) \cap (K,E)$. Then (V,E) is soft- π grp-open in X . Hence $f(V,E) \cap g(V,E) = \emptyset$ and $x \notin s\text{-}\pi$ grp-cl (F,E) . Thus (F,E) is soft- π grp-closed in X .

Theorem: 3.17

Let $f: X \rightarrow Y$ and $g: X \rightarrow Y$ is functions. If Y is soft Urysohn, f and g are soft-contra- π grp-continuous and $f = g$ on soft- π grp-dense set $(A,E) \subseteq X$, then $f = g$ on X .

Proof

Since f and g are soft-contra- π grp-continuous and Y is soft-Urysohn, by the previous theorem $(F,E) = \{x \in X : f(x) = g(x)\}$ is soft- π grp-closed in X . Then we have $f = g$ on soft- π grp-dense set $(A,E) \subseteq X$. Since $(A,E) \subseteq (F,E)$ and (A,E) is soft- π grp-dense set in X , then $X = s\text{-}\pi$ grp-cl $(A,E) \subseteq s\text{-}\pi$ grp-cl $(F,E) = (F,E)$. Hence $f = g$ on X .

4. Reference

1. Arokia Lancy and Arokiarani, on soft- β -separation axioms, International Jour. Of Math. Research and Sci. (IJMRS), 2013; 1(5):1-7.
2. Aras CG. Ayse Sonmez and Huseyin Cakalli, "On Soft mappings, arXiv: v1[Math. GM], 2013, 1305. 4545
3. Bin Chen, Soft semi-open sets and related properties in soft topological spaces, Appl. Math. Inf. Sci. 2013; 7(1):287-294.
4. christy V, Mohana K. Characterization of soft- π grp-closed sets in soft topological spaces, International Journal of Applied Research, 2016; 2(7):959-964.
5. christy V, Mohana K. On soft- π grp-continuous functions in Soft Topological Spaces, IJESC, 2016; 6(8):2605-2610.
6. Mahanta J, Das PK. On soft Topological space via Semi-open and semi-closed sets, arXiv: 2012, 1203.4133
7. Feng F. Jen YB, Zhao X. Soft semi rings, Computer and Mathematics with Application, 2008; 56:2621-2628.
8. Janaki C, Jeyanthi V. On soft π grp-closed sets in soft topological spaces. Journal of Advances in Mathematics, 4(3):478-485.
9. Janaki C, Jeyanthi V. On soft π grp-continuous functions in soft topological spaces, International Journal of Engineering Research and Technology, 2014; 3(2):762-768.
10. Kharal A, Ahmad B. Mappings of soft classes, New Math. Nat. Comput, 2011; 7(3):471- 481.
11. Maji PK, Biswas R, Roy R. An application of soft sets in a decision making problem, Comput. Math. Appl., 2003; 45:1077-1083.
12. Molodtsov D. Soft set theory first results, Comput. Math. Appl, 2003; 45:555-562.
13. Noiri T, Dontchev J. Quasi normal spaces and π grp-closed sets, Acta Math. Hungar, 2000; 89(3):211-219.
14. Shabir M, Naz M. On Soft topological spaces, Comp. And Math. with applications 2011; 61(7):1786-1799.

15. Zaitsev. On Certain classes of topological spaces and their bicompatifications, Dokl. Akad, Nauk. SSSR, 1968; 178:778-779.
16. Zorlutuna I, Akdag M, Min WK, Atmaca S. Remarks on soft topological spaces, Ann. Fuzzy Math. sInform. 2012; 3(2):171-185.