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## On soft- $\pi$ gp-separation axioms

V Christy and K Mohana

### Abstract

The aim of the paper is to study soft- $\pi$ gp-separation axioms and obtain some of their characterization.

**Keywords:** Soft- $\pi$ gp- $T_i$ -spaces. ( $i=0, 1, 2$ )

### 1. Introduction

General Topology plays a significant role in space time geometry as well as in different branches of Mathematics. Molodtsov<sup>[12]</sup> introduced soft set sets as a new mathematical tool for dealing with uncertainties which is free from the difficulties. A soft set is a collection of approximation descriptions of an object. The concept of  $\pi$ -closed sets in topological space was initiated by Zaitsav<sup>[15]</sup> and the concept of  $\pi$ g-closed set was introduced by Noiri and Dontchev<sup>[13]</sup>. Soft separation axioms for soft topological spaces were studied and introduced by Shabir *et al.*<sup>[14]</sup>. Arokia Lancy and Arockiarani. I<sup>[2]</sup> introduced soft- $\beta$ -separation axioms and derived some of its characterization. Soft semi-open sets and its properties along with soft semi-separation axioms were introduced and studied by Bin Chen<sup>[3]</sup>. Introduction of soft point along with the study on interior point, neighborhood system, continuity and compactness is found in due to Zorlutuna<sup>[16]</sup>. Many Researchers extended the result of generalization of various soft closed sets in many directions. P.K. Das<sup>[6]</sup>, Kharal *et al.*<sup>[10]</sup> developed soft function over classes of soft sets. Cigdem Gunduz Aras *et al.*,<sup>[1]</sup> in 2013 studied and discussed the properties of soft continuous mappings. V. Christy and K.Mohana<sup>[4]</sup> studied the characterization of soft- $\pi$ gp-closed sets in soft topological spaces.

In this paper, we investigate the soft- $\pi$ gp-separation axioms such as soft- $\pi$ gp- $T_i$  ( $i=0, 1, 2$ )-spaces.

### 2. Preliminaries

Let  $U$  be an initial universe set and  $E$  be a collection of all possible parameters with respect to  $U$ , where parameters are the characteristics or properties of objects in  $U$ . Let  $P(U)$  denote the power set of  $U$ , and let  $A \subseteq E$ .

**Definition: 2.1**<sup>[12]</sup> A pair  $(F, A)$  is called a soft set over  $U$ , where  $F$  is a mapping given by  $F: A \rightarrow P(U)$ . In other words, a soft set over  $U$  is a parameterized family of subsets of the universe  $U$ . For a particular  $e \in A$ ,  $F(e)$  may be considered the set of  $e$ -approximate elements of the soft set  $(F, A)$ .

**Definition: 2.2**<sup>[12]</sup> Two soft set  $(F, A)$  and  $(G, B)$  over a common universe  $U$  is said to be soft equal if  $(F, A)$  is a soft subset  $(G, B)$  and  $(G, B)$  is a soft subset of  $(F, A)$ .

**Definition: 2.3:**<sup>[11]</sup> A soft set  $(F, A)$  over  $X$  is said to be absolute soft set denoted by  $\bar{A}$  or  $X_A$ , if for all  $e \in A, F(e) = X$ . Clearly, we have  $X'_A = \phi_A$  and  $\phi'_A = X_A$

Let  $(F, E)$  be a soft set  $X$ . The soft set  $(F, E)$  is called a soft point, denoted by  $(X_e, E)$ , if for the element  $e \in E$ ,  $F(e) = \{x\}$  and  $F(e') = \phi$  for all  $e' \in E - \{e\}$ .

**Definition: 2.4**<sup>[11]</sup> For two soft sets  $(F, A)$  and  $(G, B)$  over a common universe  $U$ , Union of two soft sets of  $(F, A)$  and  $(G, B)$  is the soft set  $(H, C)$ , where  $C = A \cup B$ , and  $\forall e \in C$ ,

$$H(e) = \begin{cases} F(e) & , \text{if } e \in A - B \\ G(e) & , \text{if } e \in B - A \\ F(e) \cup G(e) & , \text{if } e \in A \cap B \end{cases}$$

We write  $(F,A) \cup (G,B) = (H,C)$ .

**Definition: 2.5** <sup>[14]</sup> Let  $\tau$  be the collection of soft sets over  $X$ , and then  $\tau$  is called a soft topology on  $X$  if  $\tau$  satisfies the following axioms:

- (i)  $\phi, \tilde{X}$  belong to  $\tau$
- (ii) The union of any number of soft sets in  $\tau$  belongs to  $\tau$ .
- (iii) The intersection of any two soft sets in  $\tau$  belongs to  $\tau$ .

The triplet  $(X, \tau, E)$  is called a soft topological space over  $X$ . Let  $(X, \tau, E)$  be a soft space over  $X$ , then the members of  $\tau$  are said to be soft open sets in  $X$ .

**Definition: 2.6** <sup>[6]</sup> Let  $(X, \tau, E)$  be a soft topological space over  $X$  and the soft interior of  $(F, E)$  denoted by  $\text{Int}(F, E)$  is the union of all soft open subsets of  $(F, E)$ . Clearly,  $(F, E)$  is the largest soft open set over  $X$  which is contained in  $(F, E)$ . The soft closure of  $(F, E)$  denoted by  $\text{Cl}(F, E)$  is the intersection of all closed sets containing  $(F, E)$ . Clearly,  $(F, E)$  is smallest soft closed set containing  $(F, E)$ .

$$\text{Int}(F, E) = \cup \{ (O, E) : (O, E) \text{ is soft open and } (O, E) \subseteq (F, E) \}.$$

$$\text{Cl}(F, E) = \cap \{ (O, E) : (O, E) \text{ is soft closed and } (F, E) \subseteq (O, E) \}.$$

**Definition: 2.7** <sup>[7]</sup> The Intersection  $(H, C)$  of two soft sets  $(F, A)$  and  $(G, B)$  over a common universe  $U$  denoted  $(F, A) \cap (G, B)$  is defined as  $C = A \cap B$  and  $H(e) = F(e) \cap G(e)$  for all  $e \in C$ .

**Definition: 2.8** <sup>[14]</sup>

- 1. a soft-semi open <sup>[9]</sup>, if  $(A, E) \subseteq \text{Cl}(\text{Int}(A, E))$
- 2. a soft-regular open <sup>[7]</sup>, if  $(A, E) = \text{Int}(\text{Cl}(A, E))$ .
- 3. a soft-pre-open <sup>[7]</sup>, if  $(A, E) \subseteq \text{Int}(\text{Cl}(A, E))$ .
- 4. a soft-clopen <sup>[9]</sup>, if  $(A, E)$  is both soft open and soft closed.

The complement of the soft semi open, soft regular open, soft pre-open sets are their respective soft semi closed, soft regular closed and soft pre-closed sets.

The finite union of soft regular open sets is called soft  $\pi$ -open set and its complement is soft- $\pi$ -closed set. The soft regular open set of  $X$  is denoted by  $\text{SRO}(X)$  or  $\text{SRO}(X, \tau, E)$ .

**Definition: 2.9** <sup>[8]</sup> Let  $(X, \tau, E)$  and  $(Y, \tau^*, E)$  be two soft topological spaces. A function  $f: (X, \tau, E) \rightarrow (Y, \tau^*, E)$  is said to be Soft-continuous <sup>[14]</sup>, if  $f^{-1}(F, E)$  is soft-open in  $(X, \tau, E)$ , for every soft-open set  $(F, E)$  of  $(Y, \tau^*, E)$ .

**Definition: 2.10** <sup>[4]</sup>

- 1. A soft subset  $(A, E)$  of a soft topological space  $X$  is called soft- $\pi$ gp-closed set in  $X$  if  $\text{spcl}(A, E) \subseteq (U, E)$  whenever  $(A, E) \subseteq (U, E)$  and  $(U, E)$  is soft- $\pi$ -open in  $X$ .
- 2. Let  $X$  and  $Y$  be two topological spaces and the function  $f: X \rightarrow Y$ . Then the function  $f$  is soft- $\pi$ gp-irresolute if  $f^{-1}(F, E)$  is soft- $\pi$ gp-open in  $X$ , for every soft- $\pi$ gp-open set  $(F, E)$  of  $Y$ .

**Definition 2.11:** <sup>[5]</sup> Let  $(X, \tau, E)$  and  $(Y, \tau^*, E)$  be two soft topological spaces and  $f: (X, \tau, E) \rightarrow (Y, \tau^*, E)$  be a function. Then the function  $f$  is said to be soft- $\pi$ gp-continuous function if  $f^{-1}(G, E)$  is soft- $\pi$ gp-closed(open) set in  $(X, \tau, E)$  for every soft-closed(open) set  $(G, E)$  of  $(Y, \tau^*, E)$ .

**Definition: 2.12** <sup>[5]</sup>: Let  $(X, \tau, E)$  and  $(Y, \tau^*, E)$  be two soft topological spaces and  $f: (X, \tau, E) \rightarrow (Y, \tau^*, E)$  be a function. Then the function  $f$  is soft-contra- $\pi$ gp-continuous if  $f^{-1}(F, E)$  is soft- $\pi$ gp-closed in  $(X, \tau, E)$  for every soft-open  $(F, E)$  in  $(Y, \tau^*, E)$

**Definition: 2.13:** <sup>[5]</sup>

- 1. The Soft  $\pi$ gp-Closure of a soft set  $(G, E)$  is defined to be the intersection of all soft  $\pi$ gp-closed sets containing the soft set  $(G, E)$  and is denoted by  $s\text{-}\pi\text{gp-cl}(G, E)$ .
- 2. The Soft  $\pi$ gp-Interior of a soft set  $(G, E)$  is defined to be the union of all soft  $\pi$ gp-open sets contained the soft set  $(G, E)$  and is denoted by  $s\text{-}\pi\text{gp-int}(G, E)$ .

**Definition: 2.14:** <sup>[9]</sup> A function  $f: (X, \tau, E) \rightarrow (Y, \tau^*, E)$  is called soft regular open if  $f((F, A))$  is soft-regular open (soft-regular-closed) if  $f(F, A)$  is soft-regular-open (soft-regular-closed) in  $Y$ , for every soft-open (soft-closed) set of  $X$ .

**Definition: 2.15:** <sup>[9]</sup>

- 1. A soft topological space  $X$  is said to be soft- $T_0$ -space if for two disjoint points  $x$  and  $y$ , there exist a soft open sets  $(F, E)$  and  $(G, E)$  containing one but not the other.
- 2. A soft topological space  $X$  is said to be soft- $T_1$ -space if for two disjoint points  $x$  and  $y$  of  $X$ , there exist a soft open sets  $(F, E)$  and  $(G, E)$  such that  $x \in (F, E)$ ,  $y \notin (F, E)$  and  $y \in (G, E)$ ,  $x \notin (G, E)$ .
- 3. A soft topological space  $X$  is said to be soft- $T_2$ -space if for two disjoint points  $x$  and  $y$  of  $X$ , there exist a disjoint open sets  $(F, E)$  and  $(G, E)$  such that  $x \in (F, E)$  and  $y \in (G, E)$ .

**Definition: 2.16:** <sup>[11]</sup> A space  $X$  is called soft-connected if  $X$  is not the union of two disjoint non-empty soft-open sets. Throughout this paper we denote  $(X, \tau, E)$ ,  $(Y, \tau^*, E)$  and  $(Z, \tau^{**}, E)$  as  $X, Y$  and  $Z$ .

### 3. Soft- $\pi$ gp-separation axioms

**Definition: 3.1**

- 1. A soft topological space  $X$  is said to be soft- $\pi$ gp- $T_0$ -space if for two disjoint points  $x$  and  $y$  of  $X$ , there exists a soft- $\pi$ gp- open sets  $(F, E)$  and  $(G, E)$  such that  $x \in (F, E)$  or  $y \in (F, E)$  and  $y \in (G, E)$  or  $x \in (G, E)$ .
- 2. A soft topological space  $X$  is said to be soft- $\pi$ gp- $T_1$ -space if for two disjoint points  $x$  and  $y$  of  $X$ , there exists a soft- $\pi$ gp- open sets  $(F, E)$  and  $(G, E)$  such that  $x \in (F, E)$  and  $y \notin (F, E)$  and  $y \in (G, E)$  and  $x \notin (G, E)$ .
- 3. A soft topological space  $X$  is said to be soft- $\pi$ gp- $T_2$ -space if for two disjoint points  $x$  and  $y$  of  $X$ , there exists a disjoint soft- $\pi$ gp- open sets  $(F, E)$  and  $(G, E)$  such that  $x \in (F, E)$  and  $y \in (G, E)$ .

**Theorem: 3.2**

A soft space  $X$  is soft- $\pi$ gp- $T_0$ -space iff soft  $\pi$ gp-closures of distinct points are distinct.

**Proof:** Let  $x$  and  $y$  be distinct points of  $X$ . Since  $X$  is a soft- $\pi$ gp- $T_0$ -space, there exists a soft  $\pi$ gp-open set  $(F, E)$  such

that  $x \in (F,E)$  and  $y \notin (F,E)$ . Consequently,  $X-(F,E)$  is a soft- $\pi$ grp-closed set containing  $y$  but not  $x$ . But soft- $\pi$ grp-cl( $y$ ) is the intersection of all soft- $\pi$ grp-closed sets containing  $y$ . Hence  $y \in$  soft- $\pi$ grp-cl( $y$ ), but  $x \notin$  soft- $\pi$ grp-cl( $y$ ) as  $x \notin X-(F,E)$ . Therefore, soft- $\pi$ grp-cl( $x$ )  $\neq$  soft- $\pi$ grp-cl( $y$ ). Conversely, let soft- $\pi$ grp-cl( $x$ )  $\neq$  soft- $\pi$ grp-cl( $y$ ) for  $x \neq y$ . Then there exists at least one point  $z \in X$  such that  $z \notin$  soft- $\pi$ grp-cl( $y$ ). We have to prove  $x \notin$  soft- $\pi$ grp-cl( $y$ ), because if  $x \in$  soft- $\pi$ grp-cl( $y$ ), then  $\{x\} \subset$  soft- $\pi$ grp-cl( $y$ )  $\Rightarrow$  soft- $\pi$ grp-cl( $x$ )  $\subset$  soft- $\pi$ grp-cl( $y$ ). So,  $z \in$  soft- $\pi$ grp-cl( $y$ ), which is a contradiction. Hence  $x \notin$  soft- $\pi$ grp-cl( $y$ )  $\Rightarrow x \in X -$  soft- $\pi$ grp-cl( $y$ ), which is a soft- $\pi$ grp-open set containing  $x$  but not  $y$ . Hence  $X$  is a soft- $\pi$ grp- $T_0$ -space.

**Theorem: 3.3**

A soft space  $X$  is soft- $\pi$ grp- $T_1$ -space iff each singleton is soft- $\pi$ grp-closed set.

**Proof:** Let  $X$  be a soft- $\pi$ grp- $T_1$ -space and  $x \in X$ . Let  $y \in X-\{x\}$ . Then for  $x \neq y$ , there exists soft- $\pi$ grp-open set  $U_y$  such that  $y \in U_y$  and  $x \notin U_y$ . Conversely,  $y \in U_y \subset X-\{x\}$ . That is  $X-\{x\} = \cup\{U_y : y \in X-\{x\}\}$ , which is soft- $\pi$ grp-open set. Hence  $\{x\}$  is soft- $\pi$ grp-closed set. Conversely, suppose  $\{x\}$  is soft- $\pi$ grp-closed set for every  $x \in X$ . Let  $x, y \in X$  with  $x \neq y$ . Now,  $x \neq y \Rightarrow y \in X-\{x\}$ . Hence  $X-\{x\}$  is soft- $\pi$ grp-open set containing  $y$  but not  $x$ . Similarly,  $X-\{y\}$  is soft- $\pi$ grp-open set containing  $x$  but not  $y$ . Therefore,  $X$  is a soft- $\pi$ grp- $T_1$ -space.

**Theorem: 3.4**

If every point ( $x,E$ ) of  $X$  is a soft- $\pi$ grp-closed, then  $X$  is a soft- $\pi$ grp- $T_1$ -space.

**Proof:** Let ( $x,E$ ) and ( $y,E$ ) be two distinct points of  $X$  and by hypothesis ( $x,E$ ) and ( $y,E$ ) are soft  $\pi$ grp-closed. Then ( $x,E$ )' and ( $y,E$ )' are soft- $\pi$ grp-open such that ( $y,E$ )  $\in$  ( $x,E$ )', ( $y,E$ )  $\notin$  ( $x,E$ ) and ( $x,E$ )  $\in$  ( $y,E$ )', ( $x,E$ )  $\notin$  ( $y,E$ )'. Hence the soft topological space  $X$  is a soft- $\pi$ grp- $T_1$ -space.

**Theorem: 3.5**

If  $f: X \rightarrow Y$  is soft- $\pi$ grp -continuous injection and  $Y$  is soft- $T_1$ -space, then  $X$  is soft- $\pi$ grp-  $T_1$ - space.

**Proof:** Let  $f: X \rightarrow Y$  be a soft- $\pi$ grp- continuous injection and  $Y$  be soft- $T_1$  -space. For any two distinct points  $x_1, x_2$  of the soft space  $X$ , there exists distinct points  $y_1, y_2$  of the soft space  $Y$  such that  $y_1 = f(x_1)$  and  $y_2 = f(x_2)$ . Since  $Y$  is soft- $T_1$ - space, there exists soft open sets ( $F, E$ ) and ( $G, E$ ) in  $Y$  such that  $y_1 \in (F,E)$  and  $y_2 \notin (F,E)$  and  $y_1 \notin (G,E)$ ,  $y_2 \in (G,E)$ . i.e.  $x_1 \in f^{-1}((F,E))$ ,  $x_1 \notin f^{-1}((G,E))$  and  $x_2 \in f^{-1}((G,E))$ ,  $x_2 \notin f^{-1}((F,E))$ . Since  $f$  is soft- $\pi$ grp - continuous,  $f^{-1}((F,E))$ ,  $f^{-1}((G,E))$  are soft- $\pi$ grp -open sets in  $X$ . Thus for two distinct points  $x_1, x_2$  of the soft space  $X$ , there exists soft- $\pi$ grp - open sets  $f^{-1}((F,E))$  and  $f^{-1}((G,E))$  such that  $x_1 \in f^{-1}((F,E))$ ,  $x_2 \notin f^{-1}((F,E))$  and  $x_2 \in f^{-1}((G,E))$ ,  $x_2 \notin f^{-1}((F,E))$ . Therefore,  $X$  is a soft- $\pi$ grp - $T_1$  - space.

**Theorem: 3.6**

If  $f : X \rightarrow Y$  be soft- $\pi$ grp -irresolute function, and  $Y$  is soft- $\pi$ grp -  $T_1$  - space, then  $X$  is soft- $\pi$ grp - $T_1$  -space.

**Proof:** Let  $x_1, x_2$  be two distinct points in a soft space  $X$ . Since  $f$  is injective, there exists distinct points  $y_1, y_2$  of a soft space  $Y$  such that  $y_1 = f(x_1)$  and  $y_2 = f(x_2)$ . Since  $Y$  is soft- $\pi$ grp - $T_1$  -space, there exists soft- $\pi$ grp- open sets ( $F,E$ ) and ( $G,E$ ) in  $Y$  such that  $y_1 \in (F,E)$  and  $y_2 \notin (F,E)$  and  $y_1 \notin (G,E)$ ,  $y_2 \in (G,E)$ . i.e.  $x_1 \in f^{-1}((F,E))$ ,  $x_1 \notin f^{-1}((G,E))$  and  $x_2 \in f^{-1}((G,E))$ ,  $x_2 \notin f^{-1}((F,E))$ . Since  $f$  is soft- $\pi$ grp- irresolute,  $f^{-1}((F,E))$ ,  $f^{-1}((G,E))$  are soft- $\pi$ grp - open sets in  $X$ . Thus, for two distinct points  $x_1, x_2$  of  $X$ , there exists soft- $\pi$ grp- open sets  $f^{-1}((F,E))$  and  $f^{-1}((G,E))$  such that  $x_1 \in f^{-1}((F,E))$ ,  $x_1 \notin f^{-1}((G,E))$  and  $x_2 \in f^{-1}((G,E))$ ,  $x_2 \notin f^{-1}((F,E))$ . Hence  $X$  is a soft- $\pi$ grp- $T_1$ - space.

**Theorem: 3.7**

If  $f : X \rightarrow Y$  be soft- $\pi$ grp -continuous injection, and  $Y$  is soft- $T_2$ -space, then  $X$  is soft- $\pi$ grp - $T_2$  -space.

**Proof:** Let  $f : X \rightarrow Y$  be soft- $\pi$ grp- continuous injection and  $Y$  be soft- $T_2$ -space. Then for any two distinct points  $x_1$  and  $x_2$  of a soft space  $X$ , there exists distinct points  $y_1, y_2$  of a soft space  $Y$  such that  $y_1 = f(x_1)$ ,  $y_2 = f(x_2)$ . Since  $Y$  is soft- $T_2$  -space, there exists disjoint soft open sets ( $F,E$ ) and ( $G,E$ ) in  $Y$  such that  $y_1 \in (F,E)$  and  $y_2 \in (G,E)$ . i.e.  $x_1 \in f^{-1}((F,E))$ ,  $x_2 \in f^{-1}((G,E))$ . Since  $f$  is soft- $\pi$ grp - continuous,  $f^{-1}((F,E))$  &  $f^{-1}((G,E))$  are soft- $\pi$ grp-open sets in  $X$ . since  $f$  is soft injective,  $f^{-1}((F,E)) \cap f^{-1}((G,E)) = f^{-1}((F,E) \cap (G,E)) = f^{-1}(\emptyset) = \emptyset$ . Thus, for two disjoint points  $x_1, x_2$  of  $X$ , there exists disjoint soft- $\pi$ grp-open sets  $f^{-1}((F,E))$  and  $f^{-1}((G,E))$  such that  $x_1 \in f^{-1}((F,E))$  and  $x_2 \in f^{-1}((G,E))$ . Hence  $X$  is soft- $\pi$ grp- $T_2$ -space.

**Theorem: 3.8**

If  $f: X \rightarrow Y$  be the soft- $\pi$ grp-irresolute injective function and  $Y$  is soft- $\pi$ grp-  $T_2$  -space, then  $X$  is soft- $\pi$ grp- $T_2$ -space.

**Proof:** Let  $x_1, x_2$  be any two distinct points in a soft space  $X$ . Since  $f$  is soft injective, there exists distinct points  $y_1, y_2$  of a soft space  $Y$  such that  $y_1 = f(x_1)$  and  $y_2 = f(x_2)$ . Since  $Y$  is soft- $\pi$ grp- $T_2$ -space, there exist disjoint soft- $\pi$ grp-open sets ( $F,E$ ) and ( $G,E$ ) in  $Y$  such that  $y_1 \in (F,E)$  and  $y_2 \in (G,E)$ . i.e.  $x_1 \in f^{-1}((F,E))$ ,  $x_2 \in f^{-1}((G,E))$ . Since  $f$  is soft- $\pi$ grp-irresolute,  $f^{-1}((F,E))$ ,  $f^{-1}((G,E))$  are disjoint soft- $\pi$ grp-open sets in  $X$ . Thus, for two distinct points  $x_1, x_2$  of  $X$ , there exists disjoint soft- $\pi$ grp-open sets  $f^{-1}((F,E))$  and  $f^{-1}((G,E))$  such that  $x_1 \in f^{-1}((F,E))$  and  $x_2 \in f^{-1}((G,E))$ . Hence  $X$  is a soft- $\pi$ grp- $T_2$ -space.

**Theorem: 3.9**

A soft topological space  $X$  is soft- $\pi$ grp- $T_2$  -space iff for distinct points  $x, y$  of  $X$ , there exists a soft- $\pi$ grp-open set ( $F,E$ ) containing  $x$  but not  $y$  such that  $y \notin$  soft- $\pi$ grp-cl( $F,E$ ).

**Proof**

Let  $x$  and  $y$  be two distinct points in a soft- $\pi$ grp- $T_2$ -space  $X$ . Then there exists distinct soft- $\pi$ grp-open sets ( $G,E$ ) and ( $H,E$ ) such that  $x \in (G,E)$  and  $y \in (H,E)$ . This implies  $x \in (H,E)'$ . So,  $(H,E)' = (F,E)$  is soft- $\pi$ grp-closed set containing  $x$  but not  $y$  and soft- $\pi$ grp-cl( $F,E$ )= $(F,E)$ . Hence  $y \notin$  soft- $\pi$ grp-cl( $F,E$ ). Let  $x$  and  $y$  be two distinct points of  $X$ . Then there exists a soft- $\pi$ grp-open set ( $F,E$ ) containing  $x$  but not  $y$  such that  $y \notin$  soft- $\pi$ grp-cl( $F,E$ )  $\Rightarrow y \in$  (soft- $\pi$ grp-cl( $F,E$ ))'. Hence ( $F,E$ ) and (soft- $\pi$ grp-cl( $F,E$ ))' are disjoint soft- $\pi$ grp- open sets containing  $x$  and  $y$  respectively. Therefore the space  $X$  is a soft- $\pi$ grp- $T_2$ -space.

**Definition: 3.10**

A space  $X$  is said to be soft Urysohn space, if each pair of distinct soft points and  $x$  and  $y$  of  $X$ , there exist two soft open sets  $(U,E)$  and  $(V,E)$  such that  $x \in (U,E)$  and  $y \in (V,E)$  and  $s\text{-cl}(U,E) \cap s\text{-cl}(V,E) = \emptyset$ .

**Theorem: 3.11**

Let  $X$  and  $Y$  are soft spaces. If for each pair of distinct soft points  $x$  and  $y$  in  $X$ , there exist a function  $f$  of  $X$  into  $Y$  such that  $f(x) \neq f(y)$ ,  $Y$  is an soft-Urysohn space and  $f$  is soft contra- $\pi$ grp-continuous at  $x$  and  $y$ , then  $X$  is  $s\text{-}\pi$ grp- $T_2$ -space.

**Proof**

Let  $x$  and  $y$  be any distinct soft points in  $X$ . Then, there exists a soft-Urysohn space  $Y$  and a function  $f: X \rightarrow Y$  such that  $f(x) \neq f(y)$  and  $f$  is soft contra- $\pi$ grp-continuous at  $x$  and  $y$ . Let  $z=f(x)$  and  $v=f(y)$ . Then  $z \neq v$ . Since  $Y$  is soft-Urysohn, there exist soft open sets  $(A,E)$  and  $(B,E)$  containing  $z$  and  $v$ , respectively such that  $s\text{-cl}(A,E) \cap s\text{-cl}(B,E) = \emptyset$ . Since  $f$  is soft-contra- $\pi$ grp-continuous at  $x$  and  $y$ , then there exists- $\pi$ grp-open sets  $(F,E)$  and  $(G,E)$  containing  $x$  and  $y$  respectively such that  $f(F,E) \subseteq s\text{-cl}(A,E)$  and  $f(G,E) \subseteq s\text{-cl}(B,E)$ . Since  $s\text{-cl}(A,E) \cap s\text{-cl}(B,E) = \emptyset$ . We have  $(F,E) \cap (G,E) = \emptyset$ . Hence  $X$  is  $s\text{-}\pi$ grp- $T_2$ -space.

**Definition: 3.12**

A space  $X$  is called soft- $\pi$ grp-connected, if  $X$  is not the union of two disjoint nonempty  $s\text{-}\pi$ grp-open sets.

**Theorem: 3.13**

A space  $X$  is soft- $\pi$ grp-connected, if every soft-contra- $\pi$ grp-continuous function from a space  $X$  into any soft  $T_0$ -space  $Y$  is constant.

**Proof**

Suppose that  $X$  is not soft- $\pi$ grp-connected and that every soft-contra- $\pi$ grp-continuous function from  $X$  into  $Y$  is constant. Since  $X$  is not soft- $\pi$ grp-connected, there exists a proper nonempty soft- $\pi$ grp-clopen subset  $(A,E)$  of  $X$ . Then  $f$  is non-constant and soft-contra- $\pi$ grp-continuous such that  $Y$  is soft  $T_0$ -space, which is a contradiction. Hence,  $X$  must be soft- $\pi$ grp-connected.

**Theorem: 3.14**

If  $f: X \rightarrow Y$  is a soft-contra- $\pi$ grp-continuous surjection and  $X$  is soft- $\pi$ grp-connected, then  $Y$  is soft connected.

**Proof**

Suppose that  $Y$  is not a soft-connected space. Then there exist nonempty disjoint open sets  $(F,E)$  and  $(G,E)$  such that  $Y = (F,E) \cup (G,E)$ . Therefore  $(F,E)$  and  $(G,E)$  are soft-clopen in  $Y$ . Since  $f$  is soft-contra- $\pi$ grp-continuous,  $f^{-1}(F,E)$  and  $f^{-1}(G,E)$  are soft- $\pi$ grp-open in  $X$ . Moreover,  $f^{-1}(F,E)$  and  $f^{-1}(G,E)$  are nonempty disjoint and  $X = f^{-1}(F,E) \cup f^{-1}(G,E)$ . This shows that  $X$  is not soft- $\pi$ grp-connected. This contradicts that  $Y$  is not soft-connected. Hence  $Y$  is soft-connected.

**Definition: 3.15**

A subset  $(A,E)$  of a soft space  $X$  is said to be soft- $\pi$ grp-dense in  $X$ , if  $s\text{-}\pi$ grp-cl $(A,E) = X$ .

**Theorem: 3.16**

If  $f: X \rightarrow Y$  and  $g: X \rightarrow Y$  are soft-contra- $\pi$ grp-continuous and  $Y$  is soft-Urysohn, then  $(F,E) = \{x \in X : f(x) = g(x)\}$  is soft- $\pi$ grp-closed in  $X$ .

**Proof**

Let  $x \in X \setminus (F,E)$ . Then  $f(x) \neq g(x)$ . Since  $Y$  is soft-Urysohn, there exist soft open sets  $(A,E)$  and  $(B,E)$  such that  $f(x) \in (A,E)$ ,  $g(x) \in (B,E)$  and  $s\text{-cl}(A,E) \cap s\text{-cl}(B,E) = \emptyset$ . Since  $f$  and  $g$  are soft-contra- $\pi$ grp-continuous,  $f^{-1}(s\text{-cl}(A,E)) \in S\pi$ GPO( $X$ ) and  $g^{-1}(s\text{-cl}(B,E)) \in S\pi$ GPO( $X$ ). Let  $(G,E) = f^{-1}(s\text{-cl}(A,E))$  and  $(K,E) = g^{-1}(s\text{-cl}(B,E))$ . Then  $(G,E)$  and  $(A,E)$  contains  $x$ . Set  $(V,E) = (G,E) \cap (K,E)$ . Then  $(V,E)$  is soft- $\pi$ grp-open in  $X$ . Hence  $f(V,E) \cap g(V,E) = \emptyset$  and  $x \notin s\text{-}\pi$ grp-cl $(F,E)$ . Thus  $(F,E)$  is soft- $\pi$ grp-closed in  $X$ .

**Theorem: 3.17**

Let  $f: X \rightarrow Y$  and  $g: X \rightarrow Y$  is functions. If  $Y$  is soft Urysohn,  $f$  and  $g$  are soft-contra- $\pi$ grp-continuous and  $f = g$  on soft- $\pi$ grp-dense set  $(A,E) \subseteq X$ , then  $f = g$  on  $X$ .

**Proof**

Since  $f$  and  $g$  are soft-contra- $\pi$ grp-continuous and  $Y$  is soft-Urysohn, by the previous theorem  $(F,E) = \{x \in X : f(x) = g(x)\}$  is soft- $\pi$ grp-closed in  $X$ . Then we have  $f = g$  on soft- $\pi$ grp-dense set  $(A,E) \subseteq X$ . Since  $(A,E) \subseteq (F,E)$  and  $(A,E)$  is soft- $\pi$ grp-dense set in  $X$ , then  $X = s\text{-}\pi$ grp-cl $(A,E) \subseteq s\text{-}\pi$ grp-cl $(F,E) = (F,E)$ . Hence  $f = g$  on  $X$ .

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