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The effect of multicollinearity in nonlinear regression models

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Abstract
Regression Analysis having many procedures for modeling and analyzing the relationship between a dependent variable and one or more explanatory variables. Linear and nonlinear regression models have extensively used in many fields of business science. One of the impartment problems in regression Analysis is multicollinearity between the explanatory variables. If there is no linear relationship between the explanatory variables, they are said to be orthogonal model. In the case of orthogonal variables, statistical assumption on the model is relatively reliable. But in day today life, fully unbound variables which are explaining the dependent variable are likely to be very low. When the explanatory variables are not orthogonal, then least squares parameter estimation method will not provide an appropriate junction, and deviations from reality will occur.

Keywords: Multicollinearity, nonlinear regression, ridge regression

1. Introduction
Non-linear Regression models provide wealthy and flexible structure that suits many analysts. Non-linear Regression analysis/models are one of the most widely used statistical techniques for establishing the relationship between variables. Nonlinear regression models having several of applications in various fields like mathematical, physical, Biological, Social Sciences, Economic and Management Sciences. A great deal of non-linear Regression problem is that, exploration of the relationship between the independent and dependent variables. A large uncultivated area of research is Business and Economics point of view, the finite sample of properties of nonlinear parameters. The inferential problems on nonlinear models have been intensively studied in the last three decades. The nonlinear estimation techniques and the error assumptions are usually parallel to those made for the linear model. There is another area that is commonly seen in the data that are best fit by a non-linear regression model.
Multicollinearity is takes place where two or more explanatory variables are correlated linearly by a strong linear relationship, that it is not easy to separate the effect of each of them from the Response variable. However, severe multicollinearity is one of the difficulties because it can increase the variance of the coefficient estimates and make the estimates very responsive to slight changes in the model. The effect is that the coefficient estimates are unstable and difficult to interpret. Multicollinearity undermines the statistical power of the analysis, can source the coefficients to switch signs, and makes it more difficult to specify the correct model.
In the present work, the main aim is to observe the effect of multicollinearity in nonlinear models.

2. General linear model
Generally the Multiple Linear Models

$$Y = X\beta + \in \qquad 2.1$$

Her Y is nx1 vector of dependent variable, X is nxp matrix of independent variable, β is px1 vector of regression coefficients and $\in$ is nx1 error term, and $\in \sim N\left(0, \sigma^2\right)$.

In order to estimate the $'\beta'$ least square method is one of the best methods. So, the sum of squares error is

$$S(\beta) = \sum_{i=1}^{n} \in^1 \in = (Y - X\beta)^1 (Y - X\beta)$$

Then $\hat{\beta} = \left(X^1X\right)^{-1} X^1Y$ 2.2

Here $\hat{\beta}$ is the Least Square Estimator of β and it the unbiased estimator with minimum variance.

3. Multicollinearly Problem

In present days multiple regression models and multicollinearity plays crucial role in observational studies. The general problem is the some of the explanatory variables are related to each other. Let $X = [X_1, X_2 .....X_P].$ So multicollinearity exist in terms of linear dependency in the columns of X. The vectors X_l, X_2…. X_P are linearly dependent and we have set of constants V_1, V_2…… V_p not all zero, such that $\sum_{i=1}^{p} V_i X_i \approx 0.$

3.1. Effect of Multicollinearity

In least square estimation of parameters the multicollinearity shows serious effect. Since the variable parameter is.

$$V(\hat{\beta}) = \sigma^2 \left(X^1X\right)^{-1} = \sigma^2 \frac{1}{1 - R_i^2} \quad \left(R_i^2 \le 1\right) \quad i=1, 2….p$$

Here $\hat{\beta}$ is least square estimator of β and $\frac{1}{1-R_i^2}$ is diagonal element of $(X^1X)^{-1}$ matrix and $'R_i^2'$ is the coefficient of multiple determination of $'X_i'$ on the remaining (p-1) regressor variables if there is strong multicollinearity exist between X_i and other regressor variable, the R_i^2 value is close to unity $\left(R_i^2 \to 1\right)$ and diagonal elements of $\left(X^1X\right)^{-1}$ is close to $\infty \left(\frac{1}{1-R_i^2} \to \infty\right)$

In the measure of multicollinearity, Mean Square Error (MSE) process is widely used to compare the different estimators of a parameters.

$$\therefore \text{MSE}\,\hat{\beta} = E\left\{\left(\hat{\beta} - \beta\right)^1 \left(\hat{\beta} - \beta\right)\right\}$$

$$= \sum_{i=1}^{p} \left(\hat{\beta}_i - \beta_i\right)^2$$

$$= \hat{\sigma}^2 \,\text{Trace}\left(X^1X\right)^{-1}$$

$$= \hat{\sigma}^2 \sum_{i=1}^{P} \frac{1}{\lambda_i} \qquad \lambda_i > 0, i = 1, 2.....p$$

Here $\hat{\sigma}^2$ is mean square error if $(X^1X)^{-1}$ is a singular, the OLS estimation tend to become wrong sign (Wethrill 1986). In order to overcome this difficulty of OLS, Hoerl and Kennard (1970) suggested the Ridge Regression Model to estimate the OLS method in regression analysis, especially multicollinearity is exist.

4. Ridge Regression

In 1970, Hoerl and Kennard were proposed the Ridge Regression Estimation Method to reduce the level of multicollinearity in the model. The main difference between the Ridge regression and OLS method is, eliminating the condition of neutrality. Based on the Gauss-Markove theorem, the least square estimator has the minimum variance between the all linear unbiased estimators, but there is no guarantee. So Ridge regression provides the parameter with minimum variance, by adding the slight bias to the estimation.

In general, MSE is

$$\text{MSE}\left(\hat{\beta}\right) = E\left(\hat{\beta} - \beta\right)^2 = \text{Var}\left(\hat{\beta}\right) + \left[E\left(\hat{\beta}\right) - \beta\right]^2$$

$$= \mathrm{Var}\left(\hat{\beta}\right) + \left[\mathrm{bais}\left(\hat{\beta}\right)\right]^2$$

Here $\hat{\beta}$ is an estimator β. If $\hat{\beta}$ is unbiased estimator of β then

MSE $\left(\hat{\beta}\right) = \mathrm{Var}\left(\hat{\beta}\right)$

The Ridge estimator β is

$$\hat{\beta}(\mathrm{k}) = \left(\mathrm{X}^1\mathrm{X} + \mathrm{KI}\right)^{-1}\mathrm{X}^1\mathrm{Y} \qquad \mathrm{K} \geq 0 \tag{4.1}$$

So

$$\left\{\mathrm{E}\left[\hat{\beta}(\mathrm{k}) - \beta\right]\right\}^2 < \mathrm{Var}\left(\hat{\beta}\right) - \mathrm{V}\left(\hat{\beta}(\mathrm{k})\right) = \mathrm{K}\left[\mathrm{Var}\left(\hat{\beta}\right) + \sigma^2\sum_{i=1}^{p}\frac{1}{\lambda_i + \mathrm{k}}\right]$$

So MSE $\left(\hat{\beta}\right) > \mathrm{MSE}\left[\hat{\beta}(\mathrm{K})\right]$

So the bias and variance of the parameters depends on the value of 'k' it is clear that, if the variance decreases, bias shows an increasing trend due to the appropriate value of 'k'. Many techniques have been developed for the selection of bias parameter 'k'. Ridge regression is one of the methods to determine the appropriate value of 'k'.

5. Estimation process in nonlinear models

The non-linear regression model is

$$\mathrm{Y} = \mathrm{f}\left(\mathrm{X}, \beta\right) + \in \tag{5.1}$$

Here is px1 vector of unknown parameter and $\in$ is error term follows $\mathrm{N}\left(0, \sigma^2\mathrm{I}\right)$. In non-linear regression models, one of the methods that can be used to estimate the parameters is least square method.

So,

$$\mathrm{S}(\beta) = \sum_{i=1}^{n}\left\{\mathrm{Y}_i - \mathrm{f}\left(\mathrm{X}_i, \beta\right)\right\}^2$$

to solve 'p' normal equations.

$$= \sum_{i=1}^{n}\left\{\mathrm{Y}_i - \mathrm{f}\left(\mathrm{X}_i, \beta\right)\right\}\left[\frac{\partial \mathrm{f}\left(\mathrm{X}_i, \beta\right)}{\partial \beta_j}\right]_{\beta - \hat{\beta}} = 0 \qquad j = 1, 2 \ldots. p$$

The normal equation system is quite difficult in nonlinear models. In order to over this difficulty, we have to use Gauss – Newton method to estimate the parameters. In this method we have to use Iterative Least Squares in order to change the linear model to approach the nonlinear model.

By using Taylor series expansion,

$$\mathrm{f}\left(\mathrm{X}_i, \beta\right) = \mathrm{f}\left(\mathrm{X}_i\beta\right) + \sum_{i=1}^{P}\left\{\frac{\partial \mathrm{f}\left(\mathrm{X}_i, \beta\right)}{\partial \beta_i}\right\}_{\beta = \beta_0}\left(\beta_i - \beta_{0i}\right) \tag{5.2}$$

$$\mathrm{f}_i^* = \mathrm{f}\left(\mathrm{x}_i, \beta\right)$$

$$\mathrm{j}_{ij}^* = \left\{\frac{\partial \mathrm{f}\left(\mathrm{X}_i\beta\right)}{\partial \beta_i}\right\}_{\beta = \beta_0}$$

$$\mathrm{Y}_i - \mathrm{f}_i^* = \mathrm{Y}_i^* \tag{5.3}$$

The non linear equation given by (5.1) can be written as linear model.

$$\mathrm{Y}^* = \mathrm{J}^*\left(\beta - \beta_o\right) + \in \tag{5.4}$$

The above is linear model and J is Jocobian Matrix of 'f' function. From (5.4)

$$\hat{\beta}_1 - \beta_0 = \left(J_0^1 J\right)^{-1} J_0 Y_0$$

$$\therefore \hat{\beta}_1 = \beta_0 + \left(J_0^1 J_0\right)^{-1} J_0 Y_0$$

$$\vdots \quad \vdots$$

$$\vdots \quad \vdots$$

$$\beta_k = \beta_{k-1} + \left(J_{k-1}^1 J_{k-1}\right)^{-1} J_{k-1} Y_{k-1}$$

6. Conclusion

According to above, in nonlinear regression models, the parameter estimation process is most complicated phenomenon. Compare with the linear model and especially in multicollinearity convergence is not suggestible to the actual parameters. Such situation Ridge regression estimation is the best method to reduce the level of multicollinearing in linear models was applied to nonlinear models. So the process of parameter estimation is going with iterative procedure until convergence is occurred. So a simulation algorithm can be developed for suitable parameter estimation to the proposed approach.

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