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**Sunita Mishra**  
 Ph.D. Research Scholar in  
 Mathematics, Mahatma  
 Gandhi Chitrakoot Gramodaya  
 Vishwavidyalaya, Chitrakoot,  
 Satna, Madhya Pradesh, India

**V Upadhyay**  
 Associate Professor of  
 Mathematics, Department of  
 Physical Sciences, Mahatma  
 Gandhi Chitrakoot Gramodaya  
 Vishwavidyalaya, Chitrakoot,  
 Satna, Madhya Pradesh, India

**AK Agrawal**  
 Associate Professor of  
 Mathematics, Department of  
 Physical Sciences, Mahatma  
 Gandhi Chitrakoot Gramodaya  
 Vishwavidyalaya, Chitrakoot,  
 Satna, Madhya Pradesh, India

**Correspondence**  
**Sunita Mishra**  
 Ph.D. Research Scholar in  
 Mathematics, Mahatma  
 Gandhi Chitrakoot Gramodaya  
 Vishwavidyalaya, Chitrakoot,  
 Satna, Madhya Pradesh, India

## A mathematical study of two phase coronary blood flow using Herschel – Bulkley model during Angina

Sunita Mishra, V Upadhyay and AK Agrawal

### Abstract

In this mathematical modeling we are considering the two phase coronary blood flow in arterioles through heart during the angina. One phase is constituted by red blood cells and other phase is that of plasma. The heart has the highest oxygen consumption per tissue mass of all human organs. The resting coronary blood flow is 250ml/min; this represents 5% of cardiac output. We have applied the Herschel-Bulkley Non-Newtonian model in two phase coronary blood flow due to angina. The role of hematocrit is explicit in the determination of blood pressure drop in the case of angina heart diseases. The hematocrit increases as the blood pressure drop increases. The overall presentation is in tensorial form and solution technique adopted is analytical as well as numerical.

**Keywords:** Two phase blood flow, coronary blood flow, angina, plasma, hematocrit, Herschel-Bulkley model, non-Newtonian, blood pressure drop

### 1. Introduction

#### 1.1 Structure & Function of Heart

The heart muscle divides it down the middle into a left half and a right half. The muscular walls called a septum. The septum is solid so that blood cannot flow back and forth between the left and right halves of the heart. Another wall separates the rounded top part of the heart from the cone shaped bottom part<sup>[1]</sup>. So there are actually four chambers inside the heart. Each top chamber is called atrium. The bottom chambers are called ventricles. The atria are often referred as holding chambers, while the ventricles are called pumping chambers. The each side of the heart from its own separate system<sup>[2]</sup>.

The blood flows into the right atrium through the tricuspid valve into the right ventricle and then is pumped into the pulmonary artery and the lung, where the blood is oxygenated. The oxygenated blood then flows from the pulmonary veins into the left atrium and through the mitral valve into left ventricle. The four valves are seated in a plane. The mitral and tricuspid valves which are opened in order to fill the ventricle with blood<sup>[3]</sup>. When the blood pressure is low and velocity is small, are relatively large in area. The aortic and pulmonary valves used in ventricular systole to pumped blood out of the ventricle at high velocity are smaller. The mitral and tricuspid valves are attached to papillary muscles, which contract in systole pull down the valves to generate systolic pressure rapidly and prevent the valves from any dangers of inversion into the atrium. The aortic and pulmonary valves have no strings attached. The closing and openings of all valves are operated by blood itself through hydrodynamic forces<sup>[4]</sup>.

#### 1.2 Constitution of Blood

Blood carries other substances around the body inside arteries veins and capillaries these include gasses (carbon dioxide and oxygen), waste products (water & urea), hormones, enzymes, nutrients (glucose, amino acid, vitamins & minerals) and blood flows in capillary system. The constituents of blood are plasma, red blood cells, white blood cells, plates and hematocrit. Hematocrit is a blood test that tells us what proportion by volumes of total blood cells is made up of red blood cells<sup>[5]</sup>.

#### 1.3 Structure & Function of Coronary Circulatory System

The coronary circulatory system consists of the blood vessels which are responsible for the blood flow within the heart muscle itself. Although blood fills the chambers of the heart or

myocardium, even it is so thick that it requires coronary blood vessels to deliver blood deep into the myocardium. The vessels that supply blood high in oxygen to the myocardium are known as coronary arteries. The coronary circulation consists of the blood vessels that supply blood to and remove [6]. These arteries when healthy are capable of auto regulation to maintain coronary blood flow at levels appropriate to need the heart.

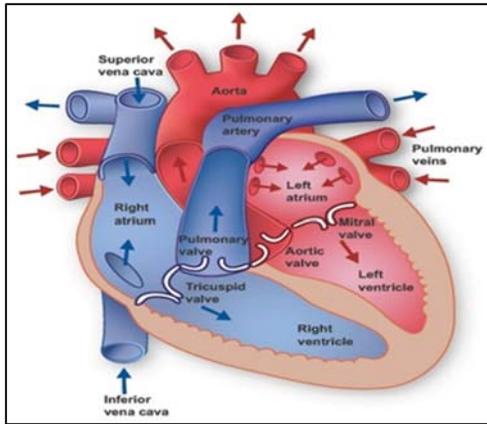


Fig 1: The Heart

**1.4 Structure & Function of Arterioles**

Arterioles are the smallest branches of arteries. They role consist in controlling the flow of blood into the capillary system. This is achieved by an action of muscle cells in their walls. In particular, they attenuate fluctuating changes in a blood pressure, thus making its value constant. Action of the arterioles prevents damaging the micro circular. Arterioles and small arteries are known as resistance vessels, because they form a main portion of the Peripherals resistance value [7].

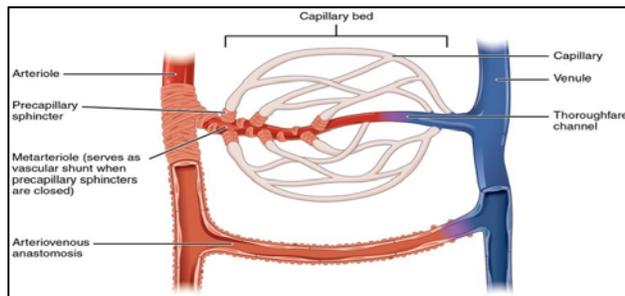


Fig 2: Arterioles

**1.5 Angina Heart Disease**

Angina is chest pain or discomforts that accruse if an area of your muscle does not get enough oxygen which blood. Angina may feel like pressure or squeezing in your chest. The pain also can occur in your solders, arms, neck, jaw or back. Angina pain may even feel like indigestion [8]. Angina is chest pain or discomfort that occurs if an area of your heart muscle doesn't get oxygen rich blood. Generally due to obstruction or spasm of coronary arteries. The main cause of angina pectoris is coronary artery disease. Due to atherosclerosis of the arteries feeding the heart [9]. In some cases angina can be extremely serious and has been know to cause death. People that suffer from average to severe cases of angina have an increased percentage of death before the

age of 55, usually around 60%. There are three types of angina [10]

1. Stable Angina
2. Unstable Angina
3. Micro vascular Angina

**2. Real Model**

**2.1 Choice of frame of Reference**

We have to select a frame of reference for mathematical modeling of the state of a moving blood· keeping in view the difficulty and generality of the problem of blood flow, we select generalized three-dimensional orthogonal curvilinear co-ordinate system, briefly prescribed as  $E^3$ , called as 3-dim Euclidean space, We interpret the quantities related to blood flow in tensorial form which is comparatively more realistic, The biophysical laws thus expressed fully hold good in any co-ordinate system, which is compulsion for the truthfulness of the law Now, let the co-ordinate axes be  $OX^i$  where O is origin and superscript  $i = 1,2,3$  let  $X^i$  be the co-ordinates of any point P in space, The mathematical description of the state if a moving blood is affected by means of functions which give the distribution of the blood velocity  $V^k = V^k(X^i, t)$ ,  $k= 1,2,3$  and of any two thermodynamic quantities pertaining to the blood, for instance the pressure  $p = p(X^i, t)$  and the density  $\rho = \rho(X^i, t)$ , As is well known, all the thermodynamic quantities are determined by the values of any two of them, together with the equate of state, Hence, if we are given five quantities, namely the three components of velocity  $V^k$ , the pressure  $p$  and the density  $\rho$ , the state of moving blood is completely determined [21] All these quantities are, in general, functions of the co-ordinates  $X^i$ ,  $i=1,2,3$  and of the time  $t$ , We emphasize that  $V^k(X^i, t)$  is the velocity of the blood at a given point  $X^i$  in space and at a given  $t$ , ie it refers to fixed points in space and not to fixed particles of the blood; in the course of time, the latter move about in space.

**2.2 Two Phase Description**

Blood is a complex fluid consisting of particulate corpuscles suspended in a non-Newtonian fluid. The particulate solids are red blood cells (RBCs), white blood cells (WBCs) and platelets. 55% of the plasma and 45% of the blood cells in a whole blood and approximately 98% of RBCs in 45% of blood cells and there are a few parts (approximately 2%) of the other cells. Which are ignorable, so one phase of the bloods plasma and 2nd phase of blood is RBCs. The first and foremost reason is that the blood is not an ideal fluid but it is a mixture of the two phases one is of plasma and other one is of blood cells. These blood cells, semi permeable packages of liquid of a density greater than that of plasma, are capable of changing their shape and size while flowing through different blood vessels. Plasma is a liquid containing semi permeable packages of RBCs. The behavior of blood is almost Newtonian at high shear rate, while at low shear rate the blood exhibits yield stress and non-Newtonian behavior [20].

**2.3 Parameters & symbols**

We are using many parameters in Herschel – Bulkley Model  $X$  = Postion of blood cells in unit volume,  $\eta_c$  = Viscosity coefficient of blood cells  $\eta_p$ = Viscosity coefficient of plasma,  $\eta_m$ = Viscosity coefficient of mixture of two phases,  $\rho_p$ = Density of blood plasma,  $\rho_c$  = Density of blood,  $\rho_m$ = Density of mixture of

two phases, Q = Blood flow flux, H = Hematocrit,  $\tau$  = Stress tensor

$e$  = Strain tensor,  $g^{ij}$  = Metrix tensor,  $\Delta P$  = Pressure difference,  $z_f - z_i$  = Length of blood vessels, R = Resistance to flow,  $T_e$  = Effective stress,  $r_p$  = Radius of plug

**2.3 Constitutive Equations**

Generally blood is non-homogeneous mixture of plasma and blood cells. Though for practical purposes it may be considered to be homogeneous two-phase mixture of plasma and blood cells. The constitutive equations proposed for whole blood mixture are as follows:

1. Newtonian equation

$\tau = \eta e$ , Where  $\eta$  is viscosity coefficient. This is found to hold good in the broad blood vessels where there is low hematocrit.

The non-Newtonian Herschel – Bulkley equation.

$$\tau = \eta e^n + \tau_0 \quad (\tau \geq \tau_0)$$

$$e = 0 \quad (\tau < \tau_0)$$

It holds good when blood shows yield stress. We notice that the yield stress arise because blood cells form aggregates in the form of rouleaux at low strain rate

If  $(\tau < \tau_0)$ , If, no blood flow-takes place. It is found that yield stress is given by the following formula:

$$\tau_0^{\frac{1}{3}} = \frac{A(H - H_m)}{100}$$

Where  $A = (0.008 \pm 0.002 \text{ dyne} / \text{c m}^2)^{1/3}$

Where, H is normal hematocrit and  $H_m$  is the hematocrit below which there is no yield stress.

**2.4 Boundary Conditions are as follows**

1. The velocity of blood flow on the axis of arterioles at  $r=0$  will be maximum and finite, say  $V_0 =$  maximum velocity,  $V = V_0$  then  $A = 0$

2. The velocity of blood flow on the wall of blood vessels at  $r=R$ , where, R is the radius of arterioles, will be zero. This condition is well known as no-slip condition.  $V = 0$  At  $r = R$

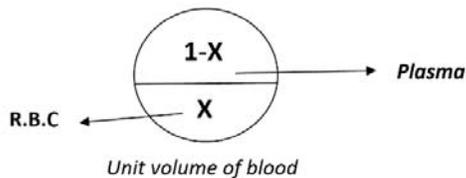
**3 Mathematical Modeling**

**3.1 Basic bio-fluid equation for two phase blood flow**

Let us the problem of systolic blood flow is deferent from the problems in cylindrical tube and select generalize three dimensional orthogonal curvilinear coordinate system (11). The bio physical laws thus expressed fully hold good in any co-coordinate system. This is a computation for the truthfulness of the laws. Blood is mixed fluid there are two phases in blood first phase is plasma and other phase is red blood cells. Thus blood can be considered as a homogeneous mixture of two phases [12].

**3.2 Equation of continuity for two phase blood flow**

The blood flow is effected by the pressure of blood and directly proportional to the volume occupied by blood cells. Let the volume portion covered by the blood cells in unit volume be X, this X be replaced by  $\frac{H}{100}$ , where H is the hematocrit the volume percentage of blood cells [13]. If the mass of blood cells to plasma is r then clearly.



$$r = \frac{X \rho_c}{(1-X)\rho_p} \quad \dots (1)$$

Where  $\rho_c$  and  $\rho_p$  are densities of blood cells and plasma respectively. Usually this mass ratio is not a constant. But in present context we have treated it constant. The both phase of blood, i.e. blood cells and plasma move with the common velocity [14]. Campbell and Pitcher has presented a model for two phase of blood. The in flow of blood in coronary arterioles is equal to out flow of that since heart behavior just like a pumping station. Hence the principle of conservation mass of coronary arterioles holds good.



Fig 3: Arterioles vessel

We get the equation of continuity for two phases as follows:-

$$\frac{\partial(X\rho_c)}{\partial t} + (X\rho_c V^i)_{,i} \text{ for blood cells phase} \quad \dots (2)$$

$$\text{and } \frac{\partial((1-X)\rho_p)}{\partial t} + ((1-X)\rho_p V^i)_{,i} = 0 \text{ for plasma phase} \quad \dots (3)$$

Where, V is the common velocity of two phase blood cells and plasma. If we define the uniform density of the blood  $\rho_m$  as follow

$$\frac{1+r}{\rho_m} = \frac{r}{\rho_c} + \frac{1}{\rho_p} \quad \dots (4)$$

Combined the equation (2) and (3)

$$\frac{\partial \rho_m}{\partial t} + (\rho_m V^i)_{,i} = 0 \quad \dots (5)$$

**3.3 Equation of Motion for two phase blood flow**

The hydro dynamical pressure p be supposed to be uniform because the both phases i.e. blood cells and plasma are always in equilibrium state in blood. Taking viscosity coefficient of blood cells to be  $\eta_c$  and applying the principle of conservation of momentum in coronary arterioles [15].



$$\frac{dp}{dt} + p - \tau = F_{ext}(0) = 0$$

$$\frac{dp}{dt} = -p + \tau$$

We get equation of motion for the two phase of blood cells as follows:-

$$X\rho_c \frac{\partial v^i}{\partial t} + (X\rho_c V^j)_{,j} v^i = -X p_{,j} g^{ij} + X \eta_c (g^{ik} v_{,k}^i)_{,j} \quad \dots (6)$$

Similarly, taking the viscosity coefficient of plasma to be. the equation of motion for plasma will be as follows:-

$$(1-X)\rho_p \frac{\partial v^i}{\partial t} + \{(1-X)\rho_p V^j\}_{,j} v^i = -(1-X) p_{,j} g^{ij} + (1-X) (g^{ik} v_{,k}^i)_{,j} \quad \dots (7)$$

Now adding equation [6, 7] and using relation [4], the equation of motion for blood flow with the both phases will be as follows:-

$$\rho_m \frac{\partial v^i}{\partial t} + (\rho_m V^j) V_{,j}^i = - p_{,j} g^{ij} + \eta_m (g^{jk} V_{,k}^i)_{,j} \quad \dots (8)$$

Where  $\eta_m = X \eta_c + (1-X) \eta_p$  is viscosity coefficient of blood as a mixture of two phase.

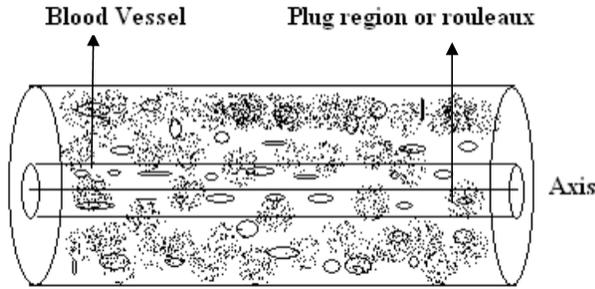


Fig 4: Herchel-Bulkley blood flow

The constitutive equation for test part of the blood vessel is

$$T = \eta_m e^n + T_p$$

$$\text{or } T - T_p = \eta_m e^n = T_e$$

where,  $T_e = \text{effective stress}$   
 whose generalized form will be as follows  
 $T^{ij} = -p g^{ij} + T_e^{ij}$  where,  $T_e^{ij} = \eta_m (e^{ij})^n$  'n ≠ 1 (Non-Newtonian)  
 While  $e^{ij} = g^{ik} V_{,k}^j$

$$\rho_m \frac{\partial v^i}{\partial t} + (\rho_m V^j) V_{,j}^i = - p_{,j} g^{ij} + \eta_m (g^{jk} V_{,k}^i)_{,j} \quad \dots (9)$$

Where the symbols have their usually meanings.

**4. Solution and Discussion**

Since, the blood vessels are cylindrical. The above governing equations have to be transformed into cylindrical co-ordinates. As we know earlier:-  
 $x^1 = r, x^2 = \Theta, x^3 = z$   
 Matrix of metric tensor in cylindrical co-ordinates is as follows:-

$$[g_{ij}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

While matrix of conjugate matrix tensor is as follow-

$$[g^{ij}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{r^2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Whereas the chritoffel's symbol of 2<sup>nd</sup> kind are as follow:-

$$\left\{ \begin{matrix} 1 \\ 2 \end{matrix} \right\}^1 = -r, \left\{ \begin{matrix} 1 \\ 2 \end{matrix} \right\}^2 = \left\{ \begin{matrix} 1 \\ 2 \end{matrix} \right\}^3 = \frac{1}{r}$$

Remaining others are zero.  
 The equation of continuity -

$$\frac{\partial v}{\partial z} = 0$$

The equation of motion -

r-component  $0 = -\frac{\partial p}{\partial z}, \Theta = \text{component} = 0$   
 z component  $0 = -\frac{\partial p}{\partial z} + \frac{\eta_m}{r} [r(\frac{\partial v_z}{\partial r})^n]$

Here, this fact has been taken in view that the blood flow is axially Symmetric in arterioles concerned, i.e.  $V_\Theta = 0$  and  $V_r, V_z$  and  $p$  do not depend upon  $\Theta$ . We get  $V_z = v(r)$  and  $p = p(z)$  and

$$0 = -\frac{dp}{dz} + \frac{\eta}{r} [r(\frac{dv}{dz})^n]$$

Since, pressure gradient  $-\frac{dp}{dz} = p$

$$r (\frac{dv}{dz})^n = -pr^2/2\eta_m + A, \text{ we apply}$$

Boundary condition: at  $r = 0, v = v_0$  then  $A = 0$

$$-\frac{dv}{dr} = \left(\frac{pr}{2\eta_m}\right)^{1/n} \quad \dots (10)$$

Replace  $r$  from  $r-r_p$

$$-\frac{dv}{dr} = \left(\frac{1/2pr - 1/2pr_p}{\eta_m}\right)^{1/n} \quad \dots (11)$$

Integrating above equation (12) under the no slip boundary condition-  $v = 0$  at  $r = R$  so as to get:

$$V = \left(\frac{P}{2\eta_m}\right)^{1/n} \frac{n}{n+1} [(R-r_p)^{1/n+1} - (r-r_p)^{1/n+1}] \quad \dots (12)$$

This is the formula for velocity of blood flow in arterioles. Putting  $r = r_p$  to get the velocity  $V_p$  of plug flow as follows:-

$$V_p = \frac{n}{n+1} \left(\frac{P}{2\eta_m}\right)^{1/n} (R-r_p)^{1/n+1} \quad \dots (14)$$

Where the value of  $r_p$  is taken from-(7)

**5. Result (Bio-physical interpretation)**

Hematocrit vs. blood pressure from an authorized City Hospital & Research centre Jabalpur. By Dr Abhishek Dubey Patient case history (Age-52 years old)

**Diagnosis- Coronary Artery Disease**

S. No.	B.P (mmhg)	B.P. Drop (mmhg)	B.P Drop (pas-sec)	Hb.	H = 3*Hb.
1.	140/80	-46.66	-6211.93912	14.0	42.0
2.	100/90	-28.33	-3771.62956	14.1	42.3
3.	140/80	-46.66	-6211.93912	14.2	42.6
4.	120/70	-40	-5325.28	14.3	42.9

**The flow flux phase blood flow in coronary arterioles**

$$Q = \int_0^{r_p} 2\pi r v_p dr + \int_{r_p}^R 2\pi r v dr$$

$$= \int_0^{r_p} 2\pi \frac{n}{n+1} \left(\frac{p}{2\eta}\right)^{1/n} [(R-r_p)^{1/n+1} - (r-r_p)^{1/n+1}] dr + \int_0^{r_p} 2\pi \frac{n}{n+1} \left(\frac{p}{2\eta}\right)^{1/n} [(R-r_p)^{1/n+1} - (R-r_p)^{1/n+1}] dr$$

Using (12) and (14)

$$= \frac{2\pi n}{n+1} \left(\frac{p}{2\eta}\right)^{1/n} \left[ \frac{r^2}{2} \right]_{r_p}^R + \frac{2\pi n}{(n+1)} \left(\frac{p}{2\eta}\right)^{1/n} \left[ \frac{r^2}{2} (R-r_p)^{1/n+1} - \frac{r(r-r_p)^{\frac{1}{n}+3}}{\frac{1}{n}+2} + \frac{(r-r_p)^{\frac{1}{n}+3}}{(\frac{1}{n}+2)(\frac{1}{n}+3)} \right]_{r_p}^R$$

$$= \frac{\pi n}{n+1} \left(\frac{p}{2\eta}\right)^{1/n} \left[ R^{\frac{1}{n}+3} \left[ \frac{r_p^2}{R^2} \left(1 - \frac{r_p}{R}\right)^{\frac{2}{n}+1} + \left(1 + \frac{r_p}{R}\right) \left(1 - \frac{r_p}{R}\right)^{\frac{1}{n}+2} - \frac{2\left(1 - \frac{r_p}{R}\right)^{\frac{1}{n}+2}}{\left(\frac{1}{n}+2\right)} + \frac{2\left(1 - \frac{r_p}{R}\right)^{\frac{1}{n}+3}}{\left(\frac{1}{n}+2\right)\left(\frac{1}{n}+3\right)} \right] \right]$$

$$Q = 250 \text{ ml/min} = 0.000004167 \text{ m}^3/\text{sec}. (16)$$

$$R=1, r_p=1/3 \eta_p = 0.0015 \text{ (pascal-sec)}, \eta_m = 0.035 \text{ (pascal-sec)} (17)$$

Length of arterioles  $z_f - z_i = 1 \text{ cm.} = 0.01 \text{ m.}$ (18)

$H = 42, P = 399967.2 \text{ pascal-sec.}$

$\eta_m = \eta_c X + \eta_p(1-X)$  Where,  $X = H/100, X = 42/100 = 0.42$   
 $0.035 = \eta_c(0.42) + 0.0015(1 - 0.42)$

$\eta_c = 0.0813 \text{ pas- sec}$

$$P = - \int \frac{dp}{dx}$$

$$= \frac{6211.93912}{0.01} = 621193.912 \text{ pascal-sec..}$$

$$\frac{27 Q}{2\pi} = \left(\frac{P}{3\eta_m}\right)^{1/n} \left[ \frac{26n^3 + 33n^2 + 9n}{6n^3 + 11n^2 + 6n + 1} \right]$$

$$0.1766674 = (5916132.495)^{1/n} \left[ \frac{26n^3 + 33n^2 + 9n}{6n^3 + 11n^2 + 6n + 1} \right]$$

We get  $n = -5.034$ , Again,  $p_f - p_i = 3\eta_m(z_f - z_i) \left(\frac{27 Q}{2\pi A}\right)^n$ ,

$$\text{Where } A = \left[ \frac{26n^3 + 33n^2 + 9n}{6n^3 + 11n^2 + 6n + 1} \right]$$

$$= 3[\eta_c X + \eta_p(1-X)](z_f - z_i) \left(\frac{27 Q}{2\pi A}\right)^n$$

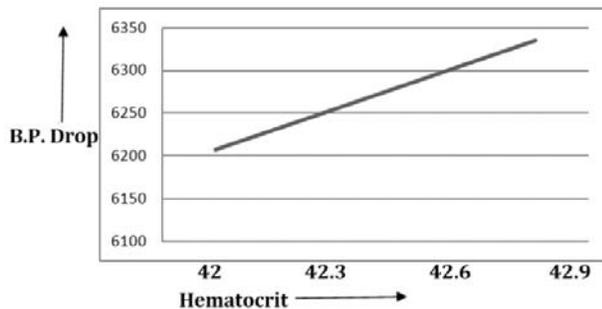
$$= 3[0.000798H + 0.0015] (0.01) (0.1766674)^{-5.034}$$

$$5.034 \left[ \frac{6n^3 + 11n^2 + 6n + 1}{26n^3 + 33n^2 + 9n} \right]^{-5.034}$$

$$= [0.002394H + 0.0045] (0.01) (0.1766674)^{-5.034} (0.2563284)^{-5.034}$$

$$= [0.002394H + 0.0045] (59091.39712)$$

Hematocrit	42	42.3	42.6	42.9
Blood pressure drop	6207.433085	6249.872526	6292.311967	6334.751409



### 6 Conclusion

A simple survey of the graph between blood pressure drop and hematocrit in cardiac patient shows that when hematocrit is increased the blood pressure drop is also increased.

### 7 Acknowledgement

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