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Selection of Bayesian single sampling plan for attributes based on prior binomial distribution

Dr. V Yuvaraj

Abstract
In Bayesian single sampling plan, the acceptance probability of lots are calculated by considering an appropriate prior distribution. The purpose of this paper is to demonstrate the use of Bayesian methods in acceptance sampling for items which are categorized according to their attributes as the defectives or non-defectives. The main thrust of the paper is to propose the model for minimizing the average cost over different combinations of (n_i, c_i) with respect to A_{2i} (the cost associated with a defective item which is accepted), for picking up the optimal Bayesian single sampling plan for attributes. The performance of Bayesian binomial single sampling plan is also discussed by determining the operating characteristic curve.

Keywords: Bayesian methods, linear cost, optimum sample size, sampling inspection, decision theory.

1. Introduction

The theory of sampling inspection is closely connected with statistical decision theory. In quality control techniques employed in industry, decisions to accept or reject the lots are based on samples drawn from different lots and constitute acceptance sampling. In the real world scenario, acceptance sampling plan played a vital role in many industries when the decision rule arises.

A well-known criterion for classifying acceptance sampling plan is based on how the features are measured. The classification of items as defective or non-defectives according to their quality characteristics leads to acceptance sampling plan for attributes. An industry receives a shipment of a particular product from a vendor, this product could be a raw material or a component that is used in the industry’s manufacturing process. Generally a sample is drawn from the lot and certain features of the items in the sample are inspected using this sample information, a decision is made regarding the acceptance or rejection of the lot. Lot – by – lot sampling inspection should be coordinated with process control. Items in the accepted lots are used in the manufacturing process. Whereas rejected lots could either be scrapped or returned to the vendor. There are various methods of inspection in quality control for improving the quality of product Li and Chang [2]. Acceptance sampling plans are practical tool for quality assurance applications involving quality contract on product orders. The sampling plans provide the vendor and buyer with decision rules for product acceptance, in order to meet the present product quality requirements.

2. Bayesian Single Sampling Plans For Attributes Based On the Prior Binomial Distribution

The theory of prior distribution is connected with theory of process control. Bayesian methods try to incorporate prior process knowledge to account for the variation in the sampling scheme. Though most of the acceptance sampling literature is based on classical methodology, there have been a few attempts to make the Bayesian paradigm to explain acceptance sampling techniques, most notably by Hald [1]. Between – lot variation in a manufacturing process may be attributed to operator, machine or raw material differences. On the other hand, difference among items sampled from within the same lot may be due to random causes. The basic idea of using Bayesian sampling techniques is to be able to identify and remove as many assignable causes of process variation, so that variation in the manufacturing process is only due to chance causes. The most commonly used distribution to describe sampling plan for attributes is a binomial distribution which models the process average probability p, of getting non-conforming items. Such plans involve

i. Drawing a single sampling of 'n' items from a lot of size 'N'
ii. Determining the number of defective items 'x' in the sample, if $x \le c$ accept the lot, otherwise reject the lot.

The probability of observing 'x', x = 0, 1, 2….n non-conforming units among the 'n' units of the sample is approximated by the prior binomial distribution. For the average process fraction non-conforming '$\bar{p}$' with 0 <$\bar{p}$< 1, the probability density function is given by

$$P(x; \overline{p}, n) = {}^{n}c_{x}\ \overline{p}^{x}\ (1-\overline{p})^{n-x} \qquad x = 0, 1, 2\ldots\ldots n$$

3. Optimum Sample Size

Statistical studies (experiments, surveys, observational studies etc.,) are always better when they are carefully planned. Good planning has many aspects: The problem should be carefully defined and operationalized. Experimental or observational units must be selected from the appropriate population the procedure must be followed carefully reliable instrument should be used to obtain measurements. Finally, the study must be of adequate size, relative to the goals of the study. Sample size is important for economic reasons: an under-sized study can be a waste of resources for not having the capability to produce useful results. An under-sized experiment exposes the subjects to potentially harmful treatments without advancing knowledge. In an over-sized experiment, a necessary number of subjects is exposed to a potentially harmful treatment, or denied a potentially beneficial one. There are several approaches to determine the sample size. A Bayesian approach can be used to optimize some utility function; it involves both precision of estimation and cost. Sample size planning is often important, and almost always difficult. It requires care in deriving scientific objectives and in obtaining suitable quantitative information prior to the study. Successful resolution of the sample size problem requires the close and the honest collaborations of statistician, and subject matter expert. Tailor [3, 4] has shown that non – Bayesian techniques are not optimal and suggest that action decisions, sample size and frequency should be determined based on the posterior probability that the process has shifted to an out-of-control state.

The main objective of this study is to determine the optimum single sampling plan (n,c) based on prior binomial distribution by minimizing the average acceptance cost K(N,n,c) subject to the condition A_2 (the cost associated with a defective item which is accepted) is very small.

That is, Single sampling attribute plans obtained by minimizing the average cost under the following assumptions

i. Sampling and decision cost are linear in lot size, sample size and the number of defectives in the lot and the sample.
ii. Sampling is without replacement.
iii. The distribution of lot quality is a binomial prior distribution.

The linear cost function Hald [1] for acceptance sampling by attributes for a single sampling plan, the product of quality p_i. That is, the product coming from product i (i = 1, 2…k) is as follows

$$\mathbf{K(p_i) = n_i (S_{1i} + S_{2i}\, p_i) + (N_i - n_i)\, [(R_{1i} + R_{2i} p_i) + (A_{2i} - R_{2i})(p_i - p_{ri}) P(p_i)]}$$

Hald [1] has suggested the following interpretation for the above constants

S_1 = Cost per item of sampling and testing.
S_2 = Repair cost for a defective item found in sampling.
A_1 = Cost per item associated with handling the N - n items not inspected in an accepted lot (frequently is 0).
A_2 = Cost associated with a defective item which is accepted (may be quite large).
R_1 = Cost per item of inspecting the remaining N - n items in a rejected lot.
R_2 = Repair cost associated with a defective item in the remaining N - n items of a rejected lot.

The minimization of the cost function $K(N_i, n_i, c_i)$ with respect to A_{2i} (i=1, 2…..k), may include costs of rework or replacement, costs of handling the defective items in assembling and disassembling, damage to other parts in the assembly and costs of renewed testing and inspection. The iterative procedure for determining the optimum values of single sampling plan (n, c) can be developed by the following steps.

4. Iterative Procedure

Step 1: Set the acceptance number c=0

Step 2: Input the lot size N and the other cost constants namely S_1, S_2, R_1, R_2& A_1.

Step 3: By setting A_2=31, find the minimum Average Acceptance Cost K(N,n,c) subject to the condition A_2 (the cost associated with defective item which is accepted) is very small.

Step 4: Select the Single Sampling Plan parameters (n, c) corresponding to the Minimum average acceptance cost K (N, n, c).

Step 5: Repeat Step 1 by giving incremental value of 'c' and find the Single Sampling Plan, (n, c) based on minimum average acceptance cost for various values of N.

On using the above iterative procedure, optimal single sampling plans (n_i, c_i) are determined by using Bayesian prior binomial distribution. Table 1 gives single sampling plans based on Bayesian prior binomial distribution using the above iterative procedure when the optimum value of A_2 (the cost associated with defective item which is accepted) is set at 31. By iterative procedure, the average acceptance cost K (N, n, c) is minimum only at that level.

Table 1: Determination of optimum sample size 'n' and acceptance number 'c'

N	c	n	p	A_2	Acceptance Cost	Rejection Cost
1000	0	70	0.07	34	387.82	77.80
	1	90	0.09	35	458.21	93.80
	2	90	0.09	35	484.22	107.48
	3	110	0.11	36	535.87	113.62
	4	110	0.11	36	551.96	122.08
	5	110	0.11	36	590.89	142.57
	6	130	0.13	37	615.54	134.50
	7	130	0.13	37	629.38	141.78
	8	290	0.058	45	1699.48	326.04

Hence the optimum single sampling plan (n_i,c_i) are picked from the Table 1 for the lot size 1000 and their acceptance and rejection cost also presented along with Operating Characteristics curve.

5. Operating Characteristic Curve (Oc)

Operating characteristic (OC) curve has always been used to judge whether a lot can be consider acceptable or not. The OC function is closely associated with acceptance sampling procedures. The OC helps us to estimate the adequate sample sizes 'n' required to obtain, with high confidence, one or more failures in our experiments. These OC function properties allow us to control the quality of our incoming products, estimate certain parameter's of interest such as the required sample sizes, which contribute to the design of better and more efficient experiment.

For a Single Sampling Plan with parameters (n.c), it follows that the average probability of accepting the lots from product of quality is equal to the binomial probability B(n,p,c). That is, P(p) = B(n,p,c) for $0 \le p \le 1$ is called OC function for the product of quality p. Therefore, the selection of optimum single sampling plan and its OC curves are given in Fig.1

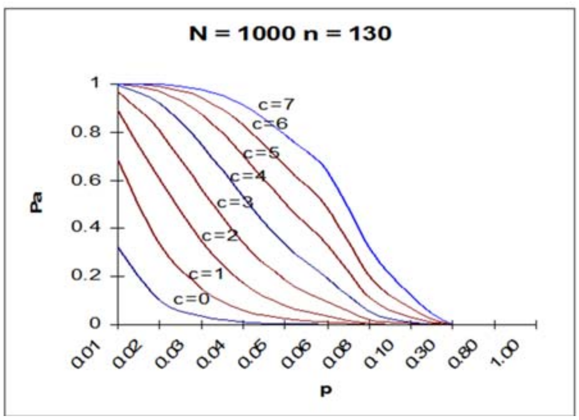


Fig 1: depicts the Operating Characteristic curves for the optimum single sampling plan based on the prior binomial distribution.

6. Simulation

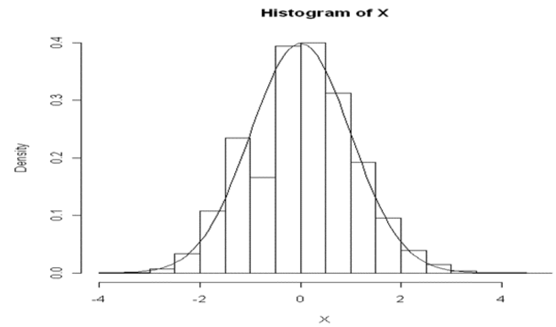
Using R-software, 1,20,000 random numbers were generated and nature of distribution is investigated which is verified by using the following histogram.

R-Program for generating the random numbers

```
>>n=130; p=0.13;
>>S = rbinom(120000,n,p)
>>X = (S-n*p)/sqrt(n*p*(1-p))
>>X
>>hist(X, Prob=T)
>>curve(dnorm(x),add=T)
```

Verification of the nature of distribution

The following histogram shows the generated data follows the normal distribution.



A better plot than the histogram for deciding if random data is approximately normal is called "normal probability plot". Therefore the above histogram indicates the data are approximately normal. Further binomial random variables were simulated 1000 times with the help of the above random numbers by using Monte Carlo technique. Based on the simulated values, mean and standard deviation were calculated and then the parameters (n, p) of the binomial distribution were obtained and are presented in Table 2.

Table 2

Parameters For Bayesian Prior Binomial Distribution			Parameters For Simulated Values	
N	n	p	n	p
1000	130	0.130	129	0.1304

The above table shows very clearly that the observed and the simulated parameters "n" and "p" were almost equal. This indicated that the selection of single sampling plan based on prior binomial distribution is strengthened to be optimum by using simulation study.

7. Conclusion

The cost A_2 arises from the fact that the buyers also use sampling inspection procedures and may return individual items or an entire lot. It is observed from the example presented in the preceding section that the Bayesian optimum single sampling plan for attribute with prior binomial distribution require minimum sample sizes 'n' and optimum Single Sampling Plan were obtained by minimizing average acceptance cost K (N, n, c) subject to the condition that the value of A_2 (the cost associated with defective item witch is accepted) is very small by setting A_2=31.

8. References

1. Hald A. Statistical theory of sampling inspection by attributes. Technometrics. Academic press inc. (London) ltd., 1981.
2. Li TF, Chang S. Classification on defective items using unidentified samples. Pattern Recognition. 2005; 38:51-58.
3. Taylor HM. Markovian Sequential Replacement Process. Annals of Mathematical Statistics, 1965; 36:1677-1694.
4. Taylor HM. Statistical control of a Gaussian Process. Technometrics, 1967; 9(1):29-41.