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## Construction of special integer triples

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### Abstract

This paper concerns with interesting Diophantine problems on triples. A search is made on finding three non-zero distinct integers, namely  $a, b, c$  such that, each of the expressions  $a + 2b, b + c$  (or)  $b - c$  is a perfect square and  $a + 2c$  is a cubical integer. Infinitely many such triples are obtained.

**Keywords:** Integer Triples, natural numbers, antiquity number theory, fewer equations

### Introduction

Number theory is that branch of mathematics which deals with properties of the natural numbers  $1, 2, 3, \dots$  also called the (+)ve integers. These numbers together with the (-) integers & zero form the set of integers. Properties of these integers have been studied since antiquity number theory is an art enjoyable & pleasing to everybody. It has fascinated and inspired both amateurs and mathematicians alike Diophantine problems have fewer equations than unknown variables and involve finding integers that work correctly for all equations. Certain Diophantine problems come from physical problems or from immediate mathematical generalizations and others come from geometry in a variety of ways. Certain Diophantine problems are neither trivial nor difficult to analyze<sup>[1-8]</sup>.

In this context one may refer<sup>[9-12]</sup>. The above problems motivated us to search for other choices of Diophantine problems on triples. This paper consists of two sections I&II. In section I, we search for three non-zero distinct integers such that  $a + 2b = \alpha^2, a + 2c = \beta^3, b + c = \gamma^2$ . In section II, we find integers  $a, b, c$  such that  $a + 2b = \alpha^2, a + 2c = \beta^3, b - c = \gamma^2$ . A few numerical examples are also given.

### Method of Analysis

#### Section-I

Let  $a, b, c$  be three non-zero distinct integers such that

$$a + 2b = \alpha^2 \quad (1)$$

$$a + 2c = \beta^3 \quad (2)$$

$$b + c = \gamma^2 \quad (3)$$

Eliminating  $a, b, c$  between the equations (1)-(3), we have,

$$a = \frac{1}{2} [\alpha^2 - 2\gamma^2 + \beta^3] \quad (4)$$

$$b = \frac{1}{4} [2\gamma^2 + \alpha^2 - \beta^3] \quad (5)$$

$$c = \frac{1}{4} [2\gamma^2 - \alpha^2 + \beta^3] \quad (6)$$

For  $a, b, c$  to be integers, the parameters  $\alpha, \beta, \gamma$  are to be chosen as follows:

I)  $\alpha, \beta, \gamma$  :even

II)  $\gamma$  :even,  $\alpha, \beta$  :odd

For the above choices, the corresponding integer values of  $a, b, c$  are presented below.

**Choice (I)**

Let  $\alpha = 2p; \beta = 2q; \gamma = 2r$

Substituting  $\alpha, \beta, \gamma$  in (4), (5), (6) we get

$$a = 2p^2 - 4r^2 + 4q^3$$

$$b = 2r^2 + p^2 - 2q^3$$

$$c = 2r^2 - p^2 + 2q^3$$

**Numerical Examples**

$p$	$q$	$r$	$a$	$b$	$c$	$a + 2b$	$a + 2c$	$b + c$
1	2	3	-2	3	33	$(2)^2$	$(4)^3$	$(6)^2$
3	5	7	322	-143	339	$(6)^2$	$(10)^3$	$(14)^2$
2	4	1	260	-122	126	$(4)^2$	$(8)^3$	$(2)^2$
2	4	6	120	-52	196	$(4)^2$	$(8)^3$	$(12)^2$

**Choice (ii)**

Let  $\alpha = 2p + 1; \beta = 4q + 1; \gamma = 2r$

Substituting  $\alpha, \beta, \gamma$  in (4), (5),(6) we get

$$a = 2p^2 + 2p - 4r^2 + 32q^3 + 24q^2 + 6q + 1$$

$$b = 2r^2 + p^2 + p - 16q^3 - 12q^2 - 3q$$

$$c = 2r^2 - p^2 - p + 16q^3 + 12q^2 + 3q$$

**Numerical Examples**

$p$	$q$	$r$	$a$	$b$	$c$	$a + 2b$	$a + 2c$	$b + c$
1	2	3	333	-162	198	$(3)^2$	$(9)^3$	$(6)^2$
3	5	7	4459	-2205	2401	$(7)^2$	$(21)^3$	$(14)^2$
2	4	1	2465	-1220	1224	$(5)^2$	$(17)^3$	$(2)^2$
2	4	6	2325	-1150	1294	$(5)^2$	$(17)^3$	$(12)^2$

**Section-II**

Let  $a, b, c$  be three non-zero distinct integers such that

$$a + 2b = \alpha^2 \tag{7}$$

$$a + 2c = \beta^3 \tag{8}$$

$$b - c = \gamma^2 \tag{9}$$

Eliminating  $a, b, c$  between the equations (7)-(9), we have,

$$\alpha^2 - 2\gamma^2 = \beta^3 \tag{10}$$

(10) Is solved through two different methods.

**Method-I**

Assume

$$\beta = M^2 - 2N^2 \tag{11}$$

Substituting  $\beta$  value in (10) and employing the method of factorizations, define  $\alpha + \sqrt{2}\gamma = (M + \sqrt{2}N)^3$ . Equating the rational and irrational parts, we get

$$\alpha = M^3 + 6MN^2$$

$$\gamma = 3M^2N + 2N^3$$

Assume  $c=T$

Substituting  $\alpha, \gamma, c$  values in (7) and (9) we get

$$a = (M^3 + 6MN^2)^2 - 2T - 2(3M^2N + 2N^3)^2$$

$$b = T + (3M^2N + 2N^3)^2$$

$$c = T$$

**Numerical Examples**

<i>M</i>	<i>N</i>	<i>T</i>	<i>a</i>	<i>b</i>	<i>c</i>	<i>a + 2b</i>	<i>a + 2c</i>	<i>b - c</i>
1	2	3	-349	487	3	(25) <sup>2</sup>	(-7) <sup>3</sup>	(22) <sup>2</sup>
3	5	7	-68935	148232	7	(477) <sup>2</sup>	(-41) <sup>3</sup>	(385) <sup>2</sup>
2	4	1	-21954	30977	1	(200) <sup>2</sup>	(-28) <sup>3</sup>	(176) <sup>2</sup>
2	4	6	-21964	30982	6	(200) <sup>2</sup>	(-28) <sup>3</sup>	(176) <sup>2</sup>

**Method-II**

Assume

$$\alpha^2 - 2\gamma^2 = \beta^3 * 1 \tag{12}$$

Where

$$1 = (3 + 2\sqrt{2})(3 - 2\sqrt{2}) \tag{13}$$

Substituting (11), (13) in (12) and following the procedure as in method I, the corresponding values of  $a, b, c$  satisfying (7)-(9) are given by

$$a = (3M^3 + 18MN^2 + 12M^2N + 8N^3)^2 - 2T - 2(9M^2N + 6N^3 + 2M^3 + 12MN^2)^2$$

$$b = T + (9M^2N + 6N^3 + 2M^3 + 12MN^2)^2$$

$$c = T$$

**Numerical Examples**

<i>M</i>	<i>N</i>	<i>T</i>	<i>a</i>	<i>b</i>	<i>c</i>	<i>a + 2b</i>	<i>a + 2c</i>	<i>b - c</i>
1	2	3	-349	13459	3	(163) <sup>2</sup>	(-7) <sup>3</sup>	(116) <sup>2</sup>
3	5	7	-68935	4447888	7	(2971) <sup>2</sup>	(-41) <sup>3</sup>	(2109) <sup>2</sup>
2	4	1	-21954	861185	1	(1304) <sup>2</sup>	(-28) <sup>3</sup>	(928) <sup>2</sup>
2	4	6	-21964	861190	6	(1304) <sup>2</sup>	(-28) <sup>3</sup>	(928) <sup>2</sup>

**Conclusion**

In this paper, we have presented infinitely many triples  $a, b, c$  such that,  $a + 2b, b + c$  (or)  $b - c$  is respectively a perfect square and  $a + 2c$  is a cubical integer. As the triples are rich in variety one may attempt to find triples with other characterization among its members.

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**References**

1. Andre Weil, Number Theory: An Approach through History, From Hammurapi to Legendre, Birkahsuser, Boston, 1987.
2. Bibhotibhusan Batta, Avadesh Narayanan Singh. History of Hindu Mathematics, Asia Publishing House, 1938.
3. Boyer CB. A History of mathematics, John Wiley& sons Inc., New York, 1968.
4. Dickson LE. History of Theory of Numbers, Vol. 11, Chelsea Publishing Company, New York, 1952.
5. Davenport, Harold The higher Arithmetic: An Introduction to the Theory of Numbers (7th ed) Cambridge University Press, 1999.
6. John Stilwell. Mathematics and its History, Springer Verlag, New York, 2004.
7. James Matteson. M.D, "A Collection of Diophantine problems with solutions, Washington, Artemas Martin, 1888.
8. Titu andreescu, Dorin Andrica, "An Introduction to Diophantine equations" GIL Publicating House, 2002.

9. Gopalan MA, Vidhyalakshmi S, Thiruniraiselvi N. On two interesting problems, Impact J Sci Tech. 2015; 9(3):51-55,
10. Gopalan MA, Thiruniraiselvi N, Vijayasankar A. On two interesting triple integer sequence, IJMA, 2015; 6(10):1-4.
11. Gopalan MA, Vidhyalakshmi S, Thiruniraiselvi N. On interesting integer pairs, IJMRME., 2015; 1(1):320-323.
12. Gopalan MA, Vidhyalakshmi S, Thiruniraiselvi N. An interesting rectangular problem”, FJMMS., 2015; 4(1):35-39.