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Ritikesh Kumar
Research Scholar, J.J.T.
University, Jhunjhnu
Rajasthan, India.

Dr. Pardeep Goel
Associate Professor M.M.
(P.G.) College, Fatehabad-
125050 (Haryana), India.

Availability modeling two units system subject to degradation post repair after complete failure using RPGT

Ritikesh Kumar, Dr. Pardeep Goel

Abstract

In this paper, availability modeling and analysis of two units system out of which one unit undergoing degradation after complete failure using Regenerative Point Graphical Technique (RPGT) is discussed. Initially both units are working at full capacity out of which one unit may have two types of failures, one is direct and second one is through partial failure mode. There is a single server (repairman), who inspects and repairs the units on each failure. On each repair unit under goes degradation if the server reports that unit is not repairable then it is replaced by a new one, which follows a general distribution other unit have perfect repair. On complete failure the unit cannot be restored to its original capacity. Fuzzy concept is used to determine failure/working state of a unit. Taking failure rates exponential, repair rates general and taking into consideration various probabilities, a transition diagram of the system is developed to find Primary circuits, Secondary circuits & Tertiary circuits and Base state. Problem is solved using RPGT to determine system parameters. System behavior is discussed with the help of graphs and tables.

Keywords: Availability, Reliability, Primary, Secondary, Tertiary Circuits, Degraded state, Base-State, Regenerative Point Graphical Technique (RPGT), Mean Time to System Failure, Busy period of the Server, Expected No. of Server's Visits, Fuzzy Logic, Steady State.

Introduction

Various Mechanical systems are assembly of a number of units in which each unit is important for the system to work efficiently. If a single unit fails then the whole system fails. Considering the importance of individual units in system Kumar, J. & Malik, S. C. [1] have discussed the concept of preventive maintenance for a single unit system. Liu., R. [2], Malik, S. C. [3], Nakagawa, T. and Osaki, S. [4] have [discussed reliability analysis of a one unit system with un-repairable spare units and its applications. Goel, P. & Singh J. [5], Gupta, P., Singh, J. & Singh, I.P. [6], Kumar, S. & Goel, P. [7], Gupta, V. K. [8], Chaudhary, Goel & Kumar [9] Sharma & Goel [10], Ritikesh & Goel [11] and Goyal & Goel [12] have discussed behavior with perfect and imperfect switch-over of systems using RPGT. Here we have discussed the behavioral analysis of two units system in which one have parallel sub units, so it can work in reduced capacity as well as in full capacity. One of the two units can fail from normal mode to complete failure directly or via partial failure. Most of the systems consist of a number of units and one of these units may be important for working of system, hence need more care over other units. Further, there are units which may not be repaired to their original capacity. Since the capability of unit after repair depends on the repair mechanism adopted and unit may have increased failure rate on subsequent failures i.e. it is degraded after each repair. The system may go under imperfect repair on complete failure and unit is degraded but operative state with reduced capacity is obtained again and again. Server inspects and repairs the unit as and when need arise. After a limiting situation, when no further repair is possible, then unit is replaced by new one, other unit which may fail directly have perfect repair upon failure. In this paper Behavioral Analysis of two units system out of which one unit undergoing degradation after post repair. Initially both units are working at full capacity out of which one unit may have two types of failures, one is direct and second one is through partial failure mode. There is a single server (repairman) who inspects and repairs the units as and when need arise and other unit having perfect repair on failure.

Correspondence
Ritikesh Kumar
Research Scholar, J.J.T.
University, Jhunjhnu
(Rajasthan), India.

The unit on complete failure cannot be restored to its original capacity. If the server reports that unit is not repairable then, it is replaced by a new one. Fuzzy concept is used to determine failure/working state of a unit. Taking failure rates exponential, repair rates general and taking into consideration various probabilities, a transition diagram of the system is drawn to determine Primary circuits, Secondary circuits & Tertiary circuits and Base state. Problem is solved using RPGT. Repair and Failure are statistically independent. Expressions for systems parameters i.e. mean time to system failure, availability, and number of server visits and busy period of the server are evaluated to study the behavior of the system for steady state. Particular cases are taken to study the effect of failure and repair rates on system parameters. Profit optimization is also discussed. System behavior is discussed with the help of graphs and tables.

Assumptions and Notations: - The following assumptions and notations are taken: -

1. A single repair facility is available.
2. The distributions of failure times and repair times are exponential and general respectively and also different. Failures and repairs are statistically independent.
3. For one of the units repair is imperfect and repaired system is not good as new one post repair.
4. Nothing can fail further when the system is in failed state.
5. The system is discussed for steady-state conditions.
6. Replacement of Un-repairable unit and repair facility is immediate.

cycle: A circuit formed through un-failed states.

m-cycle: A circuit (may be formed through regenerative or non-regenerative / failed state) whose terminals are at the regenerative state m.

m-cycle: A circuit (may be formed through un-failed regenerative or non-regenerative state) whose terminals are at the regenerative m state.

$(i \xrightarrow{sr} j)$: r-th directed simple path from i-state to j-state; r takes positive integral values for different paths from i-state to j-state.

$(\xi \xrightarrow{fff} i)$: A directed simple failure free path from ξ -state to i-state.

$V_{m,m}$: Probability factor of the state m reachable from the terminal state m of the m-cycle.

$V_{\bar{m}, \bar{m}}$: Probability factor of the state m reachable from the terminal state m of the **m-cycle**.

$R_i(t)$: Reliability of the system at time t, given that the system entered the un-failed regenerative state 'i' at t = 0.

$A_i(t)$: Probability of the system in up time at time 't', given that the system entered regenerative state 'i' at t = 0.

$B_i(t)$: Reliability that the server is busy for doing a particular job at time 't'; given that the system entered regenerative state 'i' at t = 0.

$V_i(t)$: The expected no. of server visits for doing a job in (0,t] given that the system entered regenerative state 'i' at t = 0.

' , ' denote derivative

$W_i(t)$: Probability that the server is busy doing a particular job at time t without transiting to any other regenerative state 'i' through one or more non-regenerative states, given that the system entered the regenerative state 'i' at t = 0.

μ_i : Mean sojourn time spent in state i, before visiting any other states;

$$\mu_i = \int_0^{\infty} R_i(t) dt$$

μ_i^1 : The total un-conditional time spent before transiting to any other regenerative states, given that the system entered regenerative state 'i' at t=0.

n_i : Expected waiting time spent while doing a given job, given that the system entered regenerative state 'i' at t=0;

$$\eta_i = W_i^*(0).$$

ξ : Base state of the system.

f_j : Fuzziness measure of the j-state.





λ_1 : Constant failure rate of unit A to reduced state.

λ_2 : Constant failure rate of unit A to from

λ_3 : Constant failure rate of unit B.

λ_n : $(1+\delta_{n-1})\lambda_{n-1}$ increased failure rate of unit A on complete failure. $n \geq 4$

$A/\bar{A}/a$: Unit in full capacity working / reduced state / failed state.

State	Symbol	Model
Regenerative State		0
Reduced State		3
Failed State		1,2,4,6,7
Post Repair Working		5

Taking into consideration the above assumptions and notations the Transition Diagram of the system is given in Figure 1.

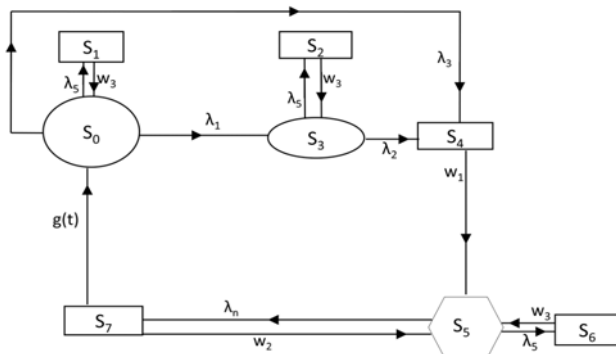


Fig 1

$S_0 = AB$ $S_1 = Ab$ $S_2 = \bar{A}b$ $S_3 = \bar{A}B$
 $S_4 = aB$ $S_5 = A^1B$ $S_6 = A^1b$ $S_7 = a^1B$
 b denotes unit B in failed state.

Simple Paths Vertex 'I' to Vertex 'j'

Table 2

Vertex	0	1	2	3	4	5	6	7
0	(0,1,0) (0,3,4,5,7,0) (0,4,5,7,0)	(0,1)	(0,3,2)	(0,3)	(0,3,4) (0,4)	(0,3,4,5) (0,4,5)	(0,3,4,5,6) (0,4,5,6)	(0,3,4,5,7)
1	(1,0)	(1,0,1) (1,0,3,4,5,7,0,1) (1,0,4,5,7,6,1)	(1,0,3,2)	(1,0,3)	(1,0,3,4) (1,0,4)	(1,0,3,4,5) (1,0,4,5)	(1,0,3,4,5,6) (1,0,4,5,6)	(1,0,3,4,5,7) (1,0,4,5,7)
2	(2,3,4,5,7,0)	(2,3,4,5,7,0,1)	(2,3,2) (2,3,4,5,7,0,3,2)	(2,3)	(2,3,4)	(2,3,4,5)	(2,3,4,5,6)	(2,3,4,5,7)
3	(3,4,5,7,0)	(3,4,5,7,0,1)	(3,2)	(3,2,3) (3,4,5,7,0,3)	(3,4)	(3,4,5)	(3,4,5,6)	(3,4,5,7)
4	(4,5,7,0)	(4,5,7,0,1)	(4,5,7,0,3,2)	(4,5,7,0,3)	(4,5,7,0,3,4) (4,5,7,0,4)	(4,5)	(4,5,6)	(4,5,7)
5	(5,7,0)	(5,7,0,1)	(5,7,0,3,2)	(5,7,0,3)	(5,7,0,3,4) (5,7,0,4) (5,7,5) (5,6,5)	(5,7,0,3,4,5) (5,7,0,4,5)	(5,6)	(5,7)
6	(6,5,7,0)	(6,5,7,0,1)	(6,5,7,0,3,2)	(6,5,7,0,3)	(6,5,7,0,3,4) (6,5,7,0,4)	(6,5)	(6,5,6) (6,5,7,0,3,4,5,6)	(6,5,7)

The possible transitions between states alongwith transition time c.d.f.'s are shown in Fig 1. Primary, Secondary and Tertiary Circuits associated with the system are given in Table 3

Table 3

Vertex i	CL1	CL2	CL3
0	(0,1,0) (0,3,4,5,7,0) (0,4,5,7,0)	Nil (3,2,3), (5,6,5) (5,7,5), (7,5,7) (5,6,5), (5,7,5) (7,5,7)	Nil (5,6,5) (5,6,5)
1	(1,0,1)	(0,3,4,5,7,0) (0,4,5,7,0)	(3,2,3), (5,6,5) (5,7,5), (7,5,7) (5,6,5), (5,7,5) (7,5,7)
2	(2,3,2)	(3,4,5,7,0,3)	(5,6,5), (5,7,5) (7,5,7), (0,1,0)
3	(3,4,5,7,0,3)	(5,6,5), (5,7,5) (7,5,7), (0,1,0)	(5,6,5)
4	(4,5,7,0,3,4) (4,5,7,0,3,4)	(5,6,5), (5,7,5) (7,5,7), (0,1,0) (3,2,3) (5,6,5), (5,7,5) (7,5,7), (0,1,0)	(5,6,5) (5,6,5)
5	(5,6,5), (5,7,5) (5,7,0,3,4,5) (5,7,0,4,5)	Nil (0,1,0), (3,2,3) (0,1,0)	Nil Nil
6	(6,5,6)	(5,7,5) (5,7,0,3,4,5) (5,7,0,4,5)	Nil (0,1,0), (3,2,3) (0,1,0)
7	(7,0,3,4,5,7) (7,5,7) (7,0,4,5,7)	(0,1,0), (3,2,3) (5,6,5) (5,6,5) (0,1,0), (5,6,5)	Nil Nil Nil

As the primary circuits vertex '5' is maximum hence the base state is 'ξ' = '5'
 Primary, Secondary, Tertiary Circuits w.r.t. the Simple Paths (Base-State 'ξ' = '5')

Table 4

Vertex j	$(5 \xrightarrow{S_r} j): (P0)$	(P1)
0	$(5 \xrightarrow{S_1} 0): \{5,7,0\}$	(0,1,0)
1	$(5 \xrightarrow{S_1} 1): \{5,7,0,1\}$	(0,1,0)
2	$(5 \xrightarrow{S_1} 2): \{5,7,0,3,2\}$	(0,1,0), (3,2,3)
3	$(5 \xrightarrow{S_1} 3): \{5,7,0,3\}$	(0,1,0), (3,2,3)
4	$(5 \xrightarrow{S_1} 4): \{5,7,0,3,4\}$	(0,1,0), (3,2,3)
	$(5 \xrightarrow{S_2} 4): \{5,7,0,4\}$	(0,1,0)
5	$(5 \xrightarrow{S_1} 5): \{5,7,0,3,4,5\}$	(0,1,0), (3,2,3)
	$(5 \xrightarrow{S_2} 5): \{5,7,0,4,5\}$	(0,1,0)
	$(5 \xrightarrow{S_3} 5): \{5,6,5\}$	Nil
	$(5 \xrightarrow{S_4} 5): \{5,7,5\}$	Nil
6	$(5 \xrightarrow{S_1} 6): \{5,6\}$	Nil
7	$(5 \xrightarrow{S_1} 7): \{5,7\}$	Nil

Transition Probability and the Mean sojourn times.

$q_{i,j}(t)$: Probability density function (p.d.f.) of the first passage time from a regenerative state 'i' to a regenerative state 'j' or to a failed state 'j' without visiting any other regenerative state in (0,t].

$p_{i,j}$: Steady state transition probability from a regenerative state 'i' to a regenerative state 'j' without visiting any other regenerative state. $p_{i,j} = q_{i,j}^*(0)$; where * denotes Laplace transformation.

Transition Probabilities

Table 5

$q_{i,j}(t)$	$P_{ij} = q_{i,j}^*(0)$
$q_{0,1} = \lambda_5 e^{-(\lambda_1 + \lambda_3 + \lambda_5)t}$	$p_{0,1} = \lambda_5 / (\lambda_1 + \lambda_3 + \lambda_5)$
$q_{0,3} = \lambda_1 e^{-(\lambda_1 + \lambda_3 + \lambda_5)t}$	$p_{0,3} = \lambda_1 / (\lambda_1 + \lambda_3 + \lambda_5)$
$q_{0,4} = \lambda_3 e^{-(\lambda_1 + \lambda_3 + \lambda_5)t}$	$p_{0,4} = \lambda_3 / (\lambda_1 + \lambda_3 + \lambda_5)$
$q_{1,0} = w_3 e^{-w_3 t}$	$p_{1,0} = 1$
$q_{2,3} = w_3 e^{-w_3 t}$	$p_{2,3} = 1$
$q_{3,2} = \lambda_5 e^{-(\lambda_2 + \lambda_5)t}$	$p_{3,2} = \lambda_5 / (\lambda_2 + \lambda_5)$
$q_{3,4} = \lambda_2 e^{-(\lambda_2 + \lambda_5)t}$	$p_{3,4} = \lambda_2 / (\lambda_2 + \lambda_5)$
$q_{4,5} = w_1 e^{-w_1 t}$	$p_{4,5} = 1$
$q_{5,6} = \lambda_5 e^{-(\lambda_5 + \lambda_n)t}$	$p_{5,6} = \lambda_5 / (\lambda_n + \lambda_5)$
$q_{5,7} = \lambda_n e^{-(\lambda_n + \lambda_5)t}$	$p_{5,7} = \lambda_n / (\lambda_n + \lambda_5)$
$q_{6,5} = w_3 e^{-w_3 t}$	$p_{6,5} = 1$
$q_{7,5} = w_2 e^{-w_2 t} \bar{g}(t)$	$p_{7,5} = 1 - g^*(w_2)$
$q_{7,0} = f(t) e^{-w_2 t}$	$p_{7,0} = g^*(w_2)$

Mean Sojourn Times

Table 6

$R_i(t)$	$\mu_i = R_i^*(0)$
$R_0^{(t)} = e^{-(\lambda_1 + \lambda_3 + \lambda_5)t}$	$\mu_0 = 1 / (\lambda_1 + \lambda_3 + \lambda_5)$
$R_1^{(t)} = e^{-w_3 t}$	$\mu_1 = 1 / w_3$
$R_2^{(t)} = e^{-w_3 t}$	$\mu_2 = 1 / w_3$
$R_3^{(t)} = e^{-(\lambda_2 + \lambda_5)t}$	$\mu_3 = 1 / (\lambda_2 + \lambda_5)$
$R_4^{(t)} = e^{-w_1 t}$	$\mu_4 = 1 / w_1$
$R_5^{(t)} = e^{-(\lambda_5 + \lambda_n)t}$	$\mu_5 = 1 / (\lambda_n + \lambda_5)$
$R_6^{(t)} = e^{-w_3 t}$	$\mu_6 = 1 / w_3$
$R_7^{(t)} = e^{-w_2 t} \bar{g}(t)$	$\mu_7 = 1 - g^*(w_2) / w_3$

Probability from state ‘5’ to different vertices are given as

$$\begin{aligned}
 V_{5,0} &= (5,7,0)/1 - (0,1,0) = p_{5,7} p_{7,0} / 1 - p_{0,1} p_{1,0} = \{(\lambda_n/\lambda_n + \lambda_5) f^* w_2\} / \{1 - \lambda_3/\lambda_1 + \lambda_3 + \lambda_5\} \\
 &= (f^* w_2 \lambda_n / (\lambda_n + \lambda_5)) (\lambda_1 + \lambda_3 + \lambda_5 / \lambda_1 + \lambda_3) = (f^* w_2 \lambda_n \lambda_1 + \lambda_3 + \lambda_5) / (\lambda_1 + \lambda_3) (\lambda_n + \lambda_5) \\
 V_{5,1} &= (5,7,0,3,1)/1 - (0,1,0) = p_{5,7} p_{7,0} p_{0,1} / (1 - p_{0,1} p_{1,0}) \\
 &= \{ \lambda_n / (\lambda_n + \lambda_5) g^* w_2 \lambda_5 / (\lambda_1 + \lambda_3 + \lambda_5) \} / \{ 1 - (\lambda_5 / (\lambda_1 + \lambda_3 + \lambda_5)) \} \\
 &= \{ (g^* w_2 \lambda_n \lambda_5) / (\lambda_n + \lambda_5) (\lambda_1 + \lambda_3 + \lambda_5) \} \{ (\lambda_1 + \lambda_3 + \lambda_5 / \lambda_1 + \lambda_3) \} \\
 &= (g^* w_2 \lambda_n \lambda_5) / (\lambda_1 + \lambda_3) (\lambda_n + \lambda_5) \\
 V_{5,2} &= (5,7,0,3,2)/1 - (0,1,0) 1 - (3,2,3) = p_{5,7} p_{7,0} p_{0,3} p_{3,2} / (1 - p_{0,1} p_{1,0}) (1 - p_{3,2} p_{2,3}) \\
 &= \{ \lambda_n / (\lambda_n + \lambda_5) g^* w_2 \lambda_1 / (\lambda_1 + \lambda_3 + \lambda_5) \lambda_5 / (\lambda_2 + \lambda_5) \} / \{ 1 - (\lambda_5 / (\lambda_1 + \lambda_3 + \lambda_5)) 1 - (\lambda_5 / (\lambda_2 + \lambda_5)) \} \\
 &= \{ (g^* w_2 \lambda_1 \lambda_n \lambda_5) / (\lambda_1 + \lambda_5) (\lambda_2 + \lambda_5) (\lambda_1 + \lambda_3 + \lambda_5) \} / \{ (\lambda_1 + \lambda_3 / \lambda_1 + \lambda_3 + \lambda_5) (\lambda_2 / \lambda_2 + \lambda_5) \} \\
 &= \{ (g^* w_2 \lambda_1 \lambda_n \lambda_5) / (\lambda_1 + \lambda_5) (\lambda_2 + \lambda_5) (\lambda_1 + \lambda_3 + \lambda_5) \} \{ (\lambda_1 + \lambda_3 + \lambda_5) (\lambda_2 + \lambda_5) / \lambda_2 (\lambda_1 + \lambda_3) \} \\
 &= (g^* \lambda_1 \lambda_n \lambda_5) / \lambda_2 (\lambda_1 + \lambda_3) (\lambda_1 + \lambda_5) \\
 V_{5,3} &= (5,7,0,3)/1 - (0,1,0) 1 - (3,2,3) = p_{5,7} p_{7,0} p_{0,3} / (1 - p_{0,1} p_{1,0}) (1 - p_{3,2} p_{2,3}) \\
 &= \{ \lambda_n / (\lambda_n + \lambda_5) g^* w_2 \lambda_1 / (\lambda_1 + \lambda_3 + \lambda_5) \} / \{ 1 - (\lambda_5 / (\lambda_1 + \lambda_3 + \lambda_5)) 1 - (\lambda_5 / (\lambda_2 + \lambda_5)) \} \\
 &= \{ (g^* w_2 \lambda_1 \lambda_n) / (\lambda_n + \lambda_5) (\lambda_1 + \lambda_3 + \lambda_5) \} / \{ (\lambda_1 + \lambda_3 / \lambda_1 + \lambda_3 + \lambda_5) (\lambda_2 / \lambda_2 + \lambda_5) \} \\
 &= \{ (g^* w_2 \lambda_1 \lambda_n) / (\lambda_n + \lambda_5) (\lambda_1 + \lambda_3 + \lambda_5) \} \{ (\lambda_1 + \lambda_3 + \lambda_5) (\lambda_2 + \lambda_5) / \lambda_2 (\lambda_1 + \lambda_3) \} \\
 &= (g^* w_2 \lambda_1 \lambda_n) (\lambda_2 + \lambda_5) / \lambda_2 (\lambda_1 + \lambda_3) (\lambda_n + \lambda_5) \\
 V_{5,4} &= (5,7,0,3,4)/1 - (0,1,0) 1 - (3,2,3) + (5,7,0,4) / (1 - 0,1,0) \\
 &= p_{5,7} p_{7,0} p_{0,3} p_{3,4} / (1 - p_{0,1} p_{1,0}) (1 - p_{3,2} p_{2,3}) + p_{5,7} p_{7,0} p_{0,4} / (1 - p_{0,1} p_{1,0}) \\
 &= \{ \lambda_n / (\lambda_n + \lambda_5) g^* w_2 \lambda_1 / (\lambda_1 + \lambda_3 + \lambda_5) \lambda_2 / (\lambda_2 + \lambda_5) \} / \{ 1 - (\lambda_5 / (\lambda_1 + \lambda_3 + \lambda_5)) 1 - (\lambda_5 / (\lambda_2 + \lambda_5)) \} \\
 &+ \{ \lambda_n / (\lambda_n + \lambda_5) g^* w_2 \lambda_3 / (\lambda_1 + \lambda_3 + \lambda_5) \} / \{ 1 - (\lambda_5 / (\lambda_1 + \lambda_3 + \lambda_5)) \} \\
 &= \{ \lambda_n / (\lambda_n + \lambda_5) g^* w_2 \lambda_1 / (\lambda_1 + \lambda_3 + \lambda_5) \lambda_2 / (\lambda_2 + \lambda_5) + \lambda_n / (\lambda_n + \lambda_5) g^* w_2 \lambda_3 / (\lambda_1 + \lambda_3 + \lambda_5) \lambda_2 / (\lambda_2 + \lambda_5) \} \\
 &/ \{ 1 - (\lambda_5 / (\lambda_1 + \lambda_3 + \lambda_5)) 1 - (\lambda_5 / (\lambda_2 + \lambda_5)) \} \\
 &= \{ (g^* w_2 \lambda_n) / (\lambda_n + \lambda_5) (\lambda_2 / \lambda_2 + \lambda_5) (\lambda_1 / \lambda_1 + \lambda_3 + \lambda_5) + (\lambda_3 / \lambda_1 + \lambda_3 + \lambda_5) \} / (\lambda_1 + \lambda_3 / \lambda_1 + \lambda_3 + \lambda_5) (\lambda_2 / \lambda_2 + \lambda_5) \\
 &= \{ (g^* w_2 \lambda_n) / (\lambda_n + \lambda_5) (\lambda_2 / \lambda_2 + \lambda_5) (\lambda_1 + \lambda_3) / (\lambda_1 + \lambda_3 + \lambda_5) (\lambda_1 + \lambda_3 + \lambda_5 / \lambda_1 + \lambda_3) (\lambda_1 + \lambda_3 / \lambda_2) \} \\
 &= (g^* \lambda_n w_2) / (\lambda_n + \lambda_5) \\
 V_{5,5} &= (5,7,0,3,4,5)/1 - (0,1,0) 1 - (3,2,3) + (5,7,0,4,5) / (1 - 0,1,0) + (5,6,5) + (5,7,5) \\
 &= p_{5,7} p_{7,0} p_{0,3} p_{0,4} p_{4,5} / (1 - p_{0,1} p_{1,0}) (1 - p_{3,2} p_{2,3}) + p_{5,7} p_{7,0} p_{0,4} p_{4,5} / (1 - p_{0,1} p_{1,0}) + p_{5,6} p_{6,5} + p_{5,7} p_{7,5} \\
 &= \{ \lambda_n / (\lambda_n + \lambda_5) g^* w_2 \lambda_1 / (\lambda_1 + \lambda_3 + \lambda_5) \lambda_2 / (\lambda_2 + \lambda_5) \} / \{ 1 - (\lambda_5 / (\lambda_1 + \lambda_3 + \lambda_5)) 1 - (\lambda_5 / (\lambda_2 + \lambda_5)) \} \\
 &+ \{ \lambda_n / (\lambda_n + \lambda_5) g^* w_2 \lambda_3 / (\lambda_1 + \lambda_3 + \lambda_5) \} / \{ 1 - (\lambda_5 / (\lambda_1 + \lambda_3 + \lambda_5)) \} + (\lambda_5 / (\lambda_n + \lambda_5)) + (\lambda_n / (\lambda_n + \lambda_5)) \\
 &= \{ g^* w_2 \lambda_n / (\lambda_n + \lambda_5) \lambda_1 / (\lambda_1 + \lambda_3 + \lambda_5) \lambda_2 / (\lambda_2 + \lambda_5) / (\lambda_1 + \lambda_3 / \lambda_1 + \lambda_3 + \lambda_5) (\lambda_2 / \lambda_2 + \lambda_5) + \{ g^* (w_2) \} \\
 &(\lambda_n / \lambda_n + \lambda_5) \lambda_3 / (\lambda_1 + \lambda_3 + \lambda_5) \} / \{ (\lambda_1 + \lambda_3) / (\lambda_1 + \lambda_3 + \lambda_5) \} + 1 - f^* w_2 (\lambda_n / \lambda_n + \lambda_5) \\
 &= \{ (g^* w_2 \lambda_n) / (\lambda_n + \lambda_5) (\lambda_1 / \lambda_1 + \lambda_3 + \lambda_5) (\lambda_2 / \lambda_2 + \lambda_5) + g^* w_2 (\lambda_n / \lambda_n + \lambda_5) (\lambda_3 / \lambda_1 + \lambda_3 + \lambda_5) (\lambda_2 / \lambda_2 + \lambda_5) \} \\
 &/ \{ (\lambda_1 + \lambda_3 / \lambda_1 + \lambda_3 + \lambda_5) (\lambda_2 / \lambda_2 + \lambda_5) \} - g^* (w_2) (\lambda_n / \lambda_n + \lambda_5) \\
 &= \{ (g^* w_2 \lambda_n / \lambda_n + \lambda_5) (\lambda_2 / \lambda_2 + \lambda_5) (\lambda_1 / \lambda_1 + \lambda_3 + \lambda_5) + (\lambda_3 / \lambda_1 + \lambda_3 + \lambda_5) \} / (\lambda_1 + \lambda_3 / \lambda_1 + \lambda_3 + \lambda_5) (\lambda_2 / \lambda_2 + \lambda_5) \\
 &- g^* (w_2) (\lambda_n / \lambda_n + \lambda_5) \\
 &= g^* (w_2) \lambda_n / (\lambda_n + \lambda_5) (\lambda_1 + \lambda_3 / \lambda_1 + \lambda_3 + \lambda_5) - (\lambda_1 + \lambda_3 / \lambda_1 + \lambda_3 + \lambda_5) + 1 \\
 &= 0 + 1 = 1 \\
 &= g^* (w_2) (\lambda_n / \lambda_n + \lambda_5) \lambda_3 (\lambda_1 \lambda_2 + \lambda_1 \lambda_5 + \lambda_2 \lambda_1 + \lambda_2 \lambda_3 + \lambda_2 \lambda_5) / \lambda_2 (\lambda_1 + \lambda_3) (\lambda_1 + \lambda_3 + \lambda_5) + (\lambda_5 + \lambda_n 1 - g^* w_2) \\
 &/ (\lambda_n + \lambda_5) \\
 V_{5,6} &= (5,6) = p_{5,6} = \lambda_5 / (\lambda_n + \lambda_5), \quad V_{5,7} = (5,7) = p_{5,7} = \lambda_n / (\lambda_n + \lambda_5), \\
 V_{0,3} &= (0,3) / (1 - 3,2,3), \quad = p_{0,3} / 1 - p_{3,2} p_{2,3}, \quad = (\lambda_1 / \lambda_1 + \lambda_3 + \lambda_5) / (1 - \lambda_5 / \lambda_2 + \lambda_5) \\
 &= (\lambda_1 / \lambda_1 + \lambda_3 + \lambda_5) (\lambda_2 + \lambda_3 / \lambda_2), \quad = (\lambda_1 \lambda_2 + \lambda_1 \lambda_3) (\lambda_1 \lambda_2 + \lambda_3 \lambda_2 + \lambda_2 \lambda_5) \\
 V_{0,0} &= (0,1,0) + (0,3,4,5,7,0) / (1 - 3,2,3) (1 - 5,7,5) (1 - 5,6,5) + (0,4,5,7,0) / (1 - 5,6,5) (1 - 5,7,5) \\
 &= p_{0,1} p_{1,0} + p_{0,3} p_{3,4} p_{4,5} p_{5,7} p_{7,0} / (1 - p_{3,2} p_{2,3}) (1 - p_{5,7} p_{7,5}) (1 - p_{5,6} p_{6,5}) + p_{0,4} p_{4,5} p_{5,7} p_{7,5} / (1 - p_{5,6} p_{6,5}) \\
 &(1 - p_{5,7} p_{7,5}) = (\lambda_1 / \lambda_1 + \lambda_3 + \lambda_5) / (1 - \lambda_5 / \lambda_2 + \lambda_5) \\
 &= (\lambda_1 / \lambda_1 + \lambda_3 + \lambda_5) (\lambda_2 + \lambda_3 / \lambda_2) = (\lambda_1 \lambda_2 + \lambda_1 \lambda_3) (\lambda_1 \lambda_2 + \lambda_3 \lambda_2 + \lambda_2 \lambda_5)
 \end{aligned}$$

MTSF (T₀): The regenerative un-failed states to which the system can transit (initial state ‘0’), before entering any failed state are: ‘i’ = 0,3 taking ‘ξ’ = ‘0’.

$$MTSF (T_0) = \left[\sum_{i,sr} \left\{ \frac{\left\{ \text{pr} \left(\xi \xrightarrow{sr(sff)} i \right) \right\} \mu_i}{\prod_{m_1 \neq \xi} \{ 1 - V_{m_1 m_1} \}} \right\} \right] \div \left[1 - \sum_{sr} \left\{ \frac{\left\{ \text{pr} \left(\xi \xrightarrow{sr(sff)} \xi \right) \right\}}{\prod_{m_2 \neq \xi} \{ 1 - V_{m_2 m_2} \}} \right\} \right]$$

$$\begin{aligned}
 T_0 &= (0,0) \mu_0 + (0,3) \mu_3 = V_{0,0} \mu_0 + V_{0,3} \mu_3 \\
 &= 1 / (\lambda_1 + \lambda_3 + \lambda_5) + \lambda_1 (\lambda_2 + \lambda_3) / \lambda_2 (\lambda_1 + \lambda_3 + \lambda_5) (1 / \lambda_2 + \lambda_5) \\
 &= \lambda_2 (\lambda_2 + \lambda_5) + \lambda_1 (\lambda_2 + \lambda_3) / \lambda_2 (\lambda_2 + \lambda_5) (\lambda_1 + \lambda_3 + \lambda_5) \\
 &= \lambda (2\lambda) + \lambda (2\lambda) / \lambda (2\lambda) + 3\lambda = 4\lambda^2 / 6\lambda^3 = (2/3) (1/\lambda)
 \end{aligned}$$

Availability of the System: The regenerative states at which the system is available are ‘j’ = 0,3,5 and the regenerative states are ‘i’ = 0 to 7 taking ‘ξ’ = ‘5’ the total fraction of time for which the system is available is given by

$$A_0 = \left[\sum_{j,sr} \left\{ \frac{\left\{ \text{pr} (\xi^{sr} \rightarrow j) \right\} f_{j, \mu_j}}{\prod_{m_1 \neq \xi} \{ 1 - V_{m_1 m_1} \}} \right\} \right] \div \left[\sum_{i,sr} \left\{ \frac{\left\{ \text{pr} (\xi^{sr} \rightarrow i) \right\} \mu_i^1}{\prod_{m_2 \neq \xi} \{ 1 - V_{m_2 m_2} \}} \right\} \right]$$

$$A_0 = \left[\sum_j V_{\xi,j}, f_j, \mu_j \right] \div \left[\sum_i V_{\xi,i}, f_j, \mu_i^1 \right]$$

$$A_0 = V_{5,5}f_5\mu_5 + V_{5,0}f_0\mu_0 + V_{5,3}f_3\mu_3 + V_{5,0}\mu_0 + V_{5,1}\mu_1 + V_{5,2}\mu_2 + V_{5,3}\mu_3 + V_{5,4}\mu_4 + V_{5,5}\mu_5 + V_{5,6}\mu_6 + V_{5,7}\mu_7$$

$$= \left[\left(\frac{1}{\lambda_n + \lambda_5} \right) + g^*(w_2)\lambda_n / (\lambda_1 + \lambda_3)(\lambda_n + \lambda_5) + g^*(w_2)\lambda_1\lambda_n / \lambda_2(\lambda_1 + \lambda_3)(\lambda_n + \lambda_5) \right] / \left[g^*(w_2)\lambda_n / (\lambda_1 + \lambda_3)(\lambda_n + \lambda_5) + g^*(w_2)\lambda_n\lambda_5 / w_3(\lambda_1 + \lambda_3)(\lambda_n + \lambda_5) + g^*(w_2)\lambda_1(\lambda_n + \lambda_5) / w_3\lambda_2(\lambda_1 + \lambda_3)(\lambda_1 + \lambda_5) + g^*(w_2)\lambda_1\lambda_n\lambda_5 / w_3\lambda_2(\lambda_1 + \lambda_3)(\lambda_1 + \lambda_5) + g^*(w_2)\lambda_1\lambda_n / \lambda_2(\lambda_1 + \lambda_3)(\lambda_n + \lambda_5) + \lambda_n g^*(w_2) / w_1(\lambda_n + \lambda_5) + 1 / w_1(\lambda_n + \lambda_5) + \lambda_n / w_3(\lambda_n + \lambda_5) + 1 - g^*(w_2)\lambda_n / w_2(\lambda_n + \lambda_5) \right]$$

$$= (3w/4w+6\lambda), \text{ for particular case}$$

Busy Period of the Server: The regenerative states where server is busy are ‘i’ = 1,2,4,6,7 taking ξ = ‘5’, the total fraction of time for which the server remains busy is

$$B_0 = \left[\sum_{j,SR} \left\{ \frac{\{pr(\xi^{SR \rightarrow j})\}n_j}{\prod_{m_1 \neq \xi} \{1 - V_{m_1 m_1}\}} \right\} \right] \div \left[\sum_{i,SR} \left\{ \frac{\{pr(\xi^{SR \rightarrow i})\}\mu_i^1}{\prod_{m_2 \neq \xi} \{1 - V_{m_2 m_2}\}} \right\} \right] = \left[\sum_j V_{\xi,j}, n_j \right] \div \left[\sum_i V_{\xi,i}, \mu_i^1 \right]$$

$$B_0 = (V_{5,6}\mu_6 + V_{5,7}\mu_7 + V_{5,1}\mu_1 + V_{5,2}\mu_2 + V_{5,4}\mu_4) / (V_{5,0}\mu_0 + V_{5,1}\mu_1 + V_{5,2}\mu_2 + V_{5,3}\mu_3 + V_{5,4}\mu_4 + V_{5,5}\mu_5 + V_{5,6}\mu_6 + V_{5,7}\mu_7)$$

$$= \left[\left(\frac{\lambda_5}{\lambda_n + \lambda_5} \right) \left(\frac{1}{w_3} \right) \left(\frac{\lambda_n}{\lambda_n + \lambda_5} \right) + 1 - g^*(w_2) / w_2 + g^*(w_2)\lambda_n\lambda_5 / w_3(\lambda_1 + \lambda_3)(\lambda_n + \lambda_5) + g^*(w_2)\lambda_1\lambda_n\lambda_5 / w_2\lambda_2(\lambda_1 + \lambda_3)(\lambda_1 + \lambda_5) + \lambda_n g^*(w_2) / w_1(\lambda_n + \lambda_5) \right] / \left[g^*(w_2)\lambda_n / (\lambda_1 + \lambda_3)(\lambda_n + \lambda_5) + g^*(w_2)\lambda_n\lambda_5 / w_3(\lambda_1 + \lambda_3)(\lambda_n + \lambda_5) + g^*(w_2)\lambda_1\lambda_n\lambda_5 / w_3\lambda_2(\lambda_1 + \lambda_3)(\lambda_1 + \lambda_5) + g^*(w_2)\lambda_1\lambda_n / \lambda_2(\lambda_1 + \lambda_3)(\lambda_n + \lambda_5) + g^*(w_2)\lambda_n / w_1(\lambda_n + \lambda_5) + 1 / (\lambda_n + \lambda_5) + \lambda_5 / w_3(\lambda_n + \lambda_5) + 1 - g^*(w_2)\lambda_n / w_2(\lambda_n + \lambda_5) \right]$$

$$= (5\lambda/4w+6\lambda), \text{ for particular case}$$

Expected Fractional Number of Inspections by the repair man: The regenerative states where the repair man do this job j = 1 the regenerative states are i = 0 to 10, Taking ‘ξ’ = ‘0’, the number of visit by the repair man is given by

$$V_0 = \left[\sum_{j,SR} \left\{ \frac{\{pr(\xi^{SR \rightarrow j})\}}{\prod_{k_1 \neq \xi} \{1 - V_{k_1 k_1}\}} \right\} \right] \div \left[\sum_{i,SR} \left\{ \frac{\{pr(\xi^{SR \rightarrow i})\}\mu_i^1}{\prod_{k_2 \neq \xi} \{1 - V_{k_2 k_2}\}} \right\} \right] = \left[\sum_j V_{\xi,j} \right] \div \left[\sum_i V_{\xi,i}, \mu_i^1 \right]$$

$$V_0 = (V_{5,7} + V_{5,1} + V_{5,2} + V_{5,4} + V_{5,6}) / (V_{5,0}\mu_0 + V_{5,1}\mu_1 + V_{5,2}\mu_2 + V_{5,3}\mu_3 + V_{5,4}\mu_4 + V_{5,5}\mu_5 + V_{5,6}\mu_6 + V_{5,7}\mu_7)$$

$$= \left[\left(\frac{\lambda_n}{\lambda_n + \lambda_5} \right) + g^*(w_2)\lambda_n\lambda_5 / (\lambda_1 + \lambda_3)(\lambda_n + \lambda_5) + g^*(w_2)\lambda_1\lambda_n\lambda_5 / \lambda_2(\lambda_1 + \lambda_3)(\lambda_1 + \lambda_5) + g^*(w_2)\lambda_n / (\lambda_n + \lambda_5) + \lambda_5 / (\lambda_n + \lambda_5) \right] / \left[g^*(w_2)\lambda_n / (\lambda_1 + \lambda_3)(\lambda_n + \lambda_5) + g^*(w_2)\lambda_n\lambda_5 / w_3(\lambda_1 + \lambda_3)(\lambda_n + \lambda_5) + g^*(w_2)\lambda_1\lambda_n\lambda_5 / w_3\lambda_2(\lambda_1 + \lambda_3)(\lambda_1 + \lambda_5) + g^*(w_2)\lambda_1\lambda_n / \lambda_2(\lambda_1 + \lambda_3)(\lambda_n + \lambda_5) + g^*(w_2)\lambda_n / w_1(\lambda_n + \lambda_5) + 1 / (\lambda_n + \lambda_5) + \lambda_5 / w_3(\lambda_n + \lambda_5) + 1 - g^*(w_2)\lambda_n / w_2(\lambda_n + \lambda_5) \right]$$

In particular case

Mean Time to System Failure $T_0 = 2/3\lambda$

Table 7

λ	0.005	0.006	0.007
T ₀	133.33	111.11	95.24

Availability of the System

Table 8

$$= w(5\lambda+1)/(4w+6\lambda),$$

λ/w	w = 0.8	w = 0.9	w = 1.0
λ = 0.05	0.74303	0.74380	0.74441
λ = 0.06	0.74165	0.74257	0.74331
λ = 0.07	0.74028	0.74135	0.74220

MTSF Graph

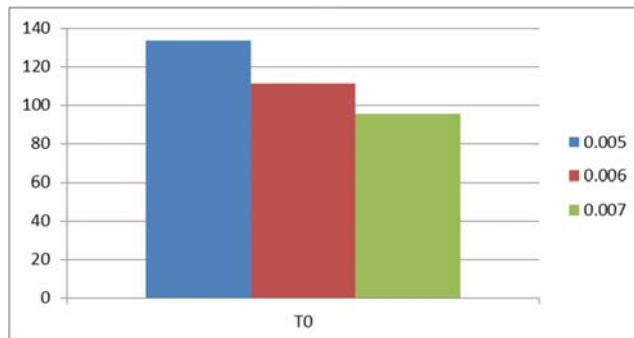


Fig 2

Availability of the System Graph

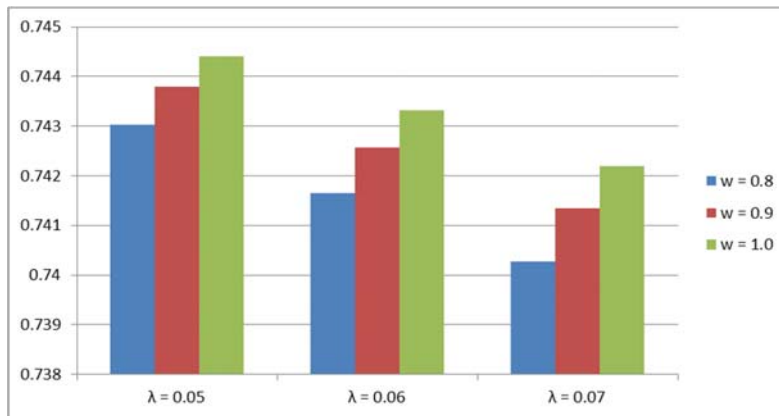


Fig 3

Busy Period of Server

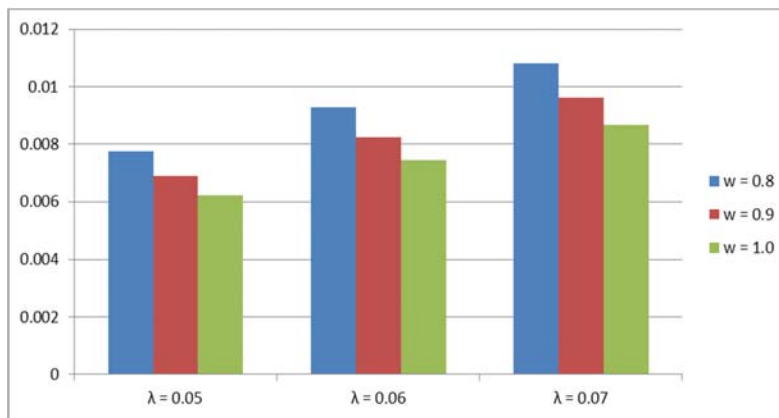


Fig 4

Expected Number of Server's Visits

$$V_0 = w(5\lambda + \lambda) / 4w + 6\lambda$$

Table 10

λ/w	$w = 0.8$	$w = 0.9$	$w = 1.0$
$\lambda = 0.05$	0.253369	0.254132	0.254342
$\lambda = 0.06$	0.254635	0.254951	0.255203
$\lambda = 0.07$	0.255398	0.255766	0.256061

Expected Number of Server's Visits Graph

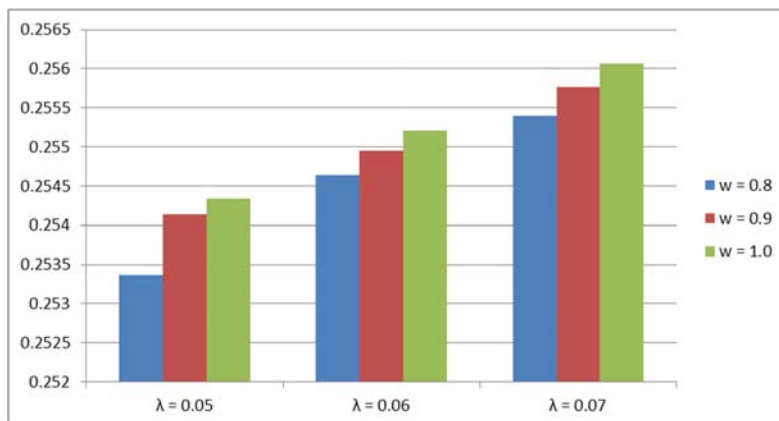


Fig 5

Profit Function

$$\text{Profit} = A_0R_0 - (B_0R_1 + V_0R_2),$$

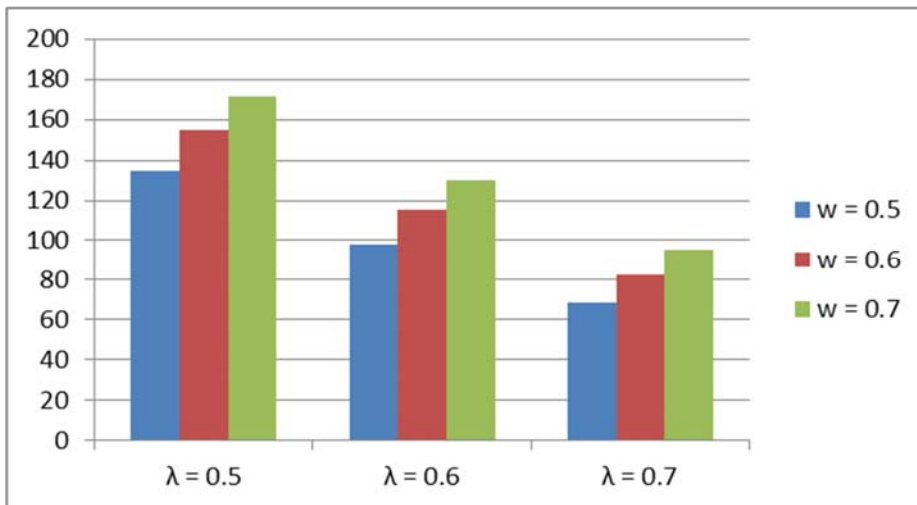
Where A_0 = Availability of System, B_0 = Busy Period of Server

V_0 = Expected Number of Inspection by the Repair Man, R_0 = Revenue

R_1 = Busy Period per Unit, R_2 = Per Visit Cost, $R_0 = 1000$, $R_1 = 50$

$R_2 = 100$

Profit Function Graph



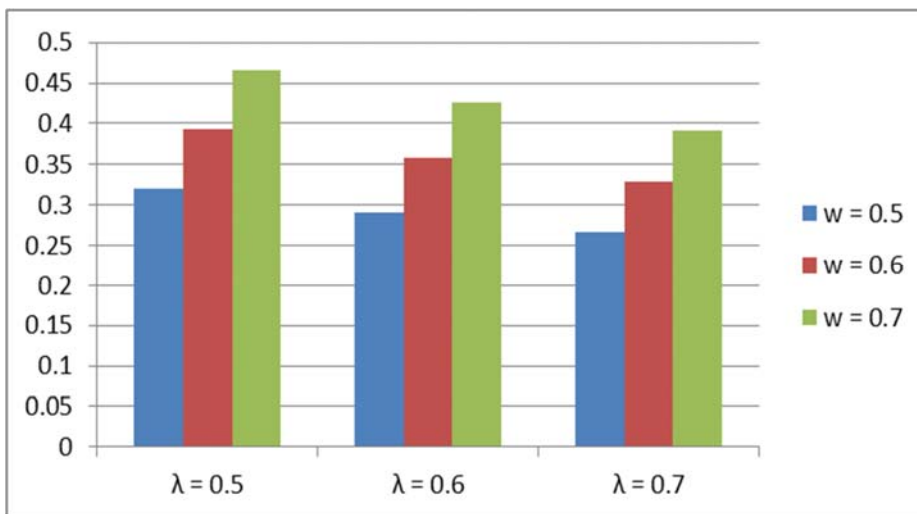
$$\text{Profit} = [(2000w - 250\lambda - 2000w\lambda) / (4w + 6\lambda)]$$

	w = 0.5	w = 0.6	w = 0.7
λ = 0.5	135.0000	154.62963	171.55172
λ = 0.6	98.21428	115.0000	129.6875
λ = 0.7	68.54839	82.57576	95.0000

Corollary: - When Intermediate repair is Feasible

$$\text{MTSF} (T_0) = [2w(w + \lambda)^2] / [2(w + \lambda)^2 + \lambda(2\lambda + w)^2]$$

MTSF Graph

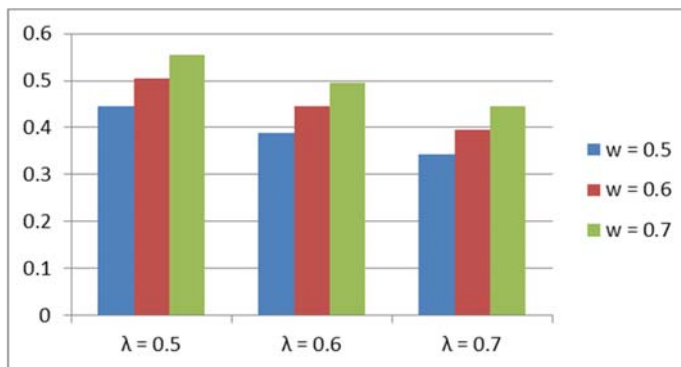


MTSF Table

T ₀	w = 0.5	w = 0.6	w = 0.7
λ = 0.5	0.320000	0.392432	0.466127
λ = 0.6	0.291286	0.358209	0.426614
λ = 0.7	0.266321	0.328155	0.391608

Availability of the System (A₀) = $[2w(w+\lambda)]/[2w^2+3\lambda w+4\lambda^2]$

Availability of the System Graph

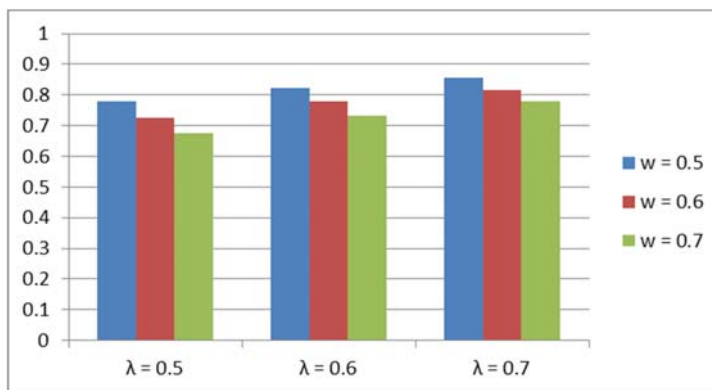


Availability of the System Table

A ₀	w = 0.5	w = 0.6	w = 0.7
λ = 0.5	0.444444	0.503817	0.554455
λ = 0.6	0.387324	0.444444	0.494565
λ = 0.7	0.341880	0.395939	0.444444

Busy Period of the Server (B₀) = $1-[2w^2/(2w^2+3\lambda w+4\lambda^2)]$

Busy Period of the Server Graph

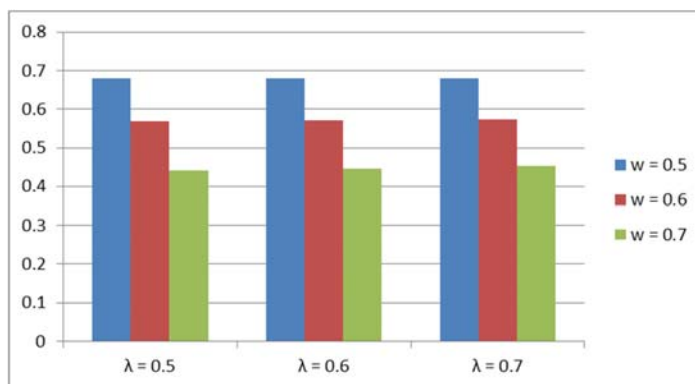


Busy Period of the Server Table

B ₀	w = 0.5	w = 0.6	w = 0.7
λ = 0.5	0.777778	0.725191	0.676568
λ = 0.6	0.823944	0.777778	0.733696
λ = 0.7	0.857550	0.817259	0.777778

Expected Number of Server's Visits (V₀) = $1-[2w(w+\lambda)]/[2(w+\lambda)^2+\lambda(2\lambda+w)^2]$

Expected Number of Server's Visits Graph

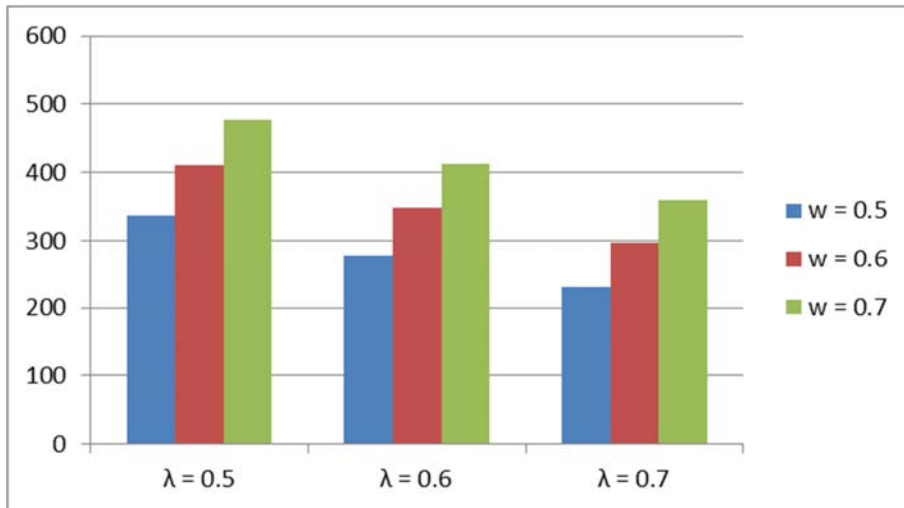


V_0	$w = 0.5$	$w = 0.6$	$w = 0.7$
$\lambda = 0.5$	0.680000	0.568325	0.440648
$\lambda = 0.6$	0.679585	0.570150	0.445402
$\lambda = 0.7$	0.680415	0.573399	0.451749

Profit Function

$$\text{Profit} = [(1900w^2 + 2000w\lambda) / (2w^2 + 3\lambda w + 4\lambda^2)] - [200w(w + \lambda) / 2(w + \lambda)^2 + \lambda(2\lambda + w)^2] - 150$$

Profit Function Graph



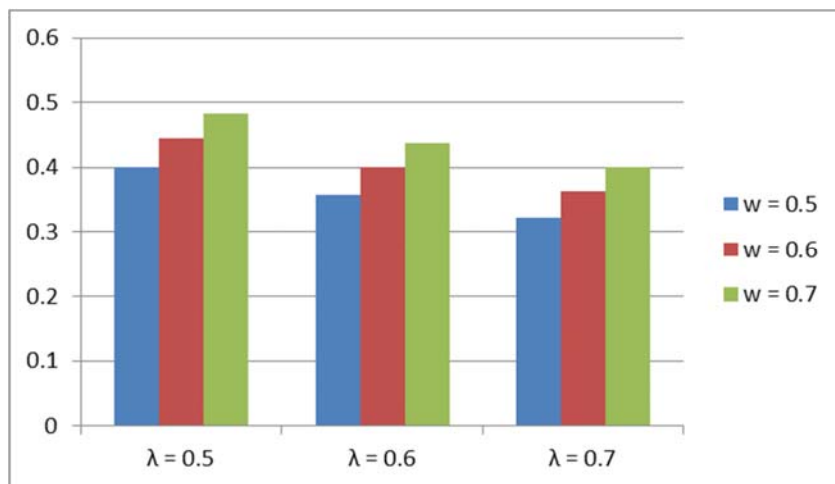
Profit Function Table

	$w = 0.5$	$w = 0.6$	$w = 0.7$
$\lambda = 0.5$	337.55551	410.72495	476.56180
$\lambda = 0.6$	278.16830	348.54055	413.34000
$\lambda = 0.7$	230.96100	297.73615	360.38021

Corollary: - Perfect repair post failure

$$\text{MTSF} (T_0) = [2w / (2w + 3\lambda)]$$

MTSF Graph

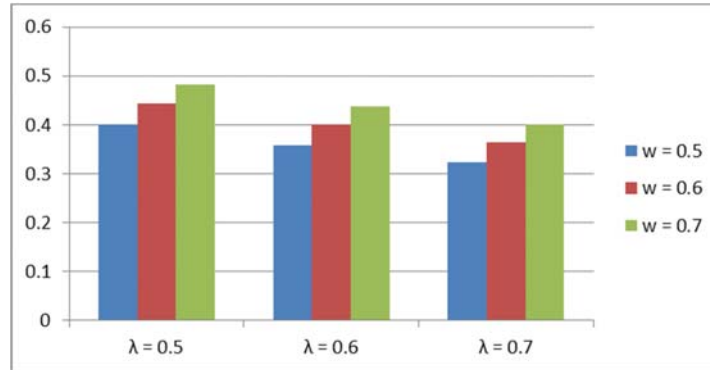


MTSF Table

T_0	$w = 0.5$	$w = 0.6$	$w = 0.7$
$\lambda = 0.5$	0.400000	0.444444	0.482759
$\lambda = 0.6$	0.357143	0.400000	0.437500
$\lambda = 0.7$	0.322581	0.363636	0.400000

Availability(A_0)

Availability of the System Graph

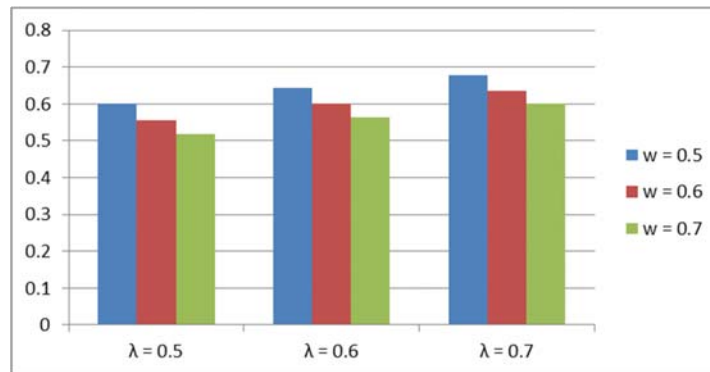


Availability of the System Table

A_0	$w = 0.5$	$w = 0.6$	$w = 0.7$
$\lambda = 0.5$	0.400000	0.444444	0.482759
$\lambda = 0.6$	0.357143	0.400000	0.437500
$\lambda = 0.7$	0.322581	0.363636	0.400000

Busy Period of the Server (B_0) = $[3\lambda/(2w+3\lambda)]$

Busy Period of the Server Graph

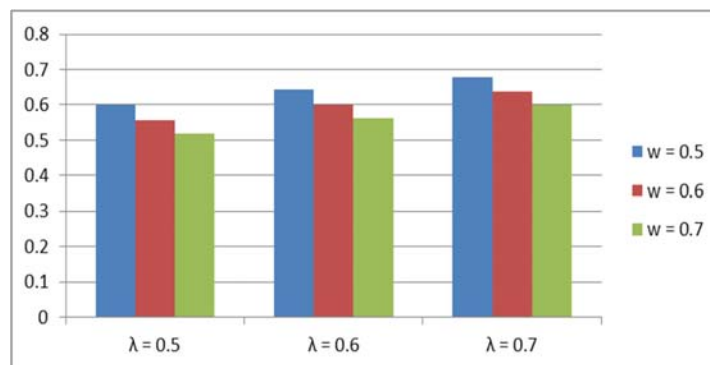


Busy Period of the Server Table

B_0	$w = 0.5$	$w = 0.6$	$w = 0.7$
$\lambda = 0.5$	0.600000	0.555556	0.517241
$\lambda = 0.6$	0.642857	0.600000	0.562500
$\lambda = 0.7$	0.677414	0.636363	0.600000

Expected Fractional Number of Server's Visits (V_0)

Expected Number of Server's Visits Graph



Expected Number of Server’s Visits Table

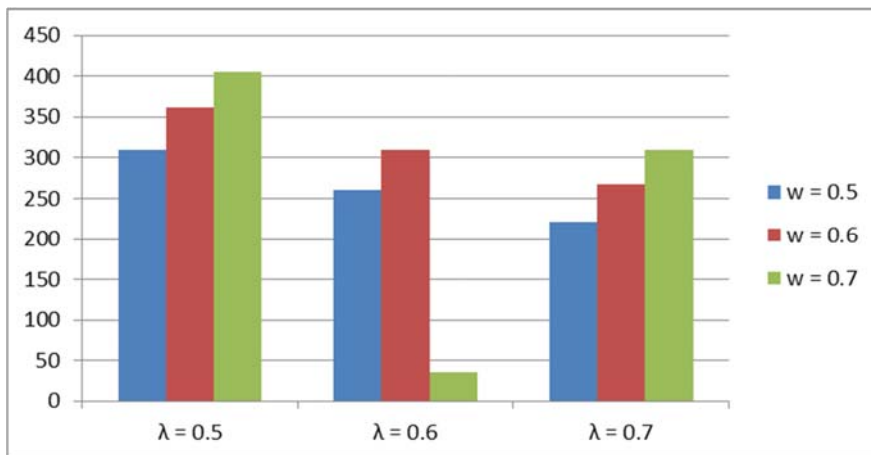
V_0	$w = 0.5$	$w = 0.6$	$w = 0.7$
$\lambda = 0.5$	0.600000	0.555556	0.517241
$\lambda = 0.6$	0.642857	0.600000	0.562500
$\lambda = 0.7$	0.677414	0.636363	0.600000

Profit Function

Profit = $A_0R_0 - B_0R_1 - V_0R_2$

Profit = $[(1900w^2 + 2000w\lambda) / (2w^2 + 3\lambda w + 4\lambda^2)] - [200w(w + \lambda) / 2(w + \lambda)^2 + \lambda(2\lambda + w)^2] - 150$

Profit Function Graph



	$w = 0.5$	$w = 0.6$	$w = 0.7$
$\lambda = 0.5$	310.0000	361.111066	405.17285
$\lambda = 0.6$	260.71445	310.00000	35.0125
$\lambda = 0.7$	220.9689	268.18155	310.000

Conclusion: From above tables and graph we see that the result obtained using Regenerative Point Graphical Technique is same as obtained by using Regenerative Point Technique and other techniques. But in Regenerative Point Graphical Technique, we obtained the results very easily and quickly without writing any state equations and without any cumbersome procedures, long calculations and simplifications. Regenerative Point Graphical Technique is applied to study the behavior and profit analysis of various process industries like soap, soft drink and dairy plant, paper industry, soap industry etc. It is hoped that the Regenerative Point Graphical Technique for the analysis of the system will be very helpful to the managements, manufactures and the personal engaged in reliability engineering and working for the behavior and profit analysis of stochastic systems.

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