International Journal of Applied Research 2016; 2(2): 788-799

ISSN Print: 2394-7500
ISSN Online: 2394-5869
Impact Factor: 5.2
IJAR 2016; 2(2): 788-799
www.allresearchjournal.com
Received: 15-12-2015
Accepted: 18-01-2016

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Availability modeling of three unit system with degradation in one unit (Post Repair)

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Abstract
In this paper, Availability Modeling of Three Unit System with Degradation in One Unit (Post Repair) undergoing degradation after complete failure using Regenerative Point Graphical Technique (RPGT) is discussed. Initially all units are working at full capacity out of which one unit may have two types of failures, one is direct and second one is through partial failure mode. There is a single server (repairman), who inspects and repairs the units on each failure. On each repair a unit under goes degradation if the server reports that unit is not repairable then it is replaced by a new one, which follows a general distribution other units have perfect repair. Fuzzy concept is used to determine failure/working state of a unit. Taking failure rates exponential, repair rates general and taking into consideration various probabilities, a transition diagram of the system is developed to find Primary circuits, Secondary circuits & Tertiary circuits and Base state. Problem is solved using RPGT to determine system parameters. System behavior is discussed with the help of graphs and tables.

Keywords: Availability, Reliability, Degraded state, Base-State, Regenerative Point Graphical Technique (RPGT), Mean Time to System Failure, Fuzzy Logic, Steady State

Introduction
Various Mechanical systems are assembly of a number of units in which each unit is important for the system to work efficiently. If a single unit fails, then the whole system fails. Considering the importance of individual units in system Kumar, J. & Malik, S. C. [1] have discussed the concept of preventive maintenance for a single unit system. Liu., R. [2], Malik, S. C. [3], Nakagawa, T. and Osaki, S. [4] have discussed reliability analysis of a one unit system with un-repairable spare units and its applications. Goel, P. & Singh J. [5], Gupta, V. K., Singh, J. & Kumar Kuldeep [6], Kumar, S. & Goel, P. [7], Gupta, V. K. [8], Chaudhary, Goel & Kumar [9] Sharma & Goel [10], Ritikesh & Goel [11] and Goyal & Goel [12] have discussed systems behavior with perfect and imperfect switch-over of systems using various techniques. Availability modeling and behavioral analysis of three unit system in which one unit undergoes degradation (post repair) while other units have perfect repair. There is a single repairman who serves the units when need arises, using Regenerative Point Graphical Technique (RPGT) is discussed for system parameters. The system consists of three online units A which undergoes degradation (post repair), B and E. Sub system A have sub components in parallel hence can work in reduced state, so the system works in reduced state. It is assumed that unit B is repaired before the failure of stand-by unit C. Fuzzy concept is used to determine failure/working state of a unit. Taking failure rates exponential, repair rates general & independent and taking into consideration various probabilities, a transition diagram of the system is developed to find Primary, Secondary & Tertiary circuits and Base state. Problem is solved using RPGT to determine system parameters. System behavior is discussed with the help of graphs and tables. Simulations are done to study the effect of failure and repair rates on system parameters. Profit optimization is also discussed. Cases for intermediate repair and perfect repair are also taken, tables and graphs drawn followed by discussion.

Assumptions and Notations
The following assumptions and notations are taken:
1. Nothing can fail further when the system is in failed state.
2. There is a single repairman for repair of failed units and in reduced states.

3. The failure rates are exponentially distributed and repair rate are general and are independent and are different for different operative units.
4. Repair of unit B & E is perfect i.e. repaired unit works as a new one.
5. The order of priority for repair is unit A> unit B>unit E
6. The system is down when any of the units is in failed state.
7. The system is discussed for steady state conditions.
8. Failure and repaired are independent.
9. Unit A may work in reduced capacity and is degraded (post repair) on complete failure.

$\overline{cycle}$: A circuit formed through un-failed states.

m-cycle : A circuit (may be formed through regenerative or non-regenerative / failed state) whose terminals are at the regenerative state m.

m-$\overline{cycle}$: A circuit (may be formed through un-failed regenerative or non-regenerative state) whose terminals are at the regenerative m state.

$\left(i \xrightarrow{sr} j\right)$: *r*-th directed simple path from *i*-state to j-state; r takes positive integral values for different paths from *i*-state to j-state.

$\left(\xi \xrightarrow{sff} i\right)$: A directed simple failure free path from ξ-state to i-state.

$V_{m,m}$: Probability factor of the state m reachable from the terminal state m of the m-cycle.

$V_{\overline{m,m}}$: Probability factor of the state m reachable from the terminal state m of the m-$\overline{cycle}$.

R_i (t) : Reliability of the system at time t, given that the system entered the un-failed regenerative state ‘i’ at t = 0.

A_i (t) : Probability of the system in up time at time ‘t’, given that the system entered regenerative state ‘i’ at t = 0.

B_i (t) : Reliability that the server is busy for doing a particulars job at time ‘t’; given that the system entered regenerative state ‘i’ at t = 0.

V_i (t) : The expected no. of server visits for doing a job in [o,t] given that the system entered regenerative state ‘i’ at t = 0.

‘,’ : denote derivative

W_i (t) : Probability that the server is busy doing a particular job at time t without transiting to any other regenerative state ‘i’ through one or more non-regenerative states, given that t system entered the regenerative state ‘i’ at t = 0.

μ_i : Mean sojourn time spent in state i, before visiting any other states;

$$\mu_i = \int_0^\infty R_i(t)dt.$$

μ_i^1 : The total un-conditional time spent before transiting to any other regenerative states, given that the system entered regenerative state ‘i’ at t=0.

n_i : Expected waiting time spent while doing a given job, given that the system entered regenerative state ‘i’ at t=0; $\eta_i = W_i^*(0)$.

ξ : Base state of the system.

f_j : Fuzziness measure of the j-state.

λ_i : Constant failure rate of sub systems A & B etc.

λ_n = $(1+\beta)\lambda_{n-1}$ where β is degradation factor on repair for sub system A.

A/$\bar{A}$/a : Unit in full capacity working / reduced state / failed state of units A, similarly for unit B.and E.

w_i are constant repair rates of units.

g(t) : Probability density function of replacement of failed unit A with new one.

Taking into consideration the above assumptions and notations the Transition Diagram of the system is given in Figure 1.

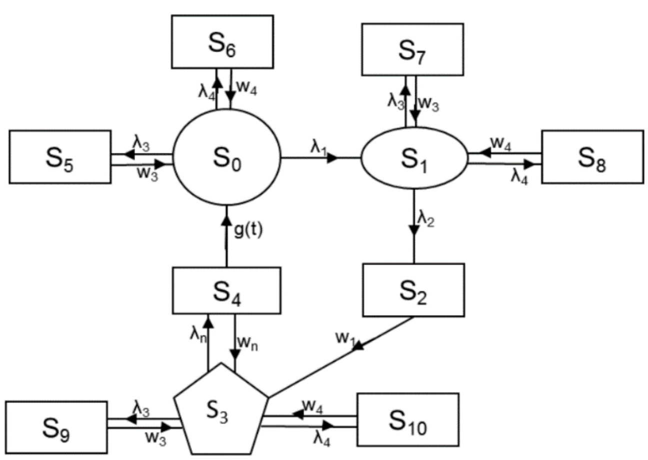
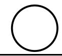
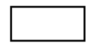
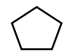


Fig 1

Table 1

State	Symbol	Model
Regenerative State		0
Reduced State		1
Failed State		2,4,5,6,7,8,9,10
Post Repair		3

$S_0 = ABE$, $S_1 = \bar{A}BE$, $S_2 = aBE$, $S_3 = A^1BE$
$S_4 = a^1BE$, $S_5 = AbE$, $S_6 = Abe$, $S_7 = \bar{A}bE$
$S_8 = \bar{A}Be$, $S_9 = A^1bE$, $S_{10} = A^1Be$

Table 2: Primary, Secondary & Tertiary Circuits at the various

Vertex i	Primary Circuits	Secondary Circuits	Tertiary Circuits
0	(0,5,0) (0,6,0) (0,1,2,3,4,0)	Nil Nil (1,7,1) (1,8,1) (3,10,3) (3,9,3) (3,4,3)	Nil Nil
1	(1,7,1) (1,8,1) (1,2,3,4,0,1)	(3,10,3) (3,9,3) (3,4,3) (0,5,0) (0,6,0)	
2	(2,3,4,0,1,2)	(3,10,3) (3,9,3) (3,4,3) (0,5,0) (0,6,0) (1,7,1) (1,8,1)	
3	(3,4,3) (3,10,3) (3,9,3) (3,4,0,1,2,3)	Nil Nil Nil (0,5,0) (0,6,0) (1,7,1) (1,8,1)	
4	(4,3,4) (4,0,1,2,3,4)	(3,10,3) (3,9,3) (3,4,3) (0,5,0) (0,6,0) (1,7,1) (1,8,1) (3,10,3) (3,9,3)	
5	(5,0,5)	(0,6,0) (0,1,2,3,4,0)	(1,7,1) (1,8,1) (3,10,3) (3,9,3) (3,4,3)
6	(6,0,6)	(0,5,0) (0,1,2,3,4,0)	(1,7,1) (1,8,1) (3,10,3) (3,9,3) (3,4,3)
7	(7,1,7)	(1,8,1) (1,2,3,4,0,1)	(3,10,3) (3,9,3) (0,5,0) (0,6,0) (3,4,3)

8	(8,1,8)	(1,7,1) (1,2,3,4,0,1)	(3,10,3) (3,9,3) (0,5,0) (0,6,0) (3,4,3)
9	(9,3,9)	(3,10,3) (3,4,3) (3,4,0,1,2,3)	(0,5,0) (0,6,0) (1,7,1) (1,8,1)
10	(10,3,10)	(3,9,3) (3,4,3) (3,4,0,1,2,3)	(0,5,0) (0,6,0) (1,7,1) (1,8,1)

As there are maximum number of primary cycles at vertx '3', hence vertx '3' is base state.

Table 3: Primary, Secondary, Tertiary Circuits w.r.t. the Simple Paths (Base-State '3')

Vertex j	$\left(3 \xrightarrow{S_r} j\right)$: (P0)	(P1)
0	$\left(3 \xrightarrow{S_1} 0\right)$: (3,4,0)	(0,6,0) (0,5,0)
1	$\left(3 \xrightarrow{S_1} 1\right)$: (3,4,0,1)	(0,6,0) (0,5,0) (1,7,1) (1,8,1)
2	$\left(3 \xrightarrow{S_1} 2\right)$: (3,4,0,1,2)	(0,6,0) (0,5,0) (1,7,1) (1,8,1)
3	$\left(3 \xrightarrow{S_1} 3\right)$: (3,9,3) $\left(3 \xrightarrow{S_2} 3\right)$: (3,10,3) $\left(3 \xrightarrow{S_3} 3\right)$: (3,4,3) $\left(3 \xrightarrow{S_4} 3\right)$: (3,4,0,1,2,3)	Nil Nil Nil (0,6,0) (0,5,0) (1,7,1) (1,8,1)
4	$\left(3 \xrightarrow{S_1} 4\right)$: (3,4)	Nil
5	$\left(3 \xrightarrow{S_1} 5\right)$: (3,4,0,5)	(0,6,0) (0,5,0)
6	$\left(3 \xrightarrow{S_1} 6\right)$: (3,4,0,6)	(0,6,0) (0,5,0)
7	$\left(3 \xrightarrow{S_1} 7\right)$: (3,4,0,1,7)	(0,6,0) (0,5,0) (1,8,1)
8	$\left(3 \xrightarrow{S_1} 8\right)$: (3,4,0,1,8)	(0,6,0) (0,5,0) (1,7,1)
9	$\left(3 \xrightarrow{S_1} 9\right)$: (3,9)	Nil
10	$\left(3 \xrightarrow{S_1} 10\right)$: (3,10)	Nil

Transition Probability and the Mean sojourn times

$q_{i,j}(t)$ **:** Probability density function (p.d.f.) of the first passage time from a regenerative state 'i' to a regenerative state 'j' or to a failed state 'j' without visiting any other regenerative state in (0,t].

$p_{i,j}$ **:** Steady state transition probability from a regenerative state 'i' to a regenerative state 'j' without visiting any other regenerative state. $p_{i,j} = q^*_{i,j}(0)$; where $*$ denotes Laplace transformation.

Transition Probabilities

$q_{i,j}(t)$	$P_{ij} = q^*_{i,j}(t)$
$q_{0,1} = \lambda_1 e^{-(\lambda_1+\lambda_3+\lambda_4)t}$ $q_{0,5} = \lambda_3 e^{-(\lambda_1+\lambda_3+\lambda_4)t}$ $q_{0,6} = \lambda_4 e^{-(\lambda_1+\lambda_3+\lambda_4)t}$	$p_{0,1} = \lambda_1/(\lambda_1+\lambda_3+\lambda_4)$ $p_{0,5} = \lambda_3/(\lambda_1+\lambda_3+\lambda_4)$ $p_{0,6} = \lambda_4/(\lambda_1+\lambda_3+\lambda_4)$
$q_{1,2} = \lambda_2 e^{-(\lambda_2+\lambda_3+\lambda_4)t}$ $q_{1,7} = \lambda_3 e^{-(\lambda_2+\lambda_3+\lambda_4)t}$ $q_{1,8} = \lambda_4 e^{-(\lambda_2+\lambda_3+\lambda_4)t}$	$p_{1,2} = \lambda_2/(\lambda_2+\lambda_3+\lambda_4)$ $p_{1,7} = \lambda_3/(\lambda_2+\lambda_3+\lambda_4)$ $p_{1,8} = \lambda_4/(\lambda_2+\lambda_3+\lambda_4)$
$q_{2,3} = w_1 e^{-w_1 t}$	$p_{2,3} = 1$
$q_{3,4} = \lambda_n e^{-(\lambda_3+\lambda_4+\lambda_n)t}$	$p_{3,4} = \lambda_n/(\lambda_3+\lambda_4+\lambda_n)$

$q_{3,9}=\lambda_3 e^{-(\lambda_3+\lambda_4+\lambda_n)t}$ $q_{3,10}=\lambda_4 e^{-(\lambda_3+\lambda_4+\lambda_n)t}$	$p_{3,9}=\lambda_3/(\lambda_3+\lambda_4+\lambda_n)$ $p_{3,10}=\lambda_4/(\lambda_3+\lambda_4+\lambda_n)$
$q_{4,0}=g(t)e^{-w_n t}$ $q_{4,3}=w_n e^{-w_n t}\overline{g(t)}$	$p_{4,0}=g^*w_n$ $p_{4,3}=1-g^*w_n$
$q_{5,0}=w_3 e^{-w_3 t}$	$p_{5,0}=1$
$q_{6,0}=w_4 e^{-w_4 t}$	$p_{6,0}=1$
$q_{7,1}=w_3 e^{-w_3 t}$	$p_{7,1}=1$
$q_{8,1}=w_4 e^{-w_4 t}$	$p_{8,1}=1$
$q_{9,3}=w_3 e^{-w_3 t}$	$p_{9,3}=1$
$q_{10,3}=w_4 e^{-w_4 t}$	$p_{10,3}=1$

Mean Sojourn Times

$R_i(t)$	$\mu_i=R_i^*(0)$
$R_0^{(t)}=e^{-(\lambda_1+\lambda_3+\lambda_4)t}$	$\mu_0=1/(\lambda_1+\lambda_3+\lambda_4)$
$R_1^{(t)}=e^{-(\lambda_2+\lambda_3+\lambda_4)t}$	$\mu_1=1/(\lambda_2+\lambda_3+\lambda_4)$
$R_2^{(t)}=e^{-(w_1)t}$	$\mu_2=1/(w_1)$
$R_3^{(t)}=e^{-(\lambda_3+\lambda_4+\lambda_n)t}$	$\mu_3=1/(\lambda_3+\lambda_4+\lambda_n)$
$R_4^{(t)}=e^{-w_4 t}\overline{g(t)}$	$\mu_4=1-g^*(w_4)/w_4$
$R_5^{(t)}=e^{-w_3 t}$	$\mu_5=1/w_3$
$R_6^{(t)}=e^{-(w_4)t}$	$\mu_6=1/(w_4)$
$R_7^{(t)}=e^{-(w_3)t}$	$\mu_7=1/(w_3)$
$R_8^{(t)}=e^{-w_4 t}$	$\mu_8=1/w_4$
$R_9^{(t)}=e^{-w_3 t}$	$\mu_9=1/w_3$
$R_{10}^{(t)}=e^{-w_4 t}$	$\mu_{10}=1/w_4$

$V_{3,0}$ = (3,4,0)/[1-(0,5,0)][1-(0,6,0)],
$= p_{3,4}p_{4,0}/(1-p_{0,5}p_{5,0})(1-p_{0,6}p_{6,0})$
$= [(\lambda_n/\lambda_3+\lambda_4+\lambda_n)g^*(w_n)][(\lambda_1+\lambda_3+\lambda_4)^2/(\lambda_1+\lambda_4)\ (\lambda_1+\lambda_3)]$

$V_{3,1}$ = (3,4,0,1)/[1-(0,5,0)][1-(0,6,0)][1-(1,7,1)][1-(1,8,1)]
$= p_{3,4}p_{4,0}p_{0,1}/(1-p_{0,5}p_{5,0})(1-p_{0,6}p_{6,0})(1-p_{1,7}p_{7,1})(1-p_{1,8}p_{8,1})$
$= [\{(\lambda_1\lambda_n\ g^*(w_n)\ (\lambda_1+\lambda_3+\lambda_4)\ (\lambda_2+\lambda_3+\lambda_4)^2\}]/[(\lambda_3+\lambda_4+\lambda_n)(\lambda_1+\lambda_4)(\lambda_1+\lambda_3)\ (\lambda_2+\lambda_3)(\lambda_2+\lambda_4)]$

$V_{3,2}$ = (3,4,0,1,2)/[1-(0,5,0)][1-(0,6,0)][1-(1,7,1)][1-(1,8,1)]
$= p_{3,4}p_{4,0}p_{0,1}p_{1,2}/(1-p_{0,5}p_{5,0})(1-p_{0,6}p_{6,0})(1-p_{1,7}p_{7,1})(1-p_{1,8}p_{8,1})$
$= [(\lambda_1\lambda_2\lambda_n\ g^*(w_n)(\lambda_1+\lambda_3+\lambda_4)(\lambda_2+\lambda_3+\lambda_4)]/[(\lambda_3+\lambda_4+\lambda_n)(\lambda_1+\lambda_4)(\lambda_1+\lambda_3)\ (\lambda_2+\lambda_3)(\lambda_2+\lambda_4)]$

$V_{3,3}$ = (3,9,3)+(3,10,3)+(3,4,3)+(3,4,0,1,2,3)/[1-(0,5,0)][1-(0,6,0)][1-(1,7,1)][1-(1,8,1)]
$= p_{3,9}p_{9,3}+p_{3,10}p_{10,3}+p_{3,4}p_{4,3}+p_{3,4}p_{4,0}p_{0,1}p_{1,2}p_{2,3}/(1-p_{0,5}p_{5,0})(1-p_{0,6}p_{6,0})(1-p_{1,7}p_{7,1})(1-p_{1,8}p_{8,1})$
$= [(1-\lambda_n g^*w_n)/(\lambda_3+\lambda_4+\lambda_n)]+[(\lambda_1\lambda_2\lambda_3 g^*w_n)(\lambda_1+\lambda_3+\lambda_4)(\lambda_2+\lambda_3+\lambda_4)]/[(\lambda_3+\lambda_4+\lambda_n)(\lambda_1+\lambda_3)(\lambda_1+\lambda_4)(\lambda_2+\lambda_4)(\lambda_2+\lambda_3)]$

$V_{3,4}$ = (3,4), $= p_{3,4}$ $= (\lambda_n/\lambda_3+\lambda_4+\lambda_n)$

$V_{3,5}$ = (3,4,0,5)/[1-(0,5,0)][1-(0,6,0)],
$= p_{3,4}p_{4,0}p_{0,5}/(1-p_{0,5}p_{5,0})(1-p_{0,6}p_{6,0})$
$= [\{(\lambda_3\lambda_n g^*(w_n)(\lambda_1+\lambda_3+\lambda_4)\}]/[(\lambda_3+\lambda_4+\lambda_n)(\lambda_1+\lambda_3)(\lambda_1+\lambda_4)]$

$V_{3,6}$ = (3,4,0,6)/[1-(0,5,0)][1-(0,6,0)],
$= p_{3,4}p_{4,0}p_{0,6}/(1-p_{0,5}p_{5,0})(1-p_{0,6}p_{6,0})$
$= [\{(\lambda_4\lambda_n g^*(w_n)(\lambda_1+\lambda_3+\lambda_4)\}]/[(\lambda_3+\lambda_4+\lambda_n)(\lambda_1+\lambda_3)(\lambda_1+\lambda_4)]$

$V_{3,7}$ = (3,4,0,1,7)/[1-(0,5,0)][1-(0,6,0)][1-(1,7,1)][1-(1,8,1)]
$= p_{3,4}p_{4,0}p_{0,1}p_{0,7}/(1-p_{0,5}p_{5,0})(1-p_{0,6}p_{6,0})(1-p_{1,7}p_{7,1})(1-p_{1,8}p_{8,1})$
$= [\{(\lambda_1\lambda_3\lambda_n\ g^*(w_n)(\lambda_1+\lambda_3+\lambda_4)\ (\lambda_2+\lambda_3+\lambda_4)\}]/[(\lambda_3+\lambda_4+\lambda_n)(\lambda_1+\lambda_4)(\lambda_1+\lambda_3)\ (\lambda_2+\lambda_3)(\lambda_2+\lambda_4)]$

$V_{3,8}$ = (3,4,0,1,8)/[1-(0,5,0)][1-(0,6,0)][1-(1,7,1)][1-(1,8,1)]
$= p_{3,4}p_{4,0}p_{0,1}p_{0,8}/(1-p_{0,5}p_{5,0})(1-p_{0,6}p_{6,0})(1-p_{1,7}p_{7,1})(1-p_{1,8}p_{8,1})$
$= [\{(\lambda_1\lambda_4\lambda_n\ g^*(w_n)(\lambda_1+\lambda_3+\lambda_4)(\lambda_2+\lambda_3+\lambda_4)\}]/[(\lambda_3+\lambda_4+\lambda_n)(\lambda_1+\lambda_4)(\lambda_1+\lambda_3)\ (\lambda_2+\lambda_3)(\lambda_2+\lambda_4)]$

$V_{3,9}$ = (3,9), $= p_{3,9}$ $= (\lambda_3/\lambda_3+\lambda_4+\lambda_n)$

$V_{3,10} = (3,10), \quad = p_{3,10} \quad = (\lambda_4/\lambda_3+\lambda_4+\lambda_n)$

$V_{0,0} = (0,5,0)+(0,6,0)+(0,1,2,3,4,0)/[1-(1,7,1)][1-(1,8,1)][1-(3,10,3)][1-(3,9,3)][1-(3,4,3)]$
$= p_{0,5}p_{5,0}+p_{0,6}p_{6,0}+p_{0,1}p_{1,2}p_{2,3}p_{3,4}p_{4,0}/(1-p_{1,7}p_{7,1})(1-p_{1,8}p_{8,1})(1-p_{3,10}p_{10,3})(1-p_{3,9}p_{9,3})(1-p_{3,4}p_{4,3})$
$= [(\lambda_3+\lambda_4)/(\lambda_1+\lambda_3+\lambda_4)]+[(\lambda_1\lambda_2\lambda_n g^*w_n)(\lambda_2+\lambda_3+\lambda_4)(\lambda_3+\lambda_4+\lambda_n)^2]/[(\lambda_1+\lambda_3+\lambda_4)(\lambda_2+\lambda_4)(\lambda_2+\lambda_3)(\lambda_3+\lambda_n)(\lambda_4+\lambda_n)(\lambda_3+\lambda_4+g^*w_n)\}]$

$V_{0,1} = (0,1)/\ )/[1-(1,7,1)][1-(1,8,1)],$
$= p_{0,1}/(1-p_{1,7}p_{7,1})(1-p_{1,8}p_{8,1})$
$= [\lambda_1(\lambda_2+\lambda_3+\lambda_4)^2/(\lambda_2+\lambda_3)(\lambda_2+\lambda_4)(\lambda_1+\lambda_3+\lambda_4)]$

$V_{0,2} = (0,1,2)\ /[1-(1,7,1)][1-(1,8,1)],$
$= p_{0,1}p_{0,2}/(1-p_{1,7}p_{7,1})(1-p_{1,8}p_{8,1})$
$= [\lambda_1\lambda_2(\lambda_2+\lambda_3+\lambda_4)/(\lambda_2+\lambda_3)(\lambda_1+\lambda_3+\lambda_4)]$

$V_{0,3} = (0,1,2,3)\ /[1-(1,7,1)][1-(1,8,1)][1-(3,10,3)][1-(3,9,3)][1-(3,4,3)]$
$= p_{0,1}p_{1,2}\ p_{2,3}/(1-p_{1,7}p_{7,1})(1-p_{1,8}p_{8,1})(1-p_{3,10}p_{10,3})(1-p_{3,9}p_{9,3})(1-p_{3,4}p_{4,3})$
$= [\lambda_1\lambda_2(\lambda_2+\lambda_3+\lambda_4)\ (\lambda_3+\lambda_4+\lambda_n)^3]/[(\lambda_1+\lambda_3+\lambda_4)(\lambda_2+\lambda_3)\ (\lambda_2+\lambda_4)(\lambda_3+\lambda_n)(\lambda_4+\lambda_n)\ (\lambda_3+\lambda_4+g^*w_n)\}]$

$V_{0,4} = (0,1,2,3,4)\ /[1-(1,7,1)][1-(1,8,1)][1-(3,10,3)][1-(3,9,3)][1-(3,4,3)]$
$= p_{0,1}p_{1,2}p_{2,3}p_{3,4}/(1-p_{1,7}p_{7,1})(1-p_{1,8}p_{8,1})(1-p_{3,10}p_{10,3})(1-p_{3,9}p_{9,3})(1-p_{3,4}p_{4,3})$
$= [\lambda_1\lambda_2\lambda_n(\lambda_2+\lambda_3+\lambda_4)(\lambda_3+\lambda_4+\lambda_n)^2]/[(\lambda_1+\lambda_3+\lambda_4)(\lambda_2+\lambda_4)(\lambda_2+\lambda_3)(\lambda_3+\lambda_n)(\lambda_4+\lambda_n)\ (\lambda_3+\lambda_4+g^*w_n)\}]$

$V_{0,5} = (0,5), \quad = p_{0,5} \quad = \lambda_3/(\lambda_1+\lambda_3+\lambda_4)$

$V_{0,6} = (0,6), \quad = p_{0,6} \quad = \lambda_4/(\lambda_1+\lambda_3+\lambda_4)$

$V_{0,7} = (0,1,7)/\ )/[1-(1,7,1)][1-(1,8,1)], \quad = p_{0,1}p_{1,7}/(1-p_{1,7}p_{7,1})(1-p_{1,8}p_{8,1})$
$= [\lambda_1\lambda_3(\lambda_2+\lambda_3+\lambda_4)/(\lambda_2+\lambda_3)(\lambda_2+\lambda_4)(\lambda_1+\lambda_3+\lambda_4)]$

$V_{0,8} = (0,1,8)/\ )/[1-(1,7,1)][1-(1,8,1)],$
$= p_{0,1}p_{1,8}/(1-p_{1,7}p_{7,1})(1-p_{1,8}p_{8,1})$
$= [\lambda_1\lambda_4(\lambda_2+\lambda_3+\lambda_4)/(\lambda_2+\lambda_3)(\lambda_2+\lambda_4)(\lambda_1+\lambda_3+\lambda_4)]$

$V_{0,9} = (0,1,2,3,9)\ /[1-(1,7,1)][1-(1,8,1)][1-(3,10,3)][1-(3,9,3)][1-(3,4,3)]$
$= p_{0,1}p_{1,2}p_{2,3}p_{3,9}/(1-p_{1,7}p_{7,1})(1-p_{1,8}p_{8,1})(1-p_{3,10}p_{10,3})(1-p_{3,9}p_{9,3})(1-p_{3,4}p_{4,3})$
$= [\lambda_1\lambda_2\lambda_3(\lambda_2+\lambda_3+\lambda_4)(\lambda_3+\lambda_4+\lambda_n)^2]/[(\lambda_1+\lambda_3+\lambda_4)(\lambda_2+\lambda_4)(\lambda_2+\lambda_3)(\lambda_3+\lambda_n)(\lambda_3+\lambda_4+g^*w_n)\ (\lambda_4+\lambda_n)]$

$V_{0,10} = (0,1,2,3,10)\ /[1-(1,7,1)][1-(1,8,1)][1-(3,10,3)][1-(3,9,3)][1-(3,4,3)]$
$= p_{0,1}p_{1,2}p_{2,3}p_{3,10}/(1-p_{1,7}p_{7,1})(1-p_{1,8}p_{8,1})(1-p_{3,10}p_{10,3})(1-p_{3,9}p_{9,3})(1-p_{3,4}p_{4,3})$
$= [\lambda_1\lambda_2\lambda_4(\lambda_2+\lambda_3+\lambda_4)(\lambda_3+\lambda_4+\lambda_n)^2]/[(\lambda_1+\lambda_3+\lambda_4)(\lambda_2+\lambda_4)(\lambda_2+\lambda_3)\ (\lambda_4+\lambda_n)(\lambda_3+\lambda_4+g^*w_n)\ (\lambda_3+\lambda_n)]$

MTSF(T_0)

The regenerative un-failed states to which the system can transit (initial state ‘0’), before entering any failed state are:

‘i’ = 0,2,3,6,7,12 taking ‘ξ’ = ‘0’.

$$\text{MTSF}\ (T_0) = \left[\sum_{i,sr}\left\{\frac{\left\{pr\left(\xi\xrightarrow{sr(sff)}i\right)\right\}\mu i}{\Pi_{m_1\neq\xi}\left\{1\text{-}V_{\overline{m_1 m_1}}\right\}}\right\}\right] \div \left[1\text{-}\sum_{sr}\left\{\frac{\left\{pr\left(\xi\xrightarrow{sr(sff)}\xi\right)\right\}}{\Pi_{m_2\neq\xi}\left\{1\text{-}V_{\overline{m_2 m_2}}\right\}}\right\}\right]$$

$= (V_{0,0}\mu_0+V_{0,1}\mu_1)/(V_{0,0}\mu_0+V_{0,1}\mu_1+V_{0,2}\mu_2+V_{0,3}\mu_3+V_{0,4}\mu_4+V_{0,5}\mu_5+V_{0,6}\mu_6+V_{0,7}\mu_7+V_{0,8}\mu_8+V_{0,9}\mu_9+V_{0,10}\mu_{10})$

Availability of the System

The regenerative states at which the system is available are ‘j’ = 0,2,3,6,7,12 and the regenerative states are ‘i’ = 0 to 14 taking ‘ξ’ = ‘3’ the total fraction of time for which the system is available is given by

$$A_0 = \left[\sum_{j,sr}\left\{\frac{\{pr(\xi^{sr\rightarrow}j)\}fj,\mu j}{\Pi_{m_1\neq\xi}\left\{1\text{-}V_{\overline{m_1 m_1}}\right\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\{pr(\xi^{sr\rightarrow}i)\}\mu_i^1}{\Pi_{m_2\neq\xi}\left\{1\text{-}V_{\overline{m_2 m_2}}\right\}}\right\}\right]$$

$= \left[\sum_j V_{\xi,j}\,, f_j, \mu_j\right] \div \left[\sum_i V_{\xi,i}\,, f_j, \mu_i^1\right]$

$= (V_{3,0}f_0\mu_0+V_{3,1}f_1\mu_1+V_{3,3}f_3\mu_3)/(V_{3,0}\mu_0+V_{3,1}\mu_1+V_{3,2}\mu_2+V_{3,3}\mu_3+V_{3,4}\mu_4+V_{3,5}\mu_5+V_{3,6}\mu_6+V_{3,7}\mu_7+V_{3,8}\mu_8+V_{3,9}\mu_9+V_{3,10}\mu_{10})$

Busy Period of the Server

The regenerative states where server is busy are ‘j’ = 2,3,4,5,6,7,8,9,10 and regenerative states are ‘i’ = 0 to 10, taking ξ = ‘0’, the total fraction of time for which the server remains busy is

$$B_0 = \left[\sum_{j,sr}\left\{\frac{\{pr(\xi^{sr}\rightarrow j)\},nj}{\Pi_{m_{1\neq\xi}}\{1\text{-}V_{\overline{m_1m_1}}\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\{pr(\xi^{sr}\rightarrow i)\}\mu_i^1}{\Pi_{m_{2\neq\xi}}\{1\text{-}V_{\overline{m_2m_2}}\}}\right\}\right] = \left[\sum_j V_{\xi,j}\,,n_j\right] \div \left[\sum_i V_{\xi,i}\,,\mu_i^1\right]$$

$=(V_{0,2}\mu_2+V_{3,4}\mu_4+V_{3,5}\mu_5+V_{3,6}\mu_6+V_{3,7}\mu_7+V_{3,8}\mu_8+V_{3,9}\mu_9+V_{3,10}\mu_{10})/(V_{3,0}\mu_0+V_{3,1}\mu_1+V_{3,2}\mu_2+V_{3,3}\mu_3+V_{3,4}\mu_4+V_{3,5}\mu_5+V_{3,6}\mu_6+V_{3,7}\mu_7+V_{3,8}\mu_8+V_{3,9}\mu_9+V_{3,10}\mu_{10})$

Expected Number of Inspections by the repair man

The regenerative states where the repair man do this job j = 1 the regenerative states are i = 0 to 10, Taking 'ξ' = '0', the number of visit by the repair man is given by

$$V_0 = \left[\sum_{j,sr}\left\{\frac{\{pr(\xi^{sr}\rightarrow j)\}}{\Pi_{k_{1\neq\xi}}\{1\text{-}V_{\overline{k_1k_1}}\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\{pr(\xi^{sr}\rightarrow i)\}\mu_i^1}{\Pi_{k_{2\neq\xi}}\{1\text{-}V_{\overline{k_2k_2}}\}}\right\}\right] = \left[\sum_j V_{\xi,j}\right] \div \left[\sum_i V_{\xi,i}\,,\mu_i^1\right]$$

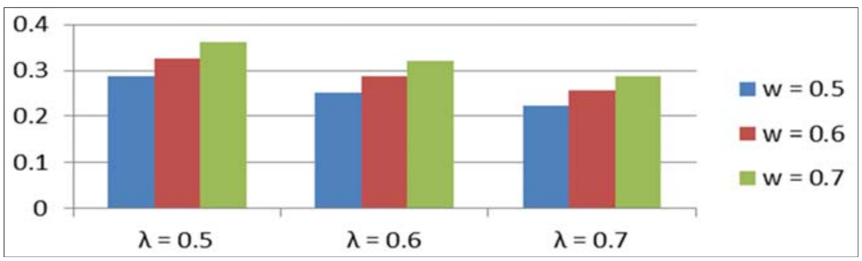
$=(V_{0,2}\mu_2+V_{0,3}\mu_3+V_{0,4}\mu_4+V_{0,5}\mu_5+V_{0,6}\mu_6+V_{0,7}\mu_7+V_{0,8}\mu_8+V_{0,9}\mu_9+V_{0,10}\mu_{10})/(V_{0,0}\mu_0+V_{0,1}\mu_1+V_{0,2}\mu_2+V_{0,3}\mu_3+V_{0,4}\mu_4+V_{0,5}\mu_5+V_{0,6}\mu_6+V_{0,7}\mu_7+V_{0,8}\mu_8+V_{0,9}\mu_9+V_{0,10}\mu_{10})$

Particular Cases: Taking failure rates and equal, we get

Availability of the System (A_0) = [53w/(53w+131λ]

Availability of the System Table

A_0	w = 0.5	w = 0.6	w = 0.7
λ = 0.5	0.288043	0.326824	0.361598
λ = 0.6	0.252141	0.288043	0.320657
λ = 0.7	0.224196	0.257490	0.288043

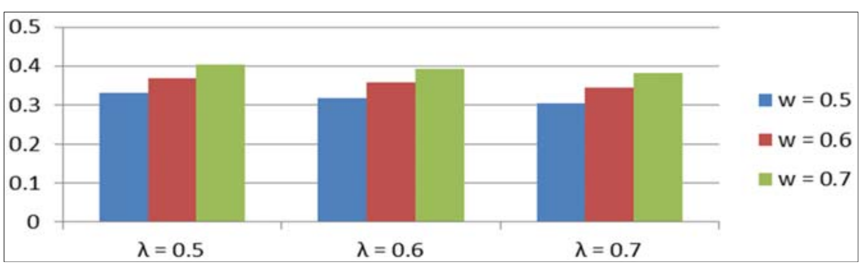


Availability of the System Graph

MTSF (T_0) = [$(144\lambda^2w+100\lambda w+16w)/(96\lambda^3+224\lambda^2+62\lambda+166\lambda w+28w)$]

MTSF Table

T_0	w = 0.5	w = 0.6	w = 0.7
λ = 0.5	0.330097	0.369565	0.404075
λ = 0.6	0.315848	0.356537	0.392670
λ = 0.7	0.303190	0.344583	0.381816

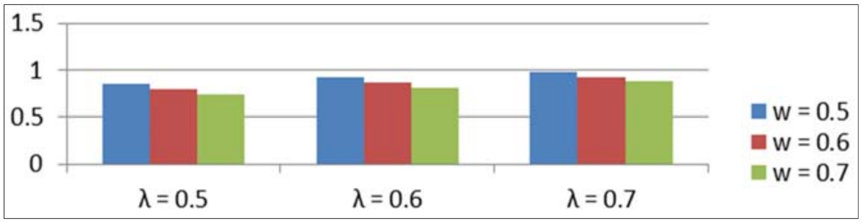


MTSF Graph

Expected Number of Inspections by the repair man (V_0) = [$(96\lambda^3+365\lambda^2+56\lambda)/(96\lambda^3+224\lambda^2+62\lambda+166\lambda w\ 28w)$]

Expected Number of Inspection by the Repair Man Table

V_0	w = 0.5	w = 0.6	w = 0.7
λ = 0.5	0.849515	0.792572	0.742784
λ = 0.6	0.917777	0.863342	0.815003
λ = 0.7	0.972075	0.920655	0.874402

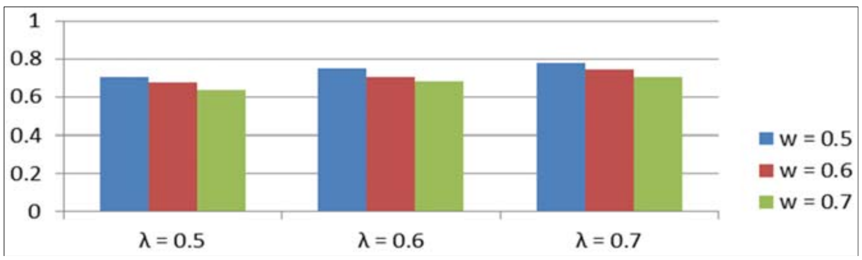


Expected Number of Server's Visit Graph

Busy Period of the Server (B_0) = $[131\lambda/(53w+131\lambda)]$

Busy Period of the Server Table

B_0	w = 0.5	w = 0.6	w = 0.7
λ = 0.5	0.704310	0.673176	0.638402
λ = 0.6	0.747859	0.704310	0.679343
λ = 0.7	0.775804	0.742510	0.704310



Busy Period of the Server Graph

Profit Function = $A_0R_0-B_0R_1-V_0R_2$

Where,

A_0 = Availability of System,

B_0 = Busy Period of Server

V_0 = Expected Number of Inspection by the Repair Man,

R_0 = Revenue,

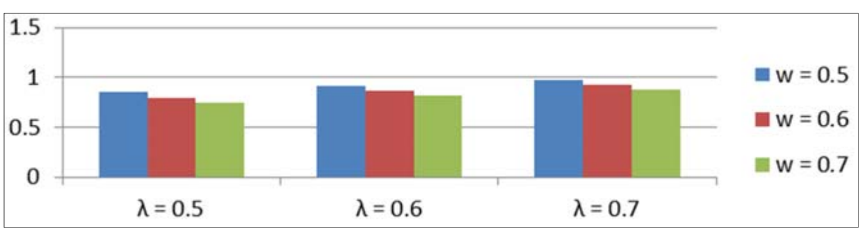
R_1 = Busy Period per Unit, R_2 = Per Visit Cost, R_0 = 1000,

R_1 = 50, R_2 = 100

Profit = $[(53000w-6550\lambda)/(53w+131\lambda)]-[(96\lambda^3+365\lambda^2+56\lambda)x100/(96\lambda^3+224\lambda^2+62\lambda+166\lambda w+28w)]$

Profit Function Table

	w = 0.5	w = 0.6	w = 0.7
λ = 0.5	167.4942	213.9128	255.39996
λ = 0.6	122.9701	166.1115	205.1894
λ = 0.7	88.1986	128.2989	165.0055



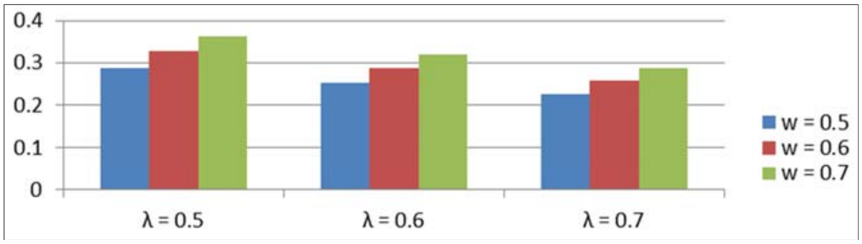
Profit Function Graph

Corollary: When Intermediate repair is feasible

Availability of the System (A_0)

Availability of the System Table

A_0	w = 0.5	w = 0.6	w = 0.7
λ = 0.5	0.288043	0.326824	0.361598
λ = 0.6	0.252141	0.288043	0.320657
λ = 0.7	0.224196	0.257490	0.288043

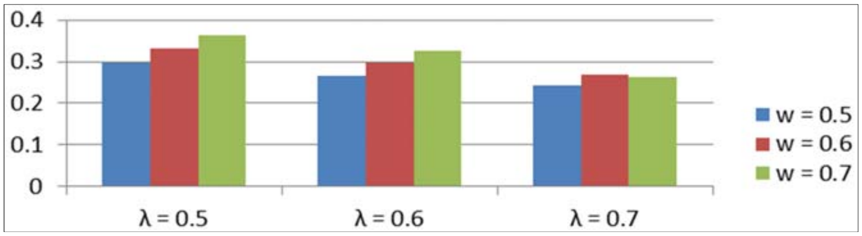


Availability of the System Graph

MTSF (T_0) = $[(w+\lambda)^3]/[(w^3+5w^2\lambda+11\lambda^2w+10\lambda^3)]$

MTSF Table

T_0	w = 0.5	w = 0.6	w = 0.7
λ = 0.5	0.296296	0.331424	0.364326
λ = 0.6	0.265404	0.296296	0.325723
λ = 0.7	0.242526	0.269902	0.262960

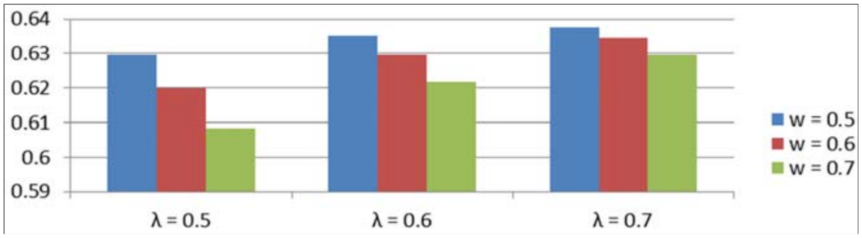


MTSF Graph

Busy Period of the Server (B_0) = $(3w^2+6\lambda^3+8\lambda^2w)/(w^3+5w^2\lambda+11\lambda^2w+10\lambda^3)$
Expected Number of Server's Visits (V_0)

Expected Number of Server's Visits Table

V_0	w = 0.5	w = 0.6	w = 0.7
λ = 0.5	0.629630	0.620019	0.608265
λ = 0.6	0.635294	0.629630	0.621744
λ = 0.7	0.637614	0.634643	0.629630

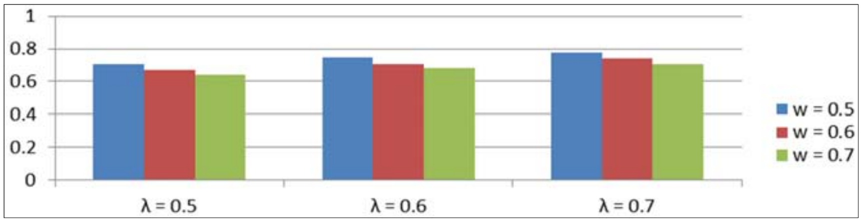


Expected Number of Server's Visits Graph

Busy Period of the Server

Busy Period of the Server Table

B_0	w = 0.5	w = 0.6	w = 0.7
λ = 0.5	0.704310	0.673176	0.638402
λ = 0.6	0.747859	0.704310	0.679343
λ = 0.7	0.775804	0.742510	0.704310

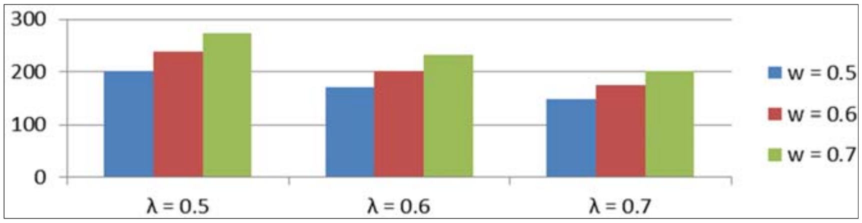


Busy Period of the Server Graph

Profit Function = $A_0R_0 - B_0R_1 - V_0R_2$
Profit = $[1000(w+\lambda)^3 - 150(3w^2\lambda + 6\lambda^3 + 8\lambda^2 w)]/(w^3 + 5w^2\lambda + 11\lambda^2 w + 10\lambda^3)$

Profit Function Table

	w = 0.5	w = 0.6	w = 0.7
λ = 0.5	201.8515	238.42115	273.08625
λ = 0.6	170.1099	201.8515	232.3032
λ = 0.7	146.8839	174.70555	201.8515

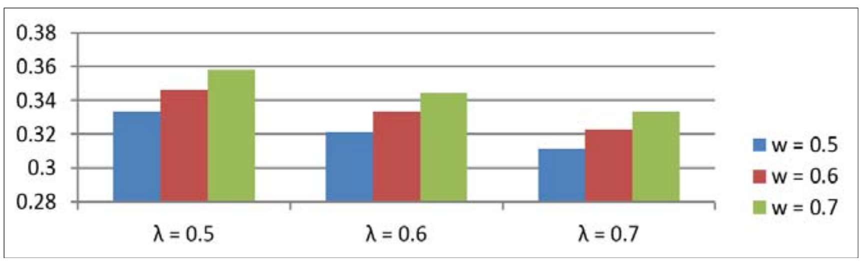


Profit Function Graph

Corollary: - Perfect repair post failure
Availability of the System (A_0)

Availability of the System Table

A_0	w = 0.5	w = 0.6	w = 0.7
λ = 0.5	0.333333	0.346457	0.358209
λ = 0.6	0.321168	0.333333	0.344371
λ = 0.7	0.311689	0.322981	0.333333

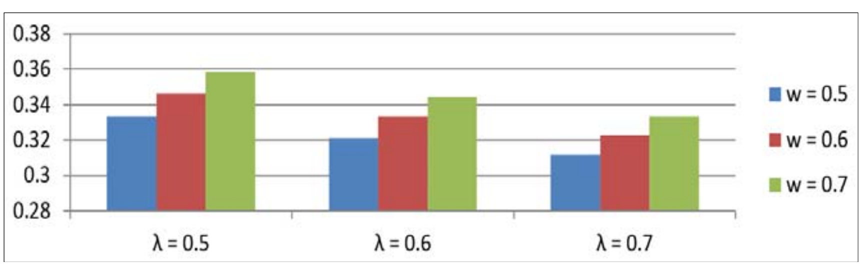


Availability of the System Graph

MTSF (T_0) = $[4(w+\lambda)/(7w+17\lambda)]$

MTSF Table

T_0	w = 0.5	w = 0.6	w = 0.7
λ = 0.5	0.333333	0.346457	0.358209
λ = 0.6	0.321168	0.333333	0.344371
λ = 0.7	0.311689	0.322981	0.333333

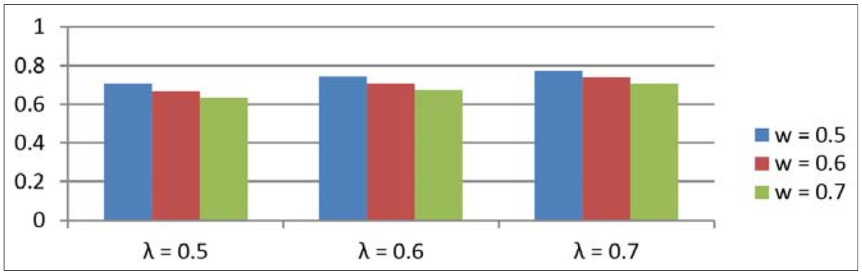


MTSF Graph

Expected Number of Server's Visits (V_0)

Expected Number of Server's Visits Table

V_0	w = 0.5	w = 0.6	w = 0.7
λ = 0.5	0.708333	0.669291	0.634328
λ = 0.6	0.744526	0.708333	0.675497
λ = 0.7	0.772727	0.739130	0.708333

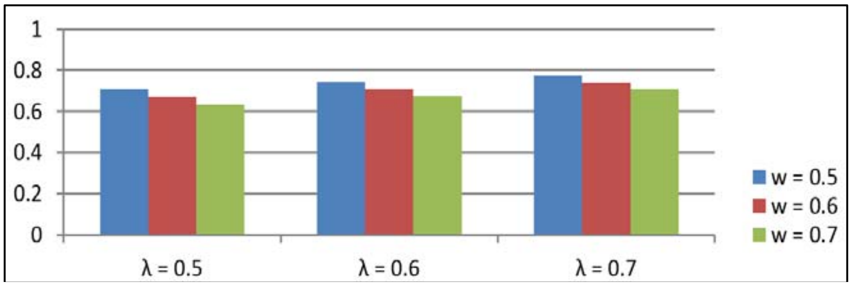


Expected Number of Server's Visits Graph

Busy Period of the Server (B_0) = [17λ(7w+17λ)]

Busy Period of the Server Table

B_0	w = 0.5	w = 0.6	w = 0.7
λ = 0.5	0.708333	0.669291	0.634328
λ = 0.6	0.744526	0.708333	0.675497
λ = 0.7	0.772727	0.739130	0.708333

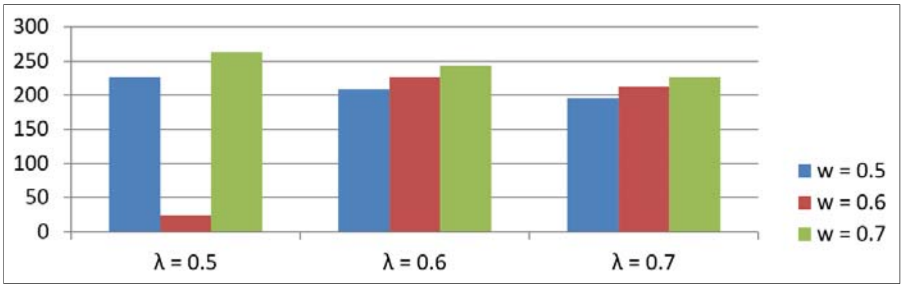


Busy Period of the Server Graph

Profit Function = A_0R_0-B_0R_1-V_0R_2

Profit = [1450λ+4000w]/[7w+17λ]

	w = 0.5	w = 0.6	w = 0.7
λ = 0.5	227.0834	24.0634	263.0598
λ = 0.6	209.4891	227.0834	243.04645
λ = 0.7	195.77995	212.1115	227.0834



Profit Function Graph

Conclusion

The results given tables and graphs depict the practical behavior as should be on increasing the failure and repair rates. From above tables and graph we see that the result obtained using Regenerative Point Graphical Technique is same as obtained by using Regenerative Point Technique and other techniques. It is hoped that the Regenerative Point Graphical Technique for the analysis of the systems having more number of units arranged in series and parallel will be very helpful to the managements, manufactures and the personal engaged in reliability engineering and working for the behavior and profit analysis of stochastic systems.

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