ISSN Print: 2394-7500
ISSN Online: 2394-5869
Impact Factor: 5.2
IJAR 2016; 2(3): 260-264
www.allresearchjournal.com
Received: 05-01-2016
Accepted: 07-02-2016

R. S. Wali
Department of Mathematics, Bhandari & Rathi College, Guledagudd-587 203, Karnataka State, India.

Vijaykumari T Chilakwad
Department of Mathematics, Government First Grade College, Dharwad, Karnataka State, India.

Correspondence
Vijaykumari T Chilakwad
Department of Mathematics, Government First Grade College, Dharwad, Karnataka State, India.

Minimal pgrω-Open sets and Maximal pgrω-Closed sets in Topological spaces

R. S. Wali, Vijaykumari T. Chilakwad

Abstract
In this paper the new sets called minimal pgrω-open sets and maximal pgrω-closed sets in a topological space are introduced which are the pgrω-open sets and pgrω-closed sets respectively. The complement of minimal pgrω-open set is a maximal pgrω-closed set. Some properties of the maximal-semi-pgrw-open sets, minimal-semi-pgrw-closed sets, Minimal pgrw-continuous maps, Maximal pgrw-continuous maps and $T_{min-pgrw}$, $T_{max-pgrw}$, Min-T_{pgrw}, Max-T_{pgrw}-Spaces are studied.

Keywords: Minimal pgrω-open set, Maximal pgrω-closed set, $T_{min-pgrw}$, $T_{max-pgrw}$, Min-T_{pgrw}, Max-T_{pgrw} –Spaces.

1. Introduction
In the years 2001 and 2003, F. Nakaoka and N. Oda [1-3] introduced and studied minimal and maximal closed sets. Seenivasan and S. Kalaiselvi [4] defined a new class of minimal and maximal sets via gsg closed sets called minimal gsg-closed, maximal gsg-closed, minimal gsg-open and maximal gsg-open sets and studied their properties. R.S Wali and V.T. Chilakwad [5] introduced and studied pgrω-closed sets and pgrω-open sets.

Definition 1.1: [1] A non-empty open proper subset U of a topological space X is said to be a minimal open set if any open set which is contained in U is either ϕ or U.

Definition 1.2: [2] A non-empty open proper subset U of a topological space X is said to be a maximal open set if any open set which contains U is either X or U.

Definition 1.3: [3] A non-empty closed proper subset F of a topological space X is said to be a minimal closed set if any closed set which is contained in F is ϕ or F.

Definition 1.4: [3] A non-empty closed proper subset F of a topological space X is said to be a maximal closed set if any closed set which contains F is either X or F.

Definition 1.5: [5] A subset A of a topological space X is called a pgrω-closed set if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is rω-open in X.

Definition 1.6: [5] A subset A of a topological space X is called a pgrω-open set in X if A^c is a pgrω-closed set in X.

Definition 1.7: A co-finite subset of a set *X* is a subset whose complement in *X* is a finite set.

Definition 1.8: [6] A map f: $(X, T_1) \to (Y, T_2)$ is called a pre generalised regular weakly-continuous map (pgrw-continuous map) if the inverse image $f^{-1}(V)$ of every closed set V in Y is pgrw-closed in X.

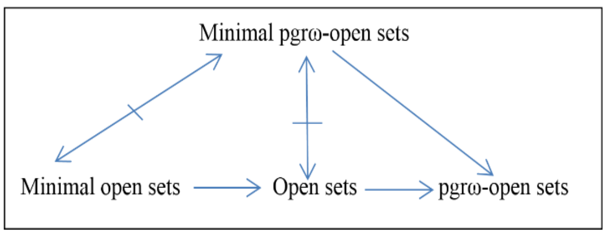
2. Minimal pgrw-open sets
Definition 2.1: A non-empty proper pgrω-open subset U of a topological space X is said to be a minimal pgrω-open set if any pgrω-open set contained in U is either ϕ or U.

Note: X and ϕ are pgrw-open, but not minimal pgrω-open sets.

Example 2.2: Let X = {a, b, c, d} and topology on X be T={X, ϕ, {a}, {b}, {a, b}, {a, b, c}}.
Minimal open sets are {a}, {b}. pgrω-open sets are X, ϕ, {a}, {b}, {c},{a,b}, {a,d}, {b,c}, {a,c}, {a,b,c}, {a, b, d}. Minimal pgrω-open sets are {a}, {b}, {c}.
Here {a, b} is open, but not minimal pgrω-open set.
{c} is a minimal pgrω-open set, but not open.

Examle: 2.3: X = {a, b, c, d}, T= ={X, ϕ, {a}, {c,d}, {a,c,d}}. Minimal open sets are {a},{c,d}.
Pgrw-open sets are X, ϕ, {a}, {b}, {c}, {d}, {a,c}, {a,d}, {c,d}, {a,b,c},{a,b,d}, {a,c,d}.
Minimal pgrw-open sets are {a}, {b}, {c}, {d}.
Here {c,d} is a minimal open set, but not a minimal pgrω-open set. And {b} is minimal pgrω-open, but not minimal open.

Remark 2.4: The above two examples show that
i) Minimal pgrω-open sets and open sets are independent and so ii) minimal pgrω -open sets and minimal-open sets are independent.
The above results are shown in the following diagram.



Theorem 2.5: Every non-empty finite pgrω-open set contains at least one minimal pgrω-open set.

Proof: Let V be a non-empty finite pgrω-open set. If V is a minimal pgrω-open set, then the statement holds true. If V is not a minimal pgrω-open set, then there exists a pgrω-open set V_1 such that ϕ≠V_1⊂ V. If V_1 is a minimal pgrω-open set, then the statement holds true. If V_1 is not a minimal pgrω-open set, then there exists a pgrω-open set V_2 such that ϕ≠V_2⊂ V_1. Continuing this process we have a sequence of pgrω-open sets $V_k \subset V_{k-1} \subset \ldots \subset V_3 \subset V_2 \subset V_1 \subset V$. Since V is a finite set, this process repeats only finitely and so finally we get a minimal pgrω-open set V_n for some positive integer n such that $V_n \subset V$.

Corollary 2.6 If V be a finite minimal open set, then there exists at least one minimal pgrω-open set U such that U⊂V.

Proof: If V is a finite minimal open set, then V is a non-empty finite pgrw-open set and so by theorem 2.5 there exists at least one minimal pgrω-open set U such that U⊂V.

3. Maximal pgrw-closed sets:

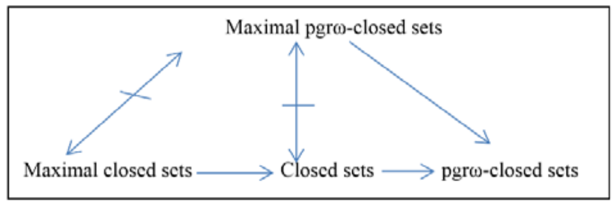
Definition 3.1: A non-empty proper pgrω-closed subset F of a topological space X is said to be a maximal pgrω-closed set if any pgrω-closed set which contains F is either X or F.

Note: X and ϕ are pgrw-closed, but not maximal pgrw-closed

Example 3.2: Let X = {a, b, c, d} and topology on X be T={X, ϕ, {a}, {b}, {a, b}, {a, b, c}}.
Closed sets are X, ϕ, {b,c,d}, {a,c,d}, {c,d}, {d}.
Maximal closed sets are {b,c,d}, {a,c,d}.
Pgrw-closed sets are X, ϕ, {c},{d}, {b,c},{c,d},{a,d},{b,d}, {b,c,d},{a,c,d}, {a,b,d}.
Maximal pgrw-closed sets are {b,c,d},{a,c,d}, {a,b,d}.
Here {d} is closed but not Maximal pgrω-closed set.
And {a,b,d} is maximal pgrw-closed, but not closed.

Example 3.3: X = {a, b, c, d}, T={X, ϕ, {a}, {c,d}, {a,c,d}}.
Maximal closed sets are {a,b}, {b,c,d}.
Maximal pgrω-closed sets are {a,b,c},{b,c,d}, {a,c,d},{a,b,d}.
{a,b} is maximal closed, but not maximal pgrω-closed and {a,b,c} is a maximal pgrw-closed set, but not maximal closed.

Remark 3.4: The above two examples show that i) Maximal pgrω-closed sets and closed sets are independent and so ii) maximal pgrω-closed sets and maximal closed sets are independent.
The above results are shown in the following diagram.



Theorem 3.5: A non-empty proper subset F of a topological space X is a maximal pgrω-closed set iff X-F is a minimal pgrω-open set.

Proof: F is a maximal pgrω-closed set. Suppose X-F is not a minimal pgrω-open set. Then there exists a pgrω-open set U≠X−F such that ϕ ≠ U ⊂ X-F that is F⊂X-U and X-U is a pgrω-closed set. This contradicts our assumption that F is a Maximal pgrω- closed set.
Conversely
Suppose X-F is a minimal pgrω-open set and F is not a Maximal pgrω-closed set. Then there exists a pgrω- closed set E other than F and X such that F⊂E that is proper ϕ ≠ X-E ⊂ X-F and X-E is a pgrω-open set. This contradicts our assumption that X-F is a minimal pgrω-open set. Therefore F is a Maximal pgrω-closed set.

Theorem 3.6: If F is a non-empty proper co-finite pgrω-closed subset of a topological space X, then there exists a co-finite maximal pgrω-closed set E such that F ⊂ E.

Proof: F is a non-empty proper co-finite pgrw- closed set in a topological space X.
=>X-F is a non-empty finite pgrw- open set.
=>there exist a finite minimal pgrw- open set U such that U⊂ X-F by th 2.5
=> there exists a co-finite Maximal closed set E= X-U such that F⊂E.

Definition 3.7: A non-empty proper pgrw-open subset A of a topological space X is said to be a maximal pgrw-open set if any pgrw-open set containing A is either X or A.

Definition 3.8: A non-empty proper pgrw-closed subset A of a topological space X is said to be a minimal pgrw-closed if any pgrw-closed set contained in A is either φ or A.

Example 3.9. Let X = {a, b, c, d}, T = {X, φ, {a}, {c, b}, {a, c,d}}.
Pgrw-open sets are X, φ, {a},{b},{c},{d},{a,c}, {a,d}, {c,d}, {a,b,c},{a,b,d}, {a,c,d}.
Maximal pgrw-open sets are {a,b,c},{a,b,d}, {a,c,d}.
Pgrw-closed sets are X, φ, {b}, {c}, {d}, {a,b},{b,c}, {b,d}, {a,b,c},{b,c,d}, {a,c,d}, {a,b,d}.
Minimal pgrw-closed sets are {b}, {c}, {d}.

4. Maximal-semi-pgrw-open sets and Minimal-semi-pgrw-closed sets:

Definition 4.1. A subset A in a topological space X is said to be maximal-semi-pgrw-open if there exists a maximal-pgrw-open set U such that U⊂ A⊂ cl(U).

Definition 4.2 A subset A in a topological space X is said to be minimal-semi-pgrw-closed if X − A is maximal-semi pgrw-open set.

Remark 4.3. Every maximal pgrw-open (minimal pgrw-closed) set is a maximal-semi-pgrw-open (minimal-semi-pgrw-closed closed) set.

Theorem 4.4. If A is a maximal-semi-pgrw-open set of a topological space X and A⊂ B⊂cl(A), then B is a maximal-semi-pgrw-open set of X.

Proof: If A is a maximal-semi-pgrw-open set of X, then there exists a maximal-pgrw-open set U of X such that U⊂ A ⊂ cl(U) and if A⊂ B⊂ cl(A), then U⊂ A ⊂ B ⊂cl(A) ⊂ cl(U). That is U ⊂ B ⊂ cl(U). Thus B is a maximal-semi-pgrw-open set of X.

Theorem 4.5. A is a minimal-semi-pgrw-closed subset in a topological space X if and only if there exists a minimal pgrw-closed set B in X such that int(B) ⊂ A ⊂ B.

Proof. Suppose A is a minimal-semi-pgrw-closed set in a topological space X. Then by definition
X – A is maximal-semi pgrw-open set of X. Then there exists a maximal pgrw-open set U such that U ⊂ X − A ⊂ cl(U). That is int(X − U) = X − cl(U) ⊂ A ⊂ X − U. Let B = X − U, so that B is a minimal pgrw-closed set in X such that int(B) ⊂ A ⊂ B.
Conversely Suppose that there exists a minimal pgrw-closed set B in X such that int(B) ⊂ A ⊂ B. Then X−B ⊂ X−A ⊂ X−int(B) = cl(X−B). That is there exists a maximal pgrw-open set U = X−B such that
U ⊂ X−A ⊂ cl(U). This implies X−A is maximal-semi-pgrw-open in X. Hence A is minimal-semi-pgrw-closed in X.

Theorem 4.6. If A and B are subsets of a topological space X such that B is minimal-semi-pgrw-closed and int(B) ⊂ A ⊂B, then A is also minimal-semi-pgrw-closed in X.

Proof. Let B be a minimal-semi-pgrw-closed set of X. Then by th. 4.5 there exists a minimal pgrw-closed set U such that int(U) ⊂ B ⊂ U and since int(B) ⊂A ⊂ B, we have int(U) ⊂ int(B) ⊂ A ⊂ B ⊂ U. That is int(U) ⊂ A ⊂ U. Therefore A is a minimal-semi-pgrw-closed set in X.

Theorem 4.7. Y is any open subspace of a topological space X and A ⊂ Y. If A is a maximal-semi pgrw-open set of X, then A is also a maximal-semi-pgrw-open set of Y.

Proof. Y is any open sub space of X and A⊂Y. If A is a maximal-semi-pgrw-open set of X, there exists a maximal pgrw-open set U of X such that U ⊂ A ⊂cl(U).As A⊂Y, U is subset of Y. Since U is maximal pgrw-open in X, Y ∩ U = U is maximal pgrw-open in Y and U = Y∩ U ⊂ Y ∩ A ⊂ Y ∩ cl((U). That is U⊂ A ⊂ cl_Y (U). Hence A is a maximal- semi-pgrw-open set of Y.

5. Minimal pgrw-continuous maps and Maximal pgrw-continuous maps

Definition 5.1. Let X and Y be topological spaces. A map f : X → Y is called
(i) Minimal pgrw-continuous (briefly min-pgrw-continuous) if f^{-1}(A) is a pgrw-closed set in X for every minimal closed set A in Y.
(ii) Maximal pgrw-continuous (briefly max-pgrw-continuous) if f^{-1}(A) is a pgrw-closed set in X for every maximal closed set A in Y.
(iii) Minimal pgrw-irresolute (briefly min-pgrw-irresolute) if f^{-1}(A) is a minimal pgrw-closed set in X for every minimal closed set A in Y.
(iv) Maximal pgrw-irresolute (briefly max-pgrw irresolute) if f^{-1}(A) is a maximal pgrw-closed set in X for every maximal closed set A in Y.
(v) Minimal maximal-pgrw-continuous (briefly min max-pgrw-continuous) if f^{-1}(A) is a maximal pgrw-closed set in X for every minimal closed set A in Y.
(vi) Maximal minimal-pgrw-continuous (briefly max min-pgrw-continuous) if f^{-1}(A) is a minimal pgrw-closed set in X for every maximal closed set A in Y.

Example: 5.2 X={a,b,c,d}, T_1={X, φ, {a,b}, {c,d}}
Y={a,b,c}, T_2 ={Y, φ, {a},{a,c}}.
Define f : X→Y by f(a)=b,f(b)=c, f(c)=a, f(d)=a. Closed sets in Y are Y, φ, {b}, {b,c}. Minimal closed set in Y is {b}. Pgrw-closed sets in X are all subsets of X. f^{-1}({b})={a} which is pgrw-closed in X. So f is a minimal pgrw-continuous map.
Maximal closed set in Y is {b,c}. f^{-1}({b,c})={a,b} which is pgrw-closed in X. So f is maximal pgrw-continuous map.
{b} is minimal closed set in Y and f^{-1}({b})={a} is minimal pgrw- closed set in X. So f is minimal pgrw-irresolute.

Example:5.3 X={a,b,c,d}, T_1={X, φ, {a}, {c,d},{a,c,d}}
Y={a,b,c,d}, T_2 ={Y, φ, {b,c},{b,c,d}, {a,b,c}}. Define f : X →Y by f(a)=b, f(b)=a, f(c)=d, f(d)=d. Maximal closed set in Y is {a,d}. Maximal pgrw-closed sets in X are {a,b,c}, {b,c,d}, {a,c,d}, {a,b,d}. So f^{-1}({a,d})={b,c,d} which is a maximal pgrw-closed set. So f is maximal pgrw-irresolute.

Theorem 5.4 Every pgrw-continuous map is minimal (maximal) pgrw-continuous.

Proof. Let f : X →Y be a pgrw-continuous map. Let A be any minimal (maximal) closed set in Y. Since every minimal (maximal) closed set is a closed set, A is a closed set in Y. Since f is pgrw-continuous, f^{-1}(A) is a pgrw-closed set in X. Hence f is a minimal (maximal) pgrw-continuous.
Converse is not true.

Example5.5: X={a.b.c}, T_1= {X, φ, {a}, {a,c}}. pgrw-closed sets in X are X, φ, {b}, {c}, {b,c}.
Y={a.b.c,d}, T_2= {Y, φ, {a}, {b}, {a,b}}.
{c,d}is the only minimal closed set in Y.
Define f :X →Y by f(a)=b, f(b)=c, f(c)=d.

$f^{-1}(\{c,d\})=\{b,c\}$ and is pgrw-closed set in X. So f is minimal pgrw-continuous. {b,c,d} is closed in Y. $f^{-1}(\{b,c,d\})=\{a,b,c\}$ is not pgrw-closed. Hence f is not pgrw-continuous.

Theorem 5.6. X and Y are two topological spaces. A map f : X → Y is minimal(maximal)pgrw-continuous if and only if the inverse image of every maximal(minimal) open set in Y is a pgrw-open set in X.

Proof:. Let A be a maximal (minimal) open set in Y. Then A^c is minimal (maximal) closed in Y. As f : X → Y is minimal(maximal)pgrw-continuous, $f^{-1}(A^c)$ is pgrw-closed in X. i.e. $[f^{-1}(A)]^c$ is pgrw-closed in X. And so $f^{-1}(A)$ is pgrw-open in X.
Conversely suppose f : X → Y is such that inverse image of every maximal(minimal) open set in Y is a pgrw-open set in X. Let A be a minimal closed set in Y. Then A^c is maximal open set in Y and so $f^{-1}(A^c)$ is pgrw-open in X. i.e. $[f^{-1}(A)]^c$ is pgrw-open in X. So $f^{-1}(A)$ is a pgrw-closed set in X. Hence f: X → Y is minimal (maximal) pgrw-continuous.

Theorem 5.7. X and Y are two topological spaces and A is a nonempty subset of X. If f: X → Y is a minimal (maximal) pgrw-continuous function, then the restriction map f_A : A →Y is minimal (maximal) pgrw-continuous.

Proof. Let f: X → Y be a minimal (maximal) pgrw-continuous map. Let B be any minimal (maximal) closed set in Y. Since f is minimal (maximal) pgrw- continuous, $f^{-1}(B)$ is a pgrw-closed set in X. But $f_A^{-1}(B) = A\cap f^{-1}(B)$ and $A\cap f^{-1}(B)$ is a pgrw-closed set in the subspace A. Therefore f_A is a minimal (maximal) pgrw-continuous.

Theorem 5.8 Every minimal (maximal) pgrw-irresolute map is a minimal (maximal) pgrw-continuous map.

Proof. Let f: X → Y be a minimal (maximal) pgrw-irresolute map. Let A be any minimal (maximal) closed set in Y. Since f is minimal (maximal) pgrw-irresolute, $f^{-1}(A)$ is a minimal(maximal) pgrw-closed set in X. That is $f^{-1}(A)$ is a pgrw-closed set in X. Hence f is minimal (maximal) pgrw-continuous.
Converse is not true.

Example 5.9: X={a,b,c,d}, T_1={X, φ, {a,b},{c,d}}. Y={a,b,c}, T_2={Y, φ, {a},{a,c}}
pgrw-closed sets in X are all subsets of X. Minimal pgrw-closed sets in X are {a},{b},{c}.
{b} is the only minimal closed set in Y.
Define f :X →Y by f(a)=b, f(b)=b, f(c)=a, f(d)=c.
$f^{-1}(\{b\})=\{a,b\}$ is pgrw-closed in X. Hence f is minimal pgrw-continuous.
{a,b} is not minimal pgrw-closed.
Hence f is not minimal pgrw-irresolute.

Theorem 5.10 X and Y are two topological spaces. A map f : X→Y is minimal maximal (maximal minimal)- pgrw-continuous if and only if the inverse image of each maximal(minimal) open set in Y is a minimal(maximal) pgrw-open set in X.

Proof: Suppose f : X → Y is minimal maximal (maximal minimal)- pgrw-continuous. Let A be a maximal (minimal) open set in Y. Then A^c is a minimal (maximal) closed set in Y. As f is minimal maximal (maximal minimal)- pgrw-continuous $f^{-1}(A^c)$ is a maxima l(minmal)-pgrw-closed set in X. So $f^{-1}(A)$ is a minimal(maximal) pgrw-open set in X.
Conversely suppose the inverse image of each maximal (minimal) open in Y is a minimal (maximal) pgrw-open set in X.
Let A be a minimal (maximal)-closed set in Y. Then A^c is maximal (minimal)- open in Y. As the inverse image of each maximal (minimal) open set in Y is a minimal (maximal) pgrw-open set in X, $f^{-1}(A^c)$ is a minimal(maximal) pgrw-open set in X. $f^{-1}(A)$ is a maximal (minimal) pgrw-closed set in X. So f:X→Y is minimal maximal (maximal minimal)-pgrw-continuous.

Theorem 5.11. Every minimal maximal (maximal minimal)-pgrw-continuous map is a minimal (maximal) pgrw-continuous map.

Proof. Let f: X → Y be a minimal maximal(maximal minimal)-pgrw-continuous map. Let A be any minimal (maximal) closed set in Y. Since f is minimal maximal (maximal minimal)-pgrw-continuous, $f^{-1}(A)$ is a maximal (minimal) pgrw -closed set in X. Since every maximal (minimal)-pgrw-closed set is a pgrw- closed set, $f^{-1}(A)$ is pgrw-closed in X. Hence f is a minimal (maximal) pgrw-continuous map.

Definition 5.12. A topological space (X,T) is called a
(i) $T_{min-pgrw}$ space if every pgrw-closed set in it is minimal closed.
(ii) $T_{max-pgrw}$ space if every pgrw-closed set in it is maximal closed.
(iii) Min-T_{pgrw} Space if every minimal pgrw-closed set in it is minimal closed.
(iv) Max-T_{pgrw}Space if every maximal pgrw-closed set in it is maximal closed.

Theorem 5.13: If f: X → Y is a minimal(maximal) pgrw-continuous map and Y is a $T_{min-pgrw}$ ($T_{max-pgrw}$) space, then f is pgrw-continuous.

Proof. f: X → Y is a minimal(maximal)-pgrw-continuous function. Let A be a closed set in Y. Since every closed set is pgrw -closed, A is pgrw -closed. By hypothesis Y is a $T_{min-pgrw}$ ($T_{max-pgrw}$) space, it follows that A is a minimal (maximal) closed set in Y. Since f is minimal (maximal) pgrw-continuous, $f^{-1}(A)$ is pgrw-closed in X. Therefore f is pgrw-continuous.

Theorem 5.14. If f: X → Y is a minimal (maximal) pgrw-irresolute map and Y is a $T_{min-pgrw}$($T_{max-pgrw}$) space, then f is pgrw-continuous.

Proof. Let A be a closed set in Y. A is pgrw-closed in Y. As Y is $T_{min-pgrw}$($T_{max-pgrw}$) space, A is minimal (maximal) closed. So $f^{-1}(A)$ is a minimal (maximal) pgrw closed set in X because f : X → Y is a minimal (maximal) pgrw-irresolute map. So $f^{-1}(A)$ is pgrw-closed. So f is pgrw-continuous.

Theorem 5.15. If f: X → Y is a minimal maximal (maximal minimal)-pgrw-continuous map and Y is a $T_{min-pgrw}$($T_{max-pgrw}$) space, then f is a pgrw-continuous.

Proof: Let A be a closed set in Y. Then A is pgrw-closed in Y. As Y is a $T_{min-pgrw}$($T_{max-pgrw}$) space, A is minimal (maximal) closed in Y. As f is a minimal maximal (maximal

minimal)-pgrw-continuous map, $f^{-1}(A)$ is a maximal pgrw-closed set in X. So $f^{-1}(A)$ is pgrw-closed.
Hence f: X → Y is pgrw-continuous.

Theorem 5.16. If f: X → Y and g: Y → Z are minimal (maximal) pgrw-irresolute maps and Y is a Min-T_{pgrw}(Max-T_{pgrw}) space, then g0f : X → Z is a minimal (maximal) pgrw-irresolute map.

Proof. Let A be any minimal (maximal) closed set in Z. Since g is minimal (maximal) pgrw-irresolute, $g^{-1}(A)$ is a minimal (maximal) pgrw-closed set inY. Since Y is a Min-T_{pgrw}(Max-T_{pgrw}) space, $g^{-1}(A)$ is a minimal (maximal) closed set in Y. Again Since f is minimal (maximal) pgrw-irresolute, $f^{-1}(g^{-1}(A))= (g0f)^{-1}(A)$ is minimal (maximal) pgrw-closed set in X. Therefore g0f is a minimal (maximal) pgrw-irresolute.

Theorem 5.17. If f: X → Y is maximal (minimal) pgrw-irresolute, Y is a Max-T_{pgrw}(Min-T_{pgrw}) space and g : Y → Z is minimal maximal (maximal minimal)-pgrw-continuous, then g0f : X → Z is a minimal- maximal (maximal minimal) pgrw-continuous map.

Proof: Let A be any minimal (maximal) closed set in Z. Since g is minimal maximal (maximal minimal)- pgrw-continuous, $g^{-1}(A)$ is a maximal(minimal) pgrw- closed set in Y. Since Y is a Max-T_{pgrw}(Min-T_{pgrw})space, $g^{-1}(A)$ is a maximal (minimal) closed set. Again since f is maximal (minimal) pgrw-irresolute,$f^{-1}(g^{-1}(A)) = (g0f)^{-1}(A)$ is a maximal (minimal) pgrw-closed set in X. Hence g0f is a minimal maximal (maximal minimal)-pgrw-continuous map.

Theorem 5.18. If f: X → Y and g: Y → Z are minimal maximal (maximal minimal)-pgrw-continuous maps and if Y is a $T_{min-pgrw}$($T_{max-pgrw}$) space, then g0f : X→ Z is a minimal maximal(maximal minimal) pgrw-continuous map.

Proof: Let A be any minimal (maximal) closed set in Z. Since g is minimal maximal (maximal minimal) pgrw-continuous, $g^{-1}(A)$ is a maximal(minimal) pgrw- closed set in Y and so $g^{-1}(A)$ is a pgrw-closed subset of Y. Since Y is $T_{min-pgrw}$ ($T_{max-pgrw}$) space, $g^{-1}(A)$ is a minimal (maximal) closed set in Y. Again since f is minimal maximal (maximal minimal) pgrw-continuous, $f^{-1}(g^{-1}(A)) = (g0f)^{-1}(A)$ is a maximal (minimal) pgrw-closed set in X. Hence g0f is a minimal maximal (maximal minimal) pgrw-continuous map.

6. References

1. Nakaoka F, Oda F. Some Application of minimal open sets, Int. J Math Math sci. 2001; 27(8):471-476.
2. Nakaoka F, Oda F. Some Properties of Maximal open sets, Int. J Math Math sci. 2001; 21(21):1331-1340.
3. Nakaoka F, Oda F. On minimal closed sets, Proceeding of topological spaces and It's application, 2003, 5(19-21).
4. Seenivasan, Kalaiselvi S. A New Class of Minimal and Maximal Sets via gsg Closed Set, Int. Journal of Math Analysis, 2013, 7(52).
5. Wali R.S., Vijaykumari T. Chilakwad, On pre generalised Regular weekly-closed sets in topological spaces, Int. J of Math. Archive. 2015; 6(1):76-85.
6. Wali R. S. and Vijaykumari T. Chilakwad, Pgrw-Continuous and Pgrw-Irresolute Maps in Topological Spaces, IOSR J of Mathematics. 2016; 12(1):01-10.