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Total magic cordial labeling and square sum total magic cordial labeling in extended duplicate graph of triangular snake

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Abstract
Cordial labeling is one of the best labeling in graph theory. In this paper, we prove the extended duplicate graph of triangular snake is total magic cordial and square sum total magic cordial.
AMS Subject Classification: 05C78

Keywords: Graph labeling, cordial labeling, magic labeling, extended duplicate graphs, square sum total magic cordial.

1. Introduction

Graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. Graph labeling provide useful mathematical models for a wide range of applications, such as data security, cryptography, astronomy, various coding theory problems, communications networks, bio-informatics and X-ray crystallography. After the introduction of graph labeling, various labeling of graphs such as graceful labeling, magic labeling, anti-magic labeling, prime labeling, cordial labeling, odd and even graceful labeling etc., have been studied in over 2100 papers [4].

One of the most famous and productive labeling of graph theory is cordial labeling. This labeling was introduced by Cahit [1] in the year 1987. Let f be a function from the vertices of G to {0, 1} and for each xy assigns the label $|f(x) - f(y)|$. The function f is said to be a cordial labeling of G if the number of vertices labeled 0 and the number of vertices labeled 1 differ by at most one, the number of edges labeled 0 and the number of edges labeled 1 differ by atmost one. In 1990, Cahit [2] has proved that every tree is cordial; K_n is cordial if and only if $n \leq 3$; $K_{m,n}$ is cordial for all m and n; all fans are cordial.

Cahit [3] has introduced the concept of total magic cordial graph. A function f is said to be total magic cordial if $f: V \cup E \rightarrow \{0,1\}$ such that (i) $\{f(x) + f(y) + f(xy)\} (\text{mod} 2) = k$, a constant and $xy \in E$. (ii) $|f(0) - f(1)| \leq 1$, where a) $f(0)$ denotes the sum of the number of the vertices labeled with 0 and the number of edges labeled with 0. b) $f(1)$ denotes the sum of the number of the vertices labeled with 1 and the number of edges labeled with 1. He also proved that all trees are total magic cordial.

Ajitha, Arumugam and Germina have introduced the concept of square sum labeling in 2009. They proved that the trees, cycles, K_n if and only if $n \leq 5$ are square sum labeling. They also proved that every strong square sum graphs except K_1, K_2 and K_3 contains a triangle

2. Preliminaries

In this section, we give the basic notions relevant to this paper. Let G = G (V, E) be a finite, simple and undirected graph with p vertices and q edges. By a labeling we mean a one-to-one mapping that carries a set of graph elements onto a set of numbers, called labels (usually the set of integers).

Definition 2.1 (Triangular snake)

A triangular snake TS_m is obtained from a path $u_1, u_2, \ldots, u_{m+1}$ by joining u_i, and u_{i+1} to a new vertex v_i for $1 \leq i \leq m$.

Definition 2.2 (Duplicate graph)
Let G (V, E) be a simple graph and the duplicate graph of G is DG = (V_1, E_1),where the vertex set $V_1 = V \cup V'$ and $V \cap V' = \phi$ and $f : V \rightarrow V'$ is bijective (for $v \in V$, we write $f(v) = v'$ for convenience) and the edge set E_1 of DG is defined as the edge *ab* is in E if and only if both ab' and $a'b$ are edges in E_1.

Definition 2.3 (Extended duplicate graph of Triangular Snake): Let DG = (V_1, E_1) be a duplicate graph of the triangular snake graph G(V, E). Extended duplicate graph of triangular snake is obtained by adding the edge $v_2 v_2'$ to the duplicate graph. It is denoted by EDG (TS_m). Clearly it has 4m+2 vertices and 6m+1 edges, where 'm' is the number of edges of the path [5-7].

Definition 2.4 (**Cordial labeling)**
A function $f : V \rightarrow \{0,1\}$ such that each edge *uv* receives the label $|f(u) - f(v)|$ is said to be cordial labeling if the number of vertices labeled '0' and the number of vertices labeled '1' differ by at most one, and the number of edges labeled '0' and the number of edges labeled '1' differ by at most one.

Definition 2.5 (**Total cordial labeling)**
A function $f : V \rightarrow \{0,1\}$ such that each edge *uv* receives the label $| f(u) - f(v)|$ is said to be total cordial labeling if the number of vertices and edges labeled '0' and the number of vertices and edges labeled '1'differ by at most one.

Definition 2.6 (Total magic cordial labeling):
A graph G (V, E) is said to admit total magic cordial labeling if
f: $V \cup E \rightarrow \{0, 1\}$ such that
(i) $\{f(x) + f(y) + f(xy)\} \pmod 2$ is a constant for all edges $xy \in E$.
(ii) for all $i, j \in \{0,1\}$
$|\{m_i(f) + n_i(f)\} - \{m_j(f) + n_j(f)\}| \leq 1, (i \neq j)$
Where mi (f) = the number of edges labeled with i and
n_i (f) = the number of vertices labeled with i
A graph which admits total magic cordial labeling is called total magic cordial graph.

Definition 2.7 (Square sum total magic cordial labeling)
A graph G (V, E) is said to admit square sum total magic cordial labeling if
f: $V \cup E \rightarrow \{0, 1\}$ such that
(i) $f^*(v_iv_j) = \{ [f(v_i)]^2 + [f(v_j)]^2 + [f(v_iv_j)]^2\} \pmod 2$ for all $v_iv_j \in E$. is a constant.
(ii) for all $i, j \in \{0,1\}$
$| m_i(f) + n_i(f)\} - \{m_j(f) + n_j(f)\}| \leq 1, (i \neq j)$
Where mi (f) = the number of edges labeled with i and
n_i (f) = the number of vertices labeled with i
A graph which admits square sum total magic cordial labeling is called square sum total magic cordial graph [8].

3. Main Results

3.1 Total magic cordial labeling:

In this section, we present an algorithm and prove the existence of total magic cordial labeling for the extended duplicate graph of Triangular Snake (TS_m).

Algorithm 1: procedure (total magic cordial labeling for EDG (TS_m)

```
V ← {v1, v2, …, v2m, v2m+1, v'1, v'2,… v'2m, v'2m+1 }
E ← {e1, e2, …, e3m, e3m+1, e'1,e'2, …,e'3m }
e3m+1 ← 1
for i = 0 to m do
    v1+2i ← 1
    v'1+2i ← 0
end for
for i = 0 to (m-1) do
    v2+2i ← 0
    v'2+2i ← 1
end for
for i = 0 to m-1 do
for j = 0 to 1 do
    e1+3i+2j ← 0
    e'1+3i+2j ← 0
        end for
for i = 0 to m-1 do
    e2+3i ← 1
    e'2+3i ← 1
end for
```

End procedure

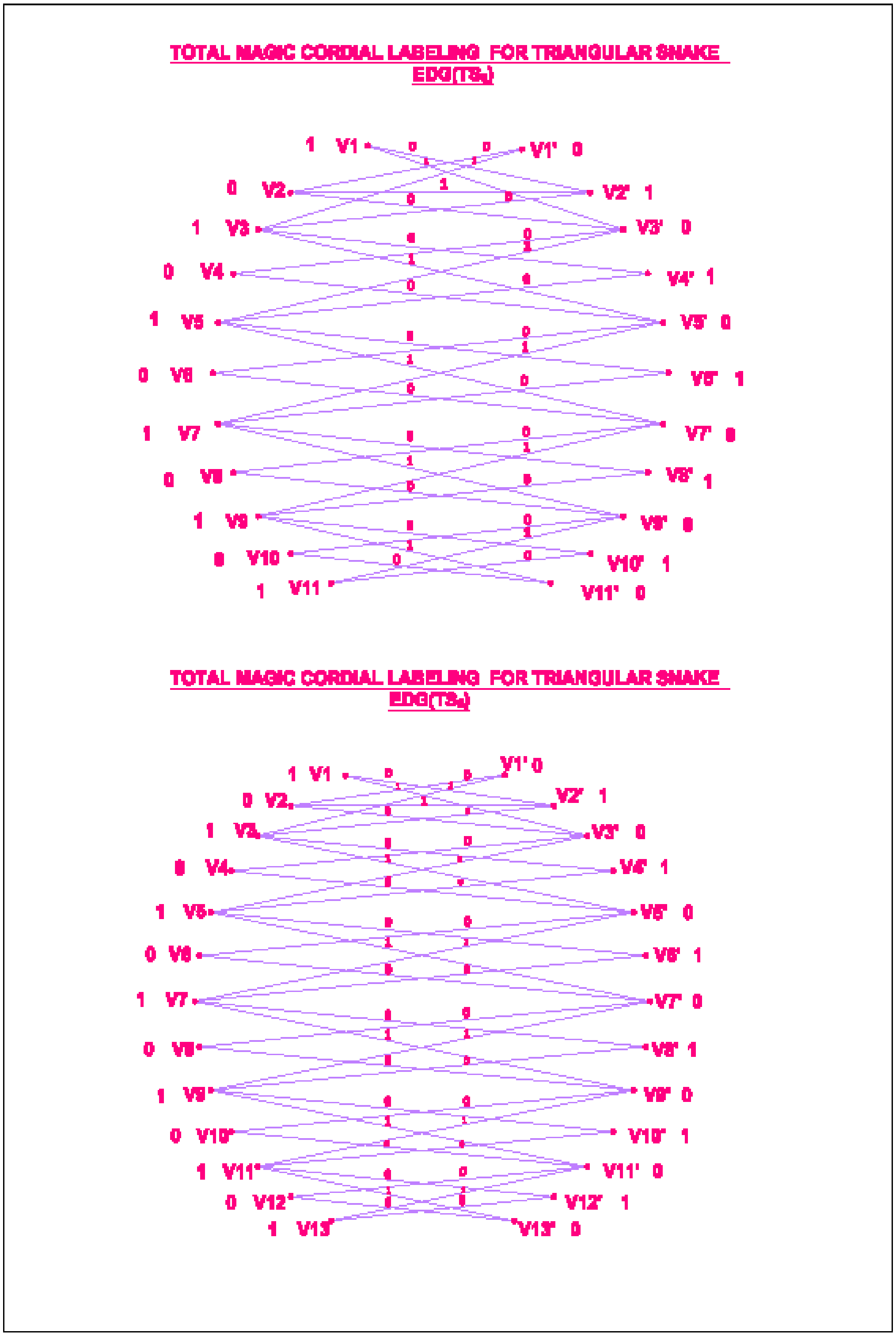
Theorem 1: The extended duplicate graph of triangular snake TS_m, $m \geq 1$ is total magic cordial.

Proof: Let TS_m, $m \geq 1$ be a triangular snake. To label the vertices and edges, define a function $f : V \cup E \rightarrow \{0,1\}$ as given in algorithm 1.
The vertices v_{1+2i} receive label '1' and v'_{1+2i} receive label '0' for $0 \leq i \leq m$; the vertices v_{2+2i} receive label '0' and v'_{2+2i} receive label '1' for $0 \leq i \leq m-1$.
Therefore the vertices $v_1, v_3, v_5, v_7, \ldots v_{2m-1}, v_{2m+1}, v'_2, v'_4, v'_6, \ldots v'_{2m-2}, v'_{2m}$ receive label '1'; the vertices $v_2, v_4, v_6, \ldots v_{2m-2}, v_{2m}, v'_1, v'_3, v'_5, \ldots v'_{2m-1}, v'_{2m+1}$ receive label '0'.
The edges $e_{1+3i+2j}$ and $e'_{1+3i+2j}$ receive label '0' for $0 \leq i \leq m-1$ and $0 \leq j \leq 1$; the edges e_{2+3i} and e'_{2+3i} receive label '1' for $0 \leq i \leq m-1$; the edge e_{3m+1} receive label '1'.
Therefore the edges $e_1, e_3, e_4, e_6, e_7, e_9, e_{10}, \ldots, e_{3m-2}, e_{3m}, e'_1, e'_3, e'_4, e'_6, e'_7, e'_9, e'_{10}, \ldots, e'_{3m-2}, e'_{3m}$ receive label '0' and the edges $e_2, e_5, e_8, \ldots, e_{3m-1}, e_{3m+1}, e'_2, e'_5, e'_8, \ldots, e'_{3m-1}$ receive label '1'. Thus the entire 4m+2 vertices and 6m+1 edges are labeled such that the number of vertices labeled '1' and '0' are same as 2m+1 and the number of edges labeled '1' is 3m+1 and the number of edges labeled '0' is 3m, which differ by one and satisfies the required condition.
The induced function $f^* : E \rightarrow \{0,1\}$ is defined as

$$f^*(v_iv_j) = \{ f(v_i) + f(v_j) + f(v_iv_j)\} \pmod 2 \text{ for all } v_iv_j \in E.$$

Thus the induced function yields a constant '0' to all the edges the total magic cordial labeling as a constant '0'.
Therefore, the function f is total magic cordial
Hence the extended duplicate graph of triangular snake TS_m, $m \geq 1$ is total magic cordial.

Example: Total magic cordial labeling for EDG (TS_5) and EDG(TS_6)



3.2: Square sum total magic cordial labeling

In this section, we present an algorithm and prove the existence of square sum total magic cordial labeling for the extended duplicate graph of Triangular Snake (TS_m)

Algorithm 2: procedure (square sum total magic cordial labeling for EDG (TS_m))

$V \leftarrow \{v_1, v_2, \ldots, v_{2m}, v_{2m+1}, v'_1, v'_2, \ldots v'_{2m}, v'_{2m+1}\}$

$E \leftarrow \{e_1, e_2, \ldots, e_{3m}, e_{3m+1}, e'_1, e'_2, \ldots, e'_{3m}\}$

$e_{3m+1} \leftarrow 1$

for i = 0 to m **do**

$\quad v_{1+2i} \leftarrow 0$

$v'_{1+2i} \leftarrow 1$

end for

for i = 0 to (m-1) **do**

$\quad v_{2+2i} \leftarrow 1$

$\quad v'_{2+2i} \leftarrow 0$

end for
for i = 0 to m-1 **do**
for j = 0 to 1 **do**
  $e_{1+3i+2j} \leftarrow 0$
  $e'_{1+3i+2j} \leftarrow 0$
  end for
end for
for i = 0 to m-1 **do**
  $e_{2+3i} \leftarrow 1$
  $e'_{2+3i} \leftarrow 1$
end for

End procedure

Theorem 2: The extended duplicate graph of triangular snake TS_m, $m \geq 1$ is square sum total magic cordial.

Proof: Let TS_m, $m \geq 1$ be a triangular snake. To label the vertices and edges, define a function f: $V \cup E \rightarrow \{0, 1\}$ as given in algorithm 2. The vertices v_{1+2i} receive label '0' and v'_{1+2i} receive label '1' for $0 \leq i \leq m$; the vertices v_{2+2i} receive label '1' and v'_{2+2i} receive label '0' for $0 \leq i \leq m-1$. Therefore the vertices $v_1, v_3, v_5, v_7, \ldots v_{2m-1}, v_{2m+1}, v'_2, v'_4, v'_6, \ldots v'_{2m-2}, v'_{2m}$ receive label '0'; the vertices $v_2, v_4, v_6, \ldots v_{2m-2}, v_{2m}, v'_1, v'_3, v'_5, \ldots v'_{2m-1}, v'_{2m+1}$ receive label '1'. The edges $e_{1+3i+2j}$ and $e'_{1+3i+2j}$ receive label '0' for $0 \leq i \leq m-1$ and $0 \leq j \leq 1$; the edges e_{2+3i} and e'_{2+3i} receive label '1' for $0 \leq i \leq m-1$; the edge e_{3m+1} receive label '1'.

Therefore the edges $e_1, e_3, e_4, e_6, e_7, e_9, e_{10}, \ldots, e_{3m-2}, e_{3m}, e'_1, e'_3, e'_4, e'_6, e'_7, e'_9, e'_{10}, \ldots, e'_{3m-2}, e'_{3m}$ receive label '0' and the edges $e_2, e_5, e_8, \ldots, e_{3m-1}, e_{3m+1}, e'_2, e'_5, e'_8, \ldots, e'_{3m-1}$ receive label '1'. Thus the entire 4m+2 vertices and 6m+1 edges are labeled such that the number of vertices labeled '1' and '0' are same as 2m+1 and the number of edges labeled '1' is 3m+1 and the number of edges labeled '0' is 3m, which differ by one and satisfies the required condition.

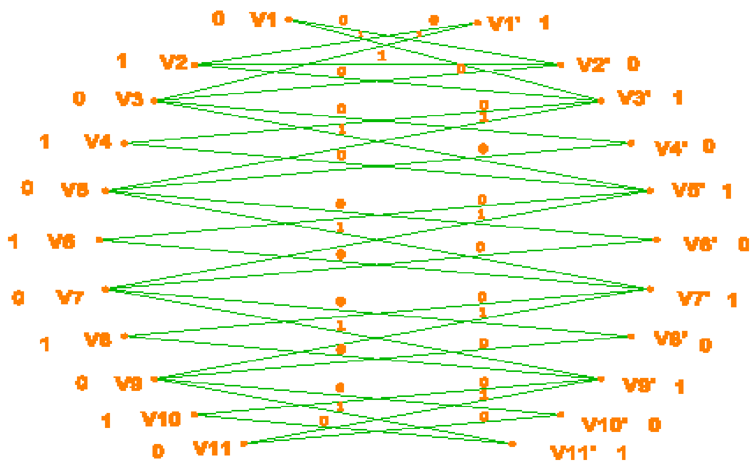
The induced function $f^*: E \rightarrow \{0, 1\}$ is defined as

$f^*(v_iv_j) = \{ [f(v_i)]^2 + [f(v_j)]^2 + [f(v_iv_j)]^2 \} \pmod 2$ for all $v_iv_j \in E$.

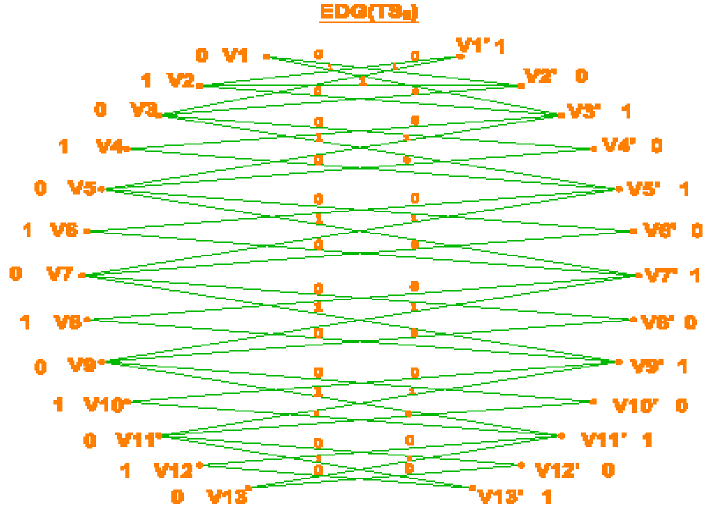
Thus the induced function yields a constant '0' to all the edges. Therefore, the function f is square sum total magic cordial. Hence the extended duplicate graph of triangular snake TS_m, $m \geq 1$ is square sum total magic cordial.

Example: Square sum total magic cordial labeling for EDG (TS_5) and EDG (TS_6)

SQUARE SUM TOTAL MAGIC CORDIAL LABELING FOR TRIANGULAR SNAKE EDG(TS_5)



SQUARE SUM TOTAL MAGIC CORDIAL LABELING FOR TRIANGULAR SNAKE EDG(TS_6)



4. Conclusion

In this paper, we have constructed the algorithms for labeling the vertices and edges. Also we have shown that the extended duplicate graph of triangular snake TS_m admits total magic cordial labeling and square sum total magic cordial labeling.

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