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On βg^* closed sets in Topological Spaces

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Abstract
The aim of this paper is to introduce the concept of βg^*closed and open set and its closure and interior. Some of the fundamental properties of this set are studied. And some of their properties are also given.

Keywords: g closed set, βg closed set, βg open set, β open.

1. Introduction

In 1970, Levine [6] introduced the concept of generalized closed set in topological spaces. Andrijevic [1] introduced the concept of semi pre open set (β open set) in general topology in 1986. In 2000, M.K.R.S Veerakumar [14] introduced the concept of g* closed set. In this paper we introduced and studied the notion of βg^* closed sets in topological spaces.

2. Preliminaries

Definition 2.1: A subset A of a topological space (X, τ) is called *β open* [1] if $A \subseteq cl(int(cl(A)))$,whenever $A \subseteq U$ and U is open in X.

Definition 2.2: A subset A of X is called pre-open set [9] if $A \subseteq int(cl(A))$ and pre-closed [7] set if $A \subseteq cl(int(A))$.

Definition 2.3: A subset A of X is called α-open [10] if $A \subseteq int(cl(int(A)))$ and α-closed [11] if $cl(int(cl(A))) \subseteq A$.

Definition 2.4: A subset A of X is called θ-closed [15] if $A= cl\theta(A)$,where $cl\theta(A)=\{x \in X:cl(U) \cap A \neq U \in A\}$

Definition 2.5: A subset A of X is called δ-closed [15] if $A=cl\delta(A)$,where $cl\delta(A)=X:int(cl(U)) \cap A \neq U \in A\}$.

Definition 2.6: A subset A of X is called regular open (briefly r-open) [8] set if $A=int(cl(A))$ and regular closed (briefly r-closed) [6] set if $A=cl(int(A))$.

Definition 2.7: A subset A of X is called generalized closed (briefly g-closed) [6] set ifcl(A)$\subseteq$U wheneverA$\subseteq$U and U is open.

Definition 2.8: A subset A of X is called Semi-generalized closed (briefly sg-closed) [2] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open in X.

Definition 2.9: A subset A of X is called Generalized α-closed (briefly gα -closed) [3] if α-cl(A)$\subseteq$U whenever $A \subseteq U$ and U is α-open in X.

Definition 2.10: A subset A of X is called Generalized semi-pre closed (briefly gsp-closed) [10] if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

Definition 2.11: A subset A of X is called Regular generalized closed (briefly rg-closed) [11] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.

Definition 2.12: A subset A of X is called θ-generalized closed (briefly θ-g-closed) [5] if clθ(A)⊆U whenever A⊆U and U is open in X.

Definition 2.13: A subset A of X is called δ-generalized closed (briefly δ-g-closed) [13] if clδ(A)⊆U whenever A⊆U and U is open in X.

Definition 2.14: A subset A of X is called Strongly generalized closed (briefly g*-closed) [9] if cl(A)⊆U whenever A⊆U and U is g-open in X.

Definition 2.15: A subset A of X is called Weakly closed (briefly w-closed) [7] if cl(A)⊆U whenever A⊆U and U is semi open in X.

Definition 2.16: A subset A of X is called Semi weakly generalized closed (briefly swg-closed) [8] if cl(int(A))⊆U whenever A⊆U and U is semi open in X.

3. On βg* closed set

Definition 3.1: A subset A of a topological space (X,τ) is called βg*closed set if gcl(A)⊆U, whenever A⊆U and U is β open in X.

Remark 3.2: ∅ and X are βg* closed subset of X.

Remark 3.3: (i).Every g closed set is βg* closed set, since whenever A⊆U, gcl (A)⊆U.
(ii).Every g* closed set is g closed. Therefore Every g* closed set is βg*closed set.

Theorem 3.4: Every closed set is βg*closed set but not conversely.

Proof: Let A be a closed set such that A⊆U and U is β open. Therefore A=Cl (A) ⊆U and this implies gcl(A) ⊆U. Hence gcl(A)⊆U whenever A⊆U and U is β-open. Therefore A is a βg* closed set.

Example 3.5: Let X={a,b,c,d}, τ={X,ϕ, {a},{b},{a,b},{a,b,c}}.Here A={a,d} is βg* closed but not closed set in X.

Remarks 3.6: (i).Every θ-closed set is a closed set. Therefore every θ-closed set is βg* closed.
(ii). Every π closed is closed. Therefore every π closed is βg*closed.
(iii). Every δ closed is closed. Therefore every δ closed is βg* closed

Theorem 3.7: Every regular closed set is βg* closed but not conversely.
Proof: Let A be a regular closed set, such that A⊆U and U is β-open. Every regular closed set is closed. By Theorem3.3, A is βg* closed set.

Example 3.8: Let X={a, b, c, d}, τ={X,ϕ,{a},{b},{a,b},{a,b,c}}.Let A={d} is βg* closed but not regular closed.

Theorem 3.9: Every gr closed set is βg* closed.

Proof: Let A be a gr closed set such that A⊆U and U is β-open. Every generalized regular closed set is closed. By Theorem 3.4, every gr closed set is a βg* closed set.

Theorem 3.10: Every w-closed set is βg* closed set, but not conversely.

Proof: Let A be a w-closed set. Therefore cl(A) ⊆U, whenever A⊆U and U is semi open. Then gcl(A) ⊆ cl(A) ⊆U. Therefore gcl(A) ⊆U, whenever A⊆U and U is semi open. Since every semi open set is β-open. Therefore every w-closed set is βg*closed set.

Example 3.11: Let X={a,b,c,d}, τ={X,ϕ, {a},{b},{a,b},{a,b,c}} here A={b,d} is βg* closed but not a closed set in X.

Remark 3.12: The Union of two βg* closed subsets of X is also βg*closed subset of X.
Proof: Assume that A and B are βg* closed sets in X, such that A∪B⊂U and U is β-open. Then A⊂U and B⊂U. Since A and B are βg*closed set. Therefore gcl(A)⊂U and gcl(B)⊂ U. Hence gcl(A∪B)=gcl(A)∪gcl(B)⊂U. That is A∪B is βg* closed set.

Theorem 3.13: Let A⊆ B⊆gcl (A) and A is a βg* closed subset of (X,τ) then B is also a βg* closed subset of (X,τ).

Proof: Since A is a βg* closed subset of (X,τ). gcl (A)⊆U, whenever A ⊆U,U is β open subset of X. Let A⊆ B⊆gcl (A).Therefore gcl(A)=gcl(B).Let V be a open subset of X such that B ⊆ V. So A ⊆ V and A being βg*closed subset of X, gcl(A) ⊆ V. That is gcl (B) ⊆ V. Hence B is also a βg* closed subset of X.

Theorem 3.14: Let A⊆ B⊆X, If A is βg* closed in X, then A is βg* closed in B.
Proof: Let A ⊆ U, where U is a β open set of X. Since U=V∩B, for Some β open set V of X and B is β open in X. Using assumption A is βg* closed in X. We have gcl(A)⊆U and so gcl(A)=cl(A)∩B ⊆ U∩B ⊆ U. Hence A is βg* closed in B.

Theorem 3.15: A subset A of X is βg*closed set iff gcl(A) ∩ Ac contains no non-empty closed set in X.

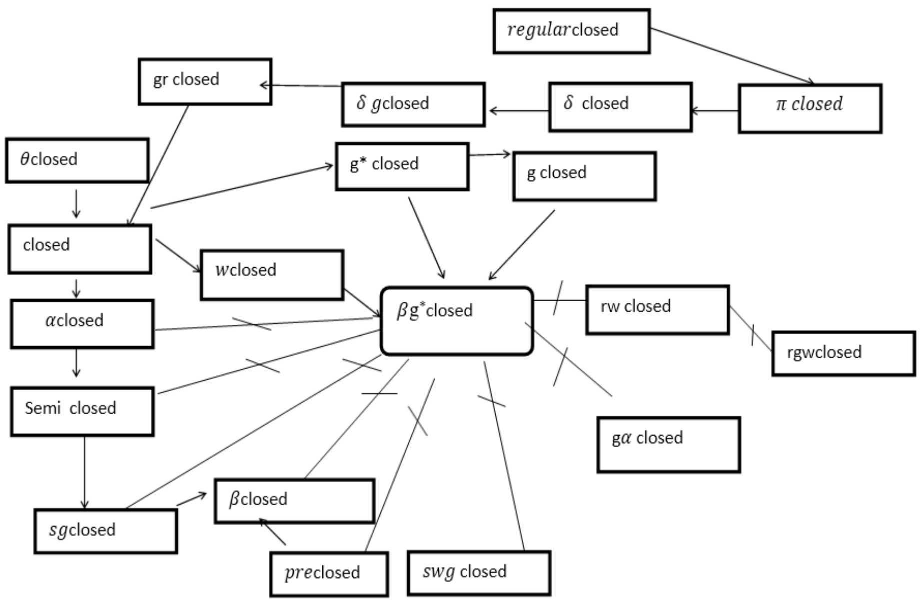
Proof: Let A be βg* closed subset of X. Also let M be closed subset of X such that M ⊆gcl(A) ∩Ac.That is M ⊆gcl(A) and M ⊆ Ac. Since M is a closed subset of X, Mc is an open subset of X ⊆ A and A being βg* open subset of X, gcl (A) ⊆ Mc. But M⊆gcl(A).So we get a contradiction. Therefore M=∅. So the condition is true. Conversely, let A ⊆N, and N is a open subset of X. Then Nc ⊆Ac, and Nc is a closed subset of X. Let gcl(A) ⊆ N. Then gcl(A) ∩ Nc is a nonzero closed subset of gcl(A) ∩ Ac, which is a contradiction. Hence A is a βg* closed subset of X.
Theorem3.16:A subset A of X is a βg* closed set in X iff gcl(A)-A contains no non-empty βclosed set in X.

Proof: Suppose that F is a non-empty β closed subset of gcl(A)-A. Now F ⊆gcl(A)-A. Then F ⊆ gcl(A) ∩ Ac. Therefore F⊆ Ac. Since Fc is β open set and A is βg* closed, gcl(A) ⊆ Fc. That is F⊆ gcl(A)c. Hence F⊆gcl(A) ∩[gcl(A)]c=∅. That is F=∅. Thus gcl(A)-A contains no non empty β closed set. Conversely assume that gcl(A)-A contains no nonempty β closed set. Let A ⊆ U,U is β open. Suppose that gcl(A) is not contained in U. Then gcl(A) ∩ Uc is a non-empty β closed set and contained in gcl(A)-A. which is a contradiction. Therefore gcl(A) ⊆ U and hence A is a βg* closed set.

Example 3.17: The figure 1 justified with the following examples.

Let X= {a, b, c, d}, be with the topology τ={X,ϕ, {a},{b},{a, b},{a, b, c}} then

1. Closed sets in X are X, ϕ, {d}, {c, d}, {a, c, d}, {b, c, d}
2. βg* closed sets in X are X, ϕ, {d}, {a, d}, {b, d}, {c, d} {a, b, d}, {a, c, d}, {b, c, d}.
3. α closed sets in X are X, ϕ, {c}, {d}, {c, d}, {a, c, d}, {b, c, d}
4. Pre closed sets in X are X, ϕ, {c}, {d}, {c, d}, {a, c, d}, {b, c, d}
5. Semi closed sets in X are X, ϕ,{a},{b},{c},{d},{a, c},{a, d},{b, c},{b, d}{c, d},{a, c, d},{b, c, d}
6. β closed sets in X are X, ϕ, {a},{b},{c},{d},{a, c},{a, d},{b, c},{b, d}{c, d},{a, c, d},{b, c, d}
7. Regular closed sets in X are X, ϕ, {a, c, d}, {b, c, d}
8. g closed sets in X are X, ϕ, {d}, {a, d}, {b, d}, {c, d} {a, b, d}, {a, c, d}, {b, c, d}.
9. g* closed sets in X are X, ϕ, {d},{a, d},{b, d},{c, d} {a, b, d},{a, c, d},{b, c, d}.
10. gα closed sets in X are X, ϕ, {c}, {d}, {c, d}, {a, c, d}, {b, c, d}.
11. gsp closed sets in X are X, ϕ,{a},{b},{c},{d},{a, c},{a, d},{b, c},{b, d},{c, d},{a, b, d}, {a, c, d}, {b, c, d}.
12. sg closed sets in X are X, ϕ, {a},{b},{c},{d},{a, c},{a, d},{b, c},{b, d},{c, d},{a, b, d},{a, c, d}, {b, c, d}.
13. swg closed sets in X are X, ϕ, {c}, {d}, {c, d}, {a, b, d}, {a, c, d}, {b, c, d}.
14. rg closed sets in X are X, ϕ, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, d}, {a, c, d}, {b, c, d}.
15. gr closed sets in X are X, ϕ, {d}, {a, d}, {b, d}, {c, d}, {a, b, d}, {a, c, d}, {b, c, d}.
16. w closed sets in X are X, ϕ, {d}, {c, d}, {a, c, d}, {b, c, d}.
17. rw closed sets in X are X, ϕ, {d}, {a, b}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}.
18. rgw closed sets in X are X, ϕ, {c}, {d}, {a, b}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}.



A ⟶ B	/means A implies B but not conversely
A —/— B	means A and B are independent of each other

4. On βg^* open set

Definition 4.1: A subset A of a topological space X is called βg* open sets if Ac is βg* closed.

Theorem 4.2: A subset A of a topological space (X,τ) is βg* open if and only if B ⊆ gint(A) whenever B is β closed in X and B⊆A.

Proof: Necessity: Suppose B ⊆ gint(A) where B is β closed in (X,τ) and B⊆A. Let Ac ⊆M where M is β open. Hence Mc ⊆A, where Mc is β closed. Hence by assumption Mc ⊆gint(A) which implies (gint(A))c ⊆M. Therefore gcl(Ac) ⊆M. Thus Ac is βg* closed, implies A is βg* open.

Sufficiency: Let A is βg* open in X with N ⊆A, Where N is β closed. We have Ac is βg* closed with Ac⊆Nc where Nc is β open. Then we have gcl(Ac)⊆Nc implies N⊆X-gcl(Ac)=gint(X-Ac)=gint(A)

Theorem 4.3: If gint(A) ⊆B ⊆A and A is βg* open subset of (X,τ) then B is also βg* open subset of (X,τ).

Proof: Let gint(A) ⊆B ⊆A implies Ac ⊆Bc ⊆gcl(Ac). Given Ac is βg*closed. By Theorem 3.13, Bc is βg* closed. Therefore B is βg* open.

Theorem 4.4: If a subset A of a topological space (X,τ) is βg*open in X then F=X, whenever F is β open and gint(A) ⊆A^c⊆ F.

Proof: Let A be a βg*open and F be β open, gint(A)∪A^c⊆ F. This gives Fc ⊆ (X-gint(A))∩A=gcl(Ac) ∩A=gcl(Ac)-Ac. Since Fc is β closed and Ac is βg* closed. By Theorem 3.16, we have Fc=∅.Thus F=X.

Theorem 4.5: If a subset A of a topological space (X,τ) is βg* closed, then gcl(A)-A is βg* open.

Proof: Let A⊆X be a βg* closed and let F be β closed such that F⊆gcl(A)-A. By Theorem 3.16, we have F=∅.So ∅= F⊆gint(gcl(A)-A).Therefore gcl(A)-A is βg*open.

Theorem 4.6: If A and B are βg* open sets in X then A∩B is also βg* open sets in X.

Proof: Let A and B be two βg* open sets in X. Then Ac and Bc are βg* closed sets in X. By Theorem3.12, Ac∪Bc is a βg* closed in X. That is (A∩B)c is a βg* closed in X. Therefore (A∩B) is a βg* open set in X.

Theorem 4.7: If A×B is a βg* open subset of (X ×Y, τ×σ), iff A is a βg* open subset in (X,τ) and B is a βg* open subset in (Y, σ).

Proof: Let A×B is a βg* open subset of (X ×Y, τ×σ). Let H be a closed subset of (X,τ) and G be a closed subset of (Y,σ) such that H⊆A,G⊆B. Then H×G is closed in (X×Y,τ× σ) such that H×G⊆A×B. By assumption A×B is a βg* open subset of (X ×Y,τ×σ) and so H×G⊆gint(A×B) ⊆gint(A) × gint(B). That is H⊆gint (A), G⊆ gint(B) and hence A is a βg* open subset in (X,τ) and B is a βg* open subset in (Y,σ). Conversely, let M be a closed subset of (X×Y, τ×σ) such that M⊆A×B. For each (X,Y) ⊆M, cl(X) ×cl(Y) ⊆cl(M)=M⊆A × B. Then the two closed sets cl(X) and cl(Y) are contained in A and B respectively. By assumption cl(X) ⊆gint(A) and cl(Y) ⊆gint(B) hold. This implies that for each (X,Y) ⊆M, (X,Y) ⊆gint(A×B). Thus AxB is a βg* open subset of (X ×Y,τ×σ).

References

1. D. Andrijevic. semi preopen sets, Mat. Vesnik, 38(1) (1986), 24-32
2. S. S. Benchalli, R.S Wali, On RW-closed sets in topological spaces, Bull. Malaysian. Math. Sci. Soc. (2) 30(2) (2007), 99-100.
3. P. Bhattacharyya, B.K. Lahiri, Semi-generalized closed sets in topology, Indian J. Math., 29 (1987), 376-382.
4. J. Dontchev, H. Maki. On θ generalized closed sets, Topology Atlass, www.Unipissing.ca/topology/p/a/b/a/08.htm.
5. Y. Gnanambal. On generalized Preregular closed sets in topological spaces, Indian J. Pure App. Math., 28(1997), 351-360.
6. N. Levine. Generalized Closed sets in Topology, rend. Cir. Mat. palermo, 2(1970), 89-96.
7. H. Maki, R. Devi, K. Balachandran. Associated topologies of generalized α sets and α- generalized closed sets, Mem. Sci. Kochi Univ. Ser. A. Math., 15(1994), 51-63.
8. H. Maki, R. Devi. K. Balachandran, Generalized α-closed sets in topology, Bull. Fukuoka Univ. Ed. Part-III 42(1993), 13-21
9. N. Nagaveni. Studies on Generalizations of Homeomorphisms in Topological Spaces, Ph. D. Thesis, Bharathiar University, Coimbatore, 1999.
10. O. Njastad. On some classes of nearly open sets, Pacific J Math., 15(1965), 961-970.
11. N. Palaniappan, K. C. Rao. Regular generalized closed sets, kyungpook math, J., 33(1993), 211-219.
12. S. Pious Missier, E. Sucila, Onμ̂-cotinuous functions in topological spaces. Vol 2 Issue 4, April, 2013, 843-856.
13. P. Sundaram, M. Sheik John. Onω-closed sets in topology, Acta Ciencia Indica 4(2000), 211-219.
14. M.K.R.S Veerakumar, Between closed sets and g-closed sets, Mem. Fac. Sci. Kochi Univ. Ser. A, Math., 21 (2000) 1-19.
15. N.V Velicko. H-closed Topological Spaces, Tran. Amer. Math, Soc., 78(1968), 103-118.