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V_4 Cordial Labeling of Fan and Globe

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Abstract
Let $< A,*>$ be any abelian group. A graph G = (V(G), E(G)) is said to be A-cordial if there is a mapping f: V(G) →A which satisfies the following two conditions with each edgee= uv is labeled as f(u)*f(v).
(i)$\left|v_f(a) - v_f(b)\right| \leq 1$, ∀ a,b∈A
(ii)$\left|e_f(a) - e_f(b)\right| \leq 1$, ∀ a,b∈A
where$v_f(a)$= the number of vertices with label a.
$v_f(b)$= the number of vertices with label b.
$e_f(a)$= the number of edges with label a.
$e_f(b)$= the number of edges with label b.
We note that if A = <V_4,*> is a multiplicative group. Then the labeling is known as V_4 Cordial Labeling. A graph is called a V_4Cordial graph if it admits a V_4 Cordial Labeling. In this paper, we proved that $F_n = P_n + K_1$and Globe (Gl(n))areV_4 Cordial graphs.
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Keywords: Cordial labeling, V_4Cordial Labeling and V_4Cordial Graph

1. Introduction
By a graph, we mean a finite undirected graph without loops or multiple edges. For graph theoretic terminology, we referred Harary[4]. For labeling of graphs, we referred Gallian[1].
A vertex labeling of a graph G is an assignment of labels to the vertices of G that induces for each edge *uv* a label depending on the vertex labels of *u* and *v*.
A graph *G* is said to be labeled if the *n* vertices are distinguished from one another by symbols such as $v_1, v_2,...,v_n$.In a labeling of a particular type, the vertices are assigned values from a given set, the edged have a prescribed induced labeling must satisfy certain properties. The concept of graceful labeling was introduced by Rosa[3] in 1967 and subsequently by Golomb[2].

2. Preliminaries
Definition 2.1: Let G = (V,E) be a simple graph. Let f:V(G)→{0,1} and for each edge uv, assign the label $|f(u) - f(v)|$. f is called a cordial labeling if the number of vertices labeled 0 and the the number of vertices labeled 1 differ by atmost 1 and also the number of edges labeled 0 and the the number of edges labeled 1 differ by atmost 1. A graph is called Cordial if it has a cordial labeling.

Definition 2.2: Let $< A,*>$ be any abelian group. A graph G = (V(G), E(G)) is said to be A-cordial if there is a mapping f: V(G) →A which satisfies the following two conditions with each edge
e= uv is labeled as f(u)*f(v).
(i)$\left|v_f(a) - v_f(b)\right| \leq 1$, ∀ a,b∈A
(ii)$\left|e_f(a) - e_f(b)\right| \leq 1$, ∀ a,b∈A
where $v_f(a)$= the number of vertices with label a.
$v_f(b)$= the number of vertices with label b.
$e_f(a)$= the number of edges with label a.
$e_f(b)$= the number of edges with label b.
We note that if A = <V_4, *> is a multiplicative group. Then the labeling is known as

V_4 Cordial Labeling. A graph is called a V_4 Cordial graph if it admits a V_4 Cordial Labeling.

Definition 2.3
FanF_n= P_n+ $K_1$is obtained from the Path P_n by joining each vertex of P_n to a vertex u.

Definition 2.4
Globe is a graph obtained from two isolated vertex are joined by n paths of length 2. It is denoted by (Gl(n)).

3. Main Results
Theorem 3.1
$F_n = P_n + K_1$is a V_4Cordial graph.

Proof: LetV_4= { 1,−1, i,−i}.
Let V($P_n + K_1$) ={u, u_i: $1 \leq i \leq n$}.
Let E ($P_n + K_1$)= {(u u_i) : $1 \leq i \leq n$}∪ {($u_i u_{i+1}$) : $1 \leq i \leq n - 1$}.
Define f :V($P_n + K_1$)→V_4
The vertex labeling are,
Let f(u)=1,

$$f(u_i) = \begin{cases} 1 \ if \ i \equiv 0,4(mod\ 8) \\ -i \ if \ i \equiv 1,6(mod\ 8) \\ i \ if \ i \equiv 2,5(mod\ 8) \\ -1 \ if \ i \equiv 3,7(mod\ 8) \end{cases}, 1 \leq i \leq n$$

The edge labeling are,

$$f(uu_i) = \begin{cases} 1 \ if \ i \equiv 0,4(mod\ 8) \\ -i \ if \ i \equiv 1,6(mod\ 8) \\ i \ if \ i \equiv 2,5(mod\ 8) \\ -1 \ if \ i \equiv 3,7(mod\ 8) \end{cases}, 1 \leq i \leq n$$

$$f(u_i u_{i+1}) = \begin{cases} -i \ if \ i \equiv 0,2(mod\ 8) \\ 1 \ if \ i \equiv 1,5(mod\ 8) \\ -1 \ if \ i \equiv 3,7(mod\ 8) \\ i \ if \ i \equiv 4,6(mod\ 8) \end{cases}, 1 \leq i \leq n-1$$

Vertex Conditions
(i)$v_f(1)$=$\frac{n}{4}$+1 and $v_f(i) = v_f(-i)$ =$v_f(-1)$ = $\frac{n}{4}$, when n ≡ 0,4 (mod 8)
(ii)$v_f(1) = v_f(-i)$ = $\frac{n-1}{4} + 1$ and $v_f(i)$ =$v_f(-1)$=$\frac{n-1}{4}$, when n ≡ 1(mod 8)
(iii)$v_f(1) = v_f(i) = v_f(-i)$ = $\frac{n-2}{4} + 1$ and $v_f(-1)$= $\frac{n-2}{4}$,whenn ≡ 2,6(mod 8)
(iv)$v_f(1)$=$v_f(i) = v_f(-i)$ =$v_f(-1)$ = $\frac{n+1}{4}$, when n ≡ 3,7 (mod 8)
(v)$v_f(1)$= $v_f(i)$ = = $\frac{n-1}{4} + 1$, and$v_f(-i)$ =$v_f(-1)$ = $\frac{n-1}{4}$, when n ≡ 5 (mod 8)
Hence, it satisfies the condition of $|v_f(a) - v_f(b)| \leq 1$, ∀ a,b∈V_4

Edge Conditions
(i)$e_f(1) = e_f(i)$ =$e_f(-1)$ =$\frac{n}{2}$, and $e_f(-i)$=$\frac{n-2}{2}$, when n ≡ 0 (mod 8)
(ii) $e_f(1) = e_f(-1) = e_f(i)$ = $\frac{n-1}{2}$and $e_f(-i)$ =$\frac{n+1}{2}$, when n ≡ 1 (mod 8)
(iii)$e_f(1) = e_f(-i)$ =$e_f(i)$ = $\frac{n}{2}$and $e_f(-1)$= $\frac{n-2}{2}$, when n ≡ 2,6 (mod 8)
(iv)$e_f(1)$ =$e_f(i) = e_f(-1)$ = $\frac{n-1}{2}$ and $e_f(-i)$ = $\frac{n+1}{2}$, when n ≡ 3 (mod 8)
(v) $e_f(1) = e_f(-i) = e_f(-1)$ = $\frac{n-1}{2}$ and $e_f(i)$ = $\frac{n+1}{2}$, when n ≡ 5,7 (mod 8)

Hence, it satisfies the condition of $|e_f(a) - e_f(b)| \leq 1$, ∀ a,b∈V_4
Hence, $F_n = P_n + K_1$is a V_4Cordial Graph.
For example, the V_4Cordial Labeling of $P_8, P_9, P_6, P_{11}, P_{13}$ and P_7 is shown in below figure 3.21-3.27.

whenn≡ 0,4 (mod 8)

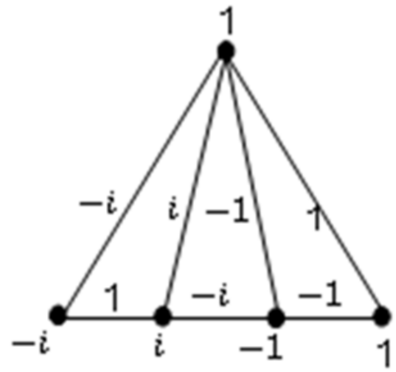

Fig3.21

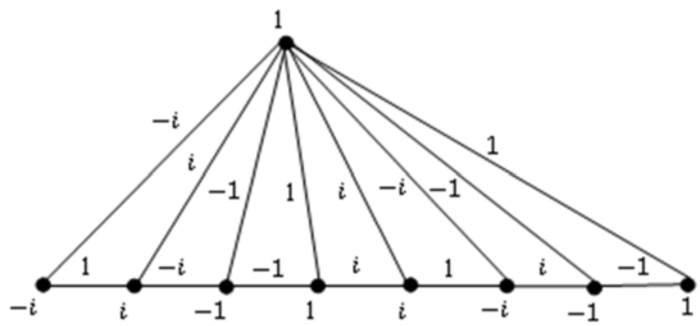

Fig 3.22

whenn≡ 1 (mod 8)

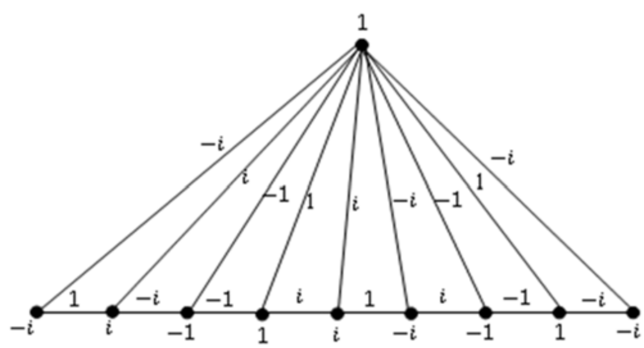

Fig 3.23

whenn≡ 2,6 (mod 8)

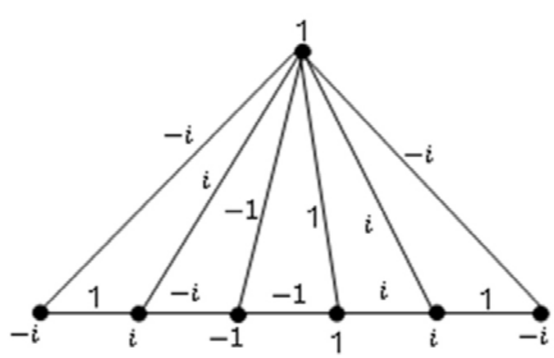

Fig3.24

whenn≡ 3,7 (mod 8)

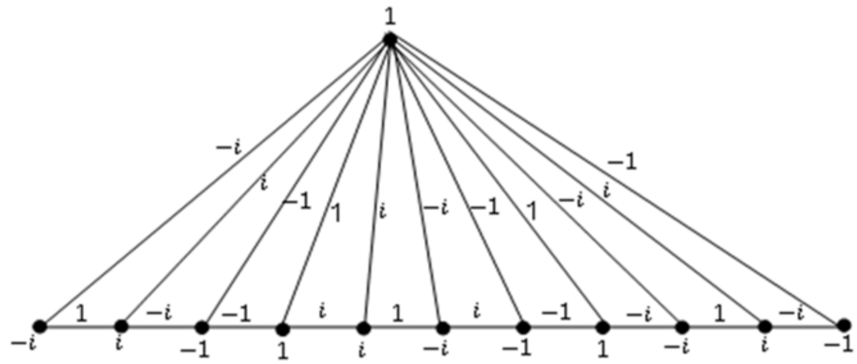

Fig 3.25

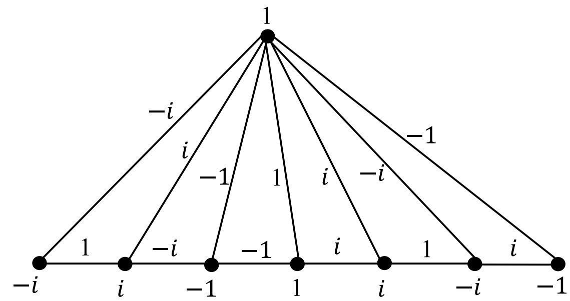

Fig 3.26

whenn≡ 5 (mod 8)

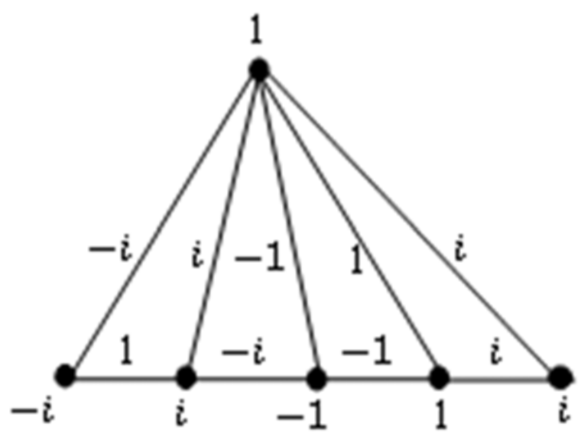

Fig3.27

Theorem 3.3
Globe (Gl(n)) is a is a V_4 Cordial graph,when n≡0,1,3(mod 4).
Proof: Let V_4= {1,−1, i,−i}.
Let V(Gl(n)) ={$u,\ v, w_i$: $1 \le i \le n$}.
Let E (Gl(n)) = {(uw_i) : $1 \le i \le n$}∪ {(vw_i) : $1 \le i \le n$}.
Define f: V(Gl(n)) →V_4.

Case(I)
When n≡0 (mod 4)
The vertex labeling are,
Let f(u) = 1, f(v)= −1

$$f(w_i) = \begin{cases} 1 \; if \; i \equiv 0 (mod\ 4) \\ -1 \; if \; i \equiv 1 (mod\ 4) \\ i \; if \; i \equiv 2 (mod\ 4) \\ -i \; if \; i \equiv 3 (mod\ 4) \end{cases}, 1 \le i \le n$$

The edge labeling are,

$$f(uw_i) = \begin{cases} 1 \; if \; i \equiv 0 (mod\ 4) \\ -1 \; if \; i \equiv 1 (mod\ 4) \\ i \; if \; i \equiv 2 (mod\ 4) \\ -i \; if \; i \equiv 3 (mod\ 4) \end{cases}, 1 \le i \le n$$

$$f(vw_i) = \begin{cases} -1 \; if \; i \equiv 0 (mod\ 4) \\ 1 \; if \; i \equiv 1 (mod\ 4) \\ -i \; if \; i \equiv 2 (mod\ 4) \\ i \; if \; i \equiv 3 (mod\ 4) \end{cases}, 1 \le i \le n$$

Vertex Conditions
(i)$v_f(1) = v_f(-1) = [\frac{n}{4}]+1$and $v_f(i) = v_f(-i) = [\frac{n}{4}]$
Hence, it satisfies the condition of $|v_f(a) - v_f(b)| \le 1$, ∀ a,b∈V_4

Edge Conditions
(i)$e_f(1) = e_f(i) = e_f(-1) = e_f(-i) = \frac{n}{2}$
Hence, it satisfies the condition of $|e_f(a) - e_f(b)| \le 1$, ∀ a,b∈V_4
Hence, (Gl(n)) is a V_4Cordial Graph.
For example, the V_4Cordial Labeling of (Gl(n)) is shown in below figure 3.41

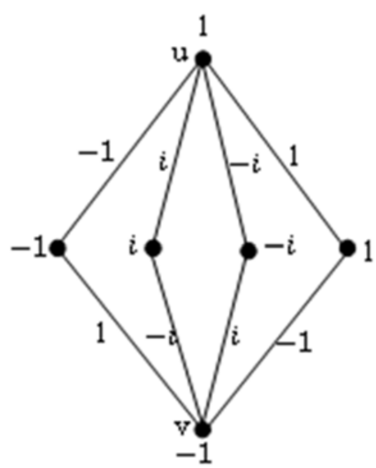

Fig3.41

Case(II)
when n≡1(mod 4)
The vertex labeling are,
Let f(u) = 1, f(v)= i

$$f(w_i) = \begin{cases} 1 \; if \; i \equiv 0 (mod\ 4) \\ -1 \; if \; i \equiv 1 (mod\ 4) \\ i \; if \; i \equiv 2 (mod\ 4) \\ -i \; if \; i \equiv 3 (mod\ 4) \end{cases}, 1 \le i \le n$$

The edge labeling are,

$$f(uw_i) = \begin{cases} 1 \; if \; i \equiv 0 (mod\ 4) \\ -1 \; if \; i \equiv 1 (mod\ 4) \\ i \; if \; i \equiv 2 (mod\ 4) \\ -i \; if \; i \equiv 3 (mod\ 4) \end{cases}, 1 \le i \le n$$

$$f(vw_i)=\begin{cases} i\ if\ i\ \equiv\ 0(mod\ 4) \\ -i\ if\ i\ \equiv\ 1(mod\ 4) \\ -1\ if\ i\ \equiv\ 2(mod\ 4) \\ 1\ if\ i\ \equiv\ 3(mod\ 4) \end{cases},1 \le i \le n$$

Vertex Conditions

(i)$v_f(1) = v_f(-1) = v_f(i) = [\frac{n}{4}]+1$ and $v_f(-i) = [\frac{n}{4}]$

Hence, it satisfies the condition of $|v_f(a) - v_f(b)| \le 1$, ∀ a,b∈V_4

Edge Conditions

(i)$e_f(1) = e_f(i) = [\frac{n}{2}]$ and $e_f(-1) = e_f(-i) = [\frac{n}{2}] + 1$

Hence, it satisfies the condition of $|e_f(a) - e_f(b)| \le 1$, ∀ a,b∈V_4

Hence, (Gl(n)) is a V_4Cordial Graph.

For example, the V_4Cordial Labeling of (Gl(n)) is shown in below figure3.42

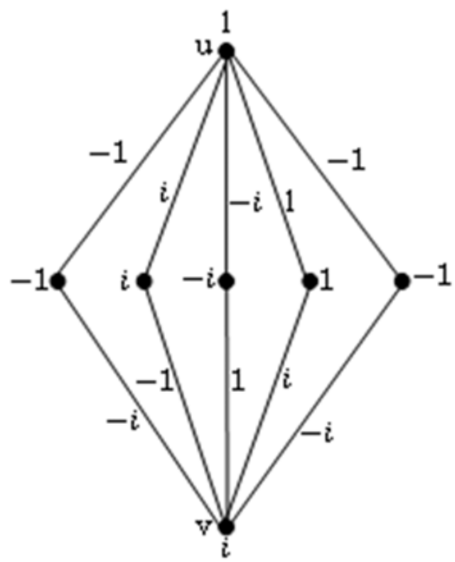

Fig3.42

Case(III)

when n≡3(mod 4)

The vertex labeling are,

Let f(u) = 1, f(v)= −i and $f(w_n)$=1

$$f(w_i)=\begin{cases} 1\ if\ i\ \equiv\ 0(mod\ 4) \\ -1\ if\ i\ \equiv\ 1(mod\ 4) \\ i\ if\ i\ \equiv\ 2(mod\ 4) \\ -i\ if\ i\ \equiv\ 3(mod\ 4) \end{cases},1 \le i \le n-1$$

The edge labeling are,

Let $f(uw_n)$=1 and $f(vw_n)=-i$

$$f(uw_i)=\begin{cases} 1\ if\ i\ \equiv\ 0(mod\ 4) \\ -1\ if\ i\ \equiv\ 1(mod\ 4) \\ i\ if\ i\ \equiv\ 2(mod\ 4) \\ -i\ if\ i\ \equiv\ 3(mod\ 4) \end{cases},1 \le i \le n-1$$

$$f(vw_i)=\begin{cases} -i\ if\ i\ \equiv\ 0(mod\ 4) \\ i\ if\ i\ \equiv\ 1(mod\ 4) \\ 1\ if\ i\ \equiv\ 2(mod\ 4) \\ -1\ if\ i\ \equiv\ 3(mod\ 4) \end{cases},1 \le i \le n-1$$

Vertex Conditions

(i)$v_f(1) = [\frac{n}{4}]+2$ and $v_f(-i)=v_f(-1) = v_f(i)= [\frac{n}{4}]+1$.

Hence, it satisfies the condition of $|v_f(a) - v_f(b)| \leq 1$, ∀ a,b∈V_4

Edge Conditions

(i)$e_f(1) = e_f(i) = [\frac{n}{2}]+1$ and $e_f(-1) = e_f(-i) = [\frac{n}{2}]$

Hence, it satisfies the condition of $|e_f(a) - e_f(b)| \leq 1$, ∀ a,b∈V_4

Hence, (Gl(n)) is a V_4Cordial Graph.

For example, the V_4Cordial Labeling of (Gl(n)) is shown in below figure 3.43

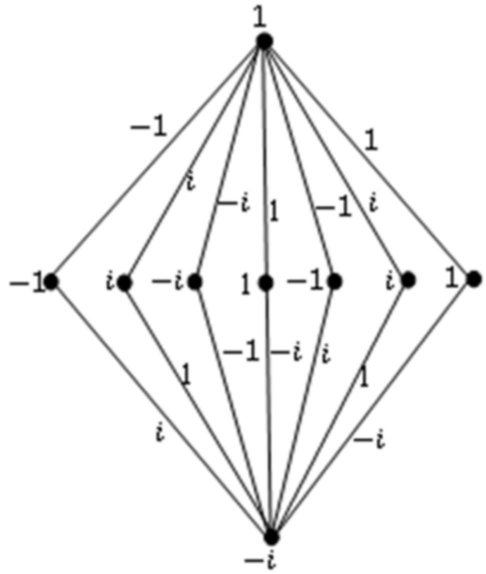

Fig 3.43

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