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On nano generalized star regular closed sets in nano topological spaces

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Abstract
The purpose of this paper is to define and study a new class of closed sets called nano generalized star regular closed sets (briefly. Ng*r) in nano topological space. Basic properties of nano g*r-closed sets are analyzed. Also investigate the Ng*r-closure and Ng*r-interior.

Keywords: Nano closed set, Nano g-closed set, Nano g*-closed set, Nano gr-closed set, Nano g*r-closed set.

Introduction
Levine [4] introduced the class of g-closed sets in 1970. Lellis Thivagar [3] introduced Nano topological space with respect to a subset X of a universe which is defined in terms of lower and upper approximations of X. He has also defined Nano closed sets Nano-interior and Nano-closure of a set. Bhuvaneswari (1, 2, & 7) introduced Nano g-closed, Nano gs-closed, Nano gr-closed and Nano rg-closed sets.

Definition 1.1 [8]: A subset A of a topological space (X,τ) is called a generalized star closed set if cl(A) ⊆U whenever A ⊆ U and U is g-open in (X, τ).

Definition 1.2 [3]: Let U be a non-empty finite set of objects called the universe and R be an equivalence relation. On U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belongings to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let X ⊆ U. Then,

(i) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and its denoted by $L_R(X)$.
That is $L_R(X)=U_{X\in U}\{R(x): R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by X.

(ii) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$.
That is $U_R(X) = U_{X\in U}\{R(x): R(x) \cap X \neq \Phi\}$

(iii) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not X with respect to R and it is denoted by $B_R(X)$.
That is $B_R(X) = U_R(X) - L_R(X)$.

Property 1.3 [3]
If (U, R) is an approximation space and X, Y ⊆ U, then

- $L_R(X) \subseteq X \subseteq U_R(X)$
- $L_R(\Phi) = U_R(\Phi) = \Phi$ and $L_R(U) = U_R(U) = U$
- $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
- $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$
- $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$
- $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
- $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever X ⊆ Y
- $U_R(X^C) = [L_R(X)]^C$ and $L_R(X^C) = [U_R(X)]^C$

- $U_R[U_R(X)] = L_R[U_R(X)] = U_R(X)$
- $L_R[L_R(X)] = U_R[L_R(X)] = L_R(X)$

Definition 1.4 [3]
Let U be the universe, R be an equivalence relation on U and $\tau_R(X)$ = {U, Φ, $U_R(X)$, $L_R(X)$, $B_R(X)$} where X ⊆ U. Then by Property 1.3, $\tau_R(X)$ satisfies the following axioms.

- U and Φ ∈ $\tau_R(X)$.
- The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.
- The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ is a topology on U called the Nano topology on U with respect to X. we call $(U, \tau_R(X))$ is called the Nano topological space. The elements of $\tau_R(X)$ are called as Nano open sets in U. Elements of $[\tau_R(X)]^C$ are called Nano closed sets.

Remark 1.5 [3]
If $\tau_R(X)$ is the Nano topology on U with respect to X. Then the set
B = {U, $L_R(X)$, $B_R(X)$} is the basis for $\tau_R(X)$.

Definition 1.6 [3]
If $(U, \tau_R(X))$ is a nano topological space with respect to X where X ⊆ U and if A ⊆ U, then

- The Nano interior of the set A is defined as the union of all nano open subsets contained in A and is denoted by Nint (A). That is Nint (A) is the largest nano open subset of A.
- The Nano closure of the set A is defined as the intersection of all nano closed sets containing A and is denoted by Ncl (A). That is Ncl (A) is the smallest nano closed set containing A.

Definition 1.7
Let $(U, \tau_R(X))$ be a nano topological space and A ⊆ U. Then A is said to be

(1) Ng-closed [1] if Ncl (A) ⊆ V whenever A ⊆ V and V is Nano-open in U.
(2) Ngr-closed [7] if Nrcl (A) ⊆ V whenever A ⊆ V and V is Nano-open in U.
(3) Ngs-closed [2] if Nscl (A) ⊆ V whenever A ⊆ V and V is Nano-open in U.
(4) Ng*-closed [5] if Ncl (A) ⊆ V whenever A ⊆ V and V is Nano g-open in U.
(5) Ng*s-closed [6], if Nscl(A) ⊆ V whenever A ⊆ V and V is Nano g-open in U

2. Nano Generalized Star Regular Closed Set

Throughout this paper (U, $\tau_R(X)$) is a nano topological space with respect to X where X ⊆ U, R is an equivalence relation on U, U/R denotes the family of equivalence classes of U by R.
In this section, we define and study the forms of nano generalized star regular closed sets in nano topological spaces.

Definition 2.1 A subset A of a nano topological space (U, $\tau_R(X)$ is called Nano generalized star regular closed sets (briefly. Ng*r-closed) if Nrcl (A) ⊆ V, whenever A⊆V and V is nano g-open.

Example 2.2 Let U= {a, b, c, d} with U/R= {{a}, {c}, {b, d}} and X= {a, b}. Then $\tau_R(X)$ = {U, Φ, {a}, {b, d}, {a, b, d}} which are open sets.

- The nano closed sets= {U, Φ, {c}, {a,c},{b,c,d}}
- The nano regular closed sets ={U, Φ, {b}, {c}, {d}, {a,c},{b, c}, {c, d},{a, b, c},{a, c, d},{b, c, d}}
- The nano generalized closed sets ={U, Φ, {c}, {a,c},{b,c}, {c,d},{a,b,c} {a,c,d},{b,c,d}}
- The nano generalized regular closed sets ={U, Φ, {c}, {b,c}, {c,d}, {a,c}, {a,b,c}, {b,c,d},{a,c,d}}
- The nano generalized star closed sets={U, Φ,{c},{a,c},{b,c},{c,d},{a,b,c},{a,c,d},{b, c, d}}
- The nano generalized star regular closed sets={Φ, U, {b}, {c}, {d}, {a,c}, {b,c}, {c,d}, {a,b,c}, {a,c,d}, {b,c,d}}

Remark 2.3 Every nano closed set is nano regular closed

Theorem 2.4 If A is nano regular closed set in (U, $\tau_R(X)$), then it is nano g*r-closed set but not conversely.
Proof. Let A be a nano regular closed set in U such that A ⊆V and V is nano open in U, since every nano open set is nano g-open set. So V is nano g-open set in U. That is Nrcl (Nint (A)) =A. Since A is nano open, Nint (A) =A. Every nano open sets is nano regular open. Therefore Nrcl (A) = A⊆V.This implies Nrcl (A) ⊆V, where V is nano g-open set in U. Therefore A is nano g*r-closed set.

Example 2.5 Let U= {a, b, c}, with U/R = {{a}, {b, c}} and X= {a, b}.Then the nano topology $\tau_R(X)$ = {U, Φ, {a},{b, c}}. Here {a, c} is nano g*r-closed set but it is not nano regular closed set.

Theorem 2.6 If A is nano g-closed set in (U, $\tau_R(X)$), then it is nano g*r-closed set but not conversely.
Proof. Let A be a nano g-closed set. Then Ncl (A)⊆V whenever A⊆V and V is nano open in U. Since every nano open set is nano g-open set. So V is nano g-open set in U. But Nrcl (A) ⊆Ncl(A) ⊆V. This implies Nrcl (A) ⊆V, A ⊆V and V is nano g-open in U. Hence A is nano g*r-closed set.

Example 2.7 In example 2.2, the set {b} is nano g*r-closed set but not nano g-closed set.

Theorem 2.8 If A is nano g*-closed set in (U, $\tau_R(X)$), then it is nano g*r-closed set but not conversely.
Proof. Let A be a nano g*-closed set of U and A⊆V, V is nano g-open in U.Since A is nano g*-closed, implies Ncl (A) ⊆V. But Nrcl (A)⊆Ncl(A) implies Nrcl(A) ⊆V, A⊆V,V is nano g-open in U.Therefore A is nano g*r-closed set.

Example 2.9 In example2.2, the set {d} is nano g*r-closed set but not nano g*-closed set.

Theorem 2.10 If A is nano gr-closed set in (U, $\tau_R(X)$), then it is nano g*r-closed set but not conversely.
Proof. Let A be a nano gr-closed set of U and A⊆V, V is nano open in U. But every nano open sets is nano g-open set. This implies V is nano g-open in U. So Nrcl (A) ⊆V, A ⊆V and V is nano g-open in U. Therefore A is Nano g*r-closed set.

Example 2.11 In example 2.2, the sets {b}, {d} are nano g*r-closed sets but not nano gr-closed set.

Theorem 2.12 The Union of two nano g*r-closed sets in (U, $\tau_R(X)$) is also a nano g*r-closed set in (U, $\tau_R(X)$).
Proof. Let A and B be two nano g*r-closed sets in (U, $\tau_R(X)$). Let V be a nano g-open set in U such that A⊆V and B⊆V. Then we have A∪B⊆V. Since A and B are nano g*r-closed in (U, $\tau_R(X)$). This Implies Nrcl(A) ⊆V and Nrcl(B) ⊆V. Now Nrcl (A∪B) = Nrcl (A) ∪ Nrcl (B) ⊆V. Thus we have Nrcl (A∪B) ⊆V, whenever A∪B⊆V, V is nano g-open set in (U, $\tau_R(X)$). This implies that A∪B is nano g*r-closed set in (U, $\tau_R(X)$).

Remark 2.13. The intersection of two nano g*r-closed sets in (U, $\tau_R(X)$) is also a nano g*r-closed set in (U, $\tau_R(X)$) as seen from the following example.

Example 2.14 Let U= {a, b, c, d} with U/R= {{a}, {d}, {b, c}} and X= {a, c}. Then the nano topology $\tau_R(X)$ = { Φ, U, {a}, {b, c}, {a, b,c}}. Then the nano g*r-closed sets are {b},{c},{d},{a, d},{b, d},{c, d},{a, b, d},{a, c, d},{b, c, d}. Here {a, c, d}∩{b, c, d} ={c, d} is also a nano g*r-closed set.

Theorem 2.15. Let A be a nano g*r-closed subset of (U, $\tau_R(X)$). If A⊆B⊆Nrcl(A), then B is also a nano g*r-closed subset of (U, $\tau_R(X)$).
Proof. Let V be a nano g-open set of a nano g*r-closed subset of $\tau_R(X)$, such that B⊆V as A⊆B, we have A⊆V, as A is nano g*r-closed set, Nrcl(A) ⊆V. Given B⊆Nrcl(A), we have Nrcl (B) ⊆Nrcl(A) ⊆V, this implies Nrcl(B) ⊆ V, this implies Nrcl(B) ⊆ B. Where B ⊆V and V is nano g-open. Hence B is also a nano g*r-closed subset of $\tau_R(X)$.

Theorem 2.16. If a set A is nano g*r-closed if and only if Nrcl (A)-A contains no non-empty, nano generalized closed set.
Proof. Necessity: Let F be a nano generalized closed set in (U, $\tau_R(X)$), such that F⊆Nrcl (A)-A. Then A⊆X-F. Since A is nano g*r-closed set and X-F is nano generalized open then Nrcl (A)⊆X-F. That is F⊆X-Nrcl(A). So F⊆[X-Nrcl (A)]∩[Nrcl(A)-A].Therefore F=Φ.
Sufficiency: Let us assume that Nrcl (A)-A contains no non-empty nano generalized closed set. Let A⊆V, V is nano generalized open. Suppose that Nrcl(A) is not contained in V, Nrcl(A)∩V^c is non-empty, nano generalized closed set of Nrcl(A)-A, which is contradiction, therefore Nrcl(A)-A. Hence A is nano g*r-closed set.

3. Nano Generalized Star Regular Open Sets

Definition 3.1. A subset A of a nano topological space (U, $\tau_R(X)$) is called nano generalized star regular open (briefly, nano g*r-open), if A^c is nano g*r-closed.

Theorem 3.2
i) If A is nano regular open set in (U, $\tau_R(X)$), then it is nano g*r-open.
ii) If A is nano g*open set in (U, $\tau_R(X)$), then it is nano g*r-open.
iii) If A is nano gr-open set in (U, $\tau_R(X)$), then it is nano g*r-open.
Proof. Proof follows from the theorem 2.4, 2.8 and 2.10.

Remark 3.3. For a subset A of a nano topological space (U, $\tau_R(X)$),
i) U-Ng*r int (A) =Ng*rcl (U-A)
ii) U-Ng*rcl (A) =Ng*r int (U-A)

Theorem 3.4. A subset A⊆U is nano g*r-open set if and only if F⊆Nrint (A), whenever F is nano g-closed set and F⊆A.
Proof. Let A be nano g*r-open set and suppose F⊆A, where F is nano g-closed set. Then U-A is nano g*r-closed set contained in the nano g-open set U-F. Hence Nrcl (U-A) ⊆U-F and U-Nrint (A) ⊆U-F. Thus F⊆Nrint (A). Conversely, if F is nano g-closed set with F⊆Nrint (A) and F⊆A. Thus U-Nrint (A) ⊆U-F.Thus Nrcl(U-A) ⊆U-F. Hence U-A is nano g*r-closed set and A is nano g*r-open set.

Theorem 3.5. If Nrint (A) ⊆B⊆A and if A is nano g*r-open, then B is nano g*r-open.
Proof. Let Nrint (A) ⊆B⊆A, then A^c⊆B^c⊆Nrcl(A^c),where A^c is nano g*r-closed and hence B^c is also nano g*r-closed by theorem 2.15. Therefore, B is nano g*r-open.

Remark 3.6. If A and B are nano g*r-open subsets of a nano topological space U, then A∪B is also nano g*p-open in U, as seen from the following example.

Example 3.7.Let U={a, b, c} with U/R={{a},{b, c}} and X={a, b}, then the nano topology $\tau_R(X)$ = {U, Φ,{a},{b,c}}. The sets {a},{b},{c},{a,b},{a,c},{b,c} are nano g*r-open sets.Then {a}∪{b}={a,b} is also nano g*r-open sets.

Theorem 3.8. If A is nano g*r-closed. Then Nrcl (A)-A is nano g*r-open.
Proof. Let A be nano g*r-closed. Let F be a nano g-closed set, such that F⊆Nrcl (A)-A. Then F= Φ.Since Nrcl (A)-A cannot have any non-empty nano g-closed set. Therefore F ⊆Nrint (Nrcl (A)-A). Hence Nrcl (A)-A is nano g*r-open.

4. Nano Generalized Star Regular-Interior and Nano Generalized Star Regular-Closure

Definition 4.1. Let U be a nano topological space and let x∈U. A subset N of U is said to be Ng*r-neighborhood of x if there exists a Ng*r-open set G such that x∈ G ⊂N.

Definition 4.2.
i) Ng*r-int (A) =U {B: B is nano g*r-open set and B⊂ A}
ii) Ng*r-cl (A) = ∩{B: B is nano g*r closed set and A⊂ B}

Theorem 4.3. If A be a subset of U. Then Ng*r-int (A) = U {B: B is nano g*r-open set and B⊂ A}

Proof. Let A be a subset of U. x∈ Ng*r-int (A)
⇔x is a Ng*r-interior point of A.
⇔A is a Ng*r-neighborhood of point x.
⇔ There exists Ng*r-open set B such that x∈B⊂A
⇔x∈U {B: B is Ng*r-open set and B⊂A}
Hence, Ng*r-int (A) =U {B: B is nano g*r-open set and B⊂A}

Theorem 4.4. Let A and B be subsets of U. Then
i) Ng*r-int (U) =U and Ng*r-int (Φ)= Φ.
ii) Ng*r-int (A) ⊂A
iii) If B is any Ng*r- open sets contained in A, then B⊂Ng*r-int (A)
iv) If A⊂B, then Ng*r-int (A) ⊂Ng*r-int (B)
V) Ng*r-int (Ng*r-int (A)) =Ng*r-int (A).

Proof.
i) Since, U and Φ are ng*r-open sets, by theorem 4.3
Ng*r-int (U) =U {B: B is Ng*r-open and G⊂U}

=U U {A: A is a Ng*r-open set}
=> Ng*r-int (U) =U
Since, Φ is the only Ng*r-open set contained in Φ, Ng*r int (Φ) =Φ
ii) Let x∈Ng*r-int (A) =>x is a Ng*r-interior point of A.
=>A is a Ng*r neighborhood of x.
=>x∈A
Thus, X∈Ng*r-int (A)⊂ A.
(iii) Let B be any Ng*r-open sets such that B⊂ A. Let X∈B, then since, B is a Ng*r- open set contained in A, x is a Ng*r-interior point of A. That is B is a Ng*r-int (A). Hence B⊂Ng*r-int (A).
(iv) Let A and B be subsets of U such that A⊂ B. Let x∈Ng*r-int (A).Then x is a Ng*r-interior point of A and so A is Ng*r neighbourhood of x. This implies that x∈Ng*r-int (B). Thus we have shown that X∈Ng*r-int (B). Hence, Ngr*-int (A)⊂ Ng*r-int(B).
(v) Let A be any subset of U. By definition of Ng*r-interior, Ng*r-int(A)=∩{A⊂F∈Ng*rc(U)} if A⊂ F ∈Ng*rc(U),then Ng*r-int (A) ⊂F.Since F is a Ng*r closed set containing Ng*r-int(A). By (iii), Ng*r-int(Ng*r-int(A)) ⊂F. Hence Ng*r-int (Ng*r-int(A)) ⊂∩{A⊂F∈Ng*r$c(U)$} =Ng*rcl(A). That is, Ng*r-int(Ng*r-int(A))=Ng*r-int(A).

Theorem 4.5. If a subset A of a space U is Ng*r-open then Ng*r-int (A) =A

Proof. Let A be a Ng*r-open subset of U. We know that Ng*r-int (A) ⊂A. Also A is Ng*r-open set contained in A. From theorem 4.4(iii), A⊂Ng*r-int (A). Hence, Ng*r-int (A) =A.

Theorem 4.6. If A and B are subsets of U, then Ng*r-int (A) U Ng*r-int(B) ⊂Ng*r-int(AUB).

Proof. We know that A⊂AUB and B⊂AUB we have by Theorem 4.4(iv), Ng*r int (A)⊂Ng*r-int (AUB) and Ng*r-int(B) ⊂Ng*r-int(AUB). This implies that Ng*r-int (A) ∪Ng*r-int (B) ⊂Ng*r-int (AUB).

Theorem 4.7. If A and B are subsets of space U, then Ng*r-int A∩B = (Ng*r-int (A) ∩Ng*r-int (B)

Proof. We know that A∩ B ⊂A and A∩B⊂B.We have,by theorem4.4(iv),Ng*r-int(A∩ B) ⊂ Ng*r-int(A) and Ng*r-int(A∩B) ⊂Ng*r-int(B).This implies that Ng*r-int(A∩B) ⊂Ng*r-int (A) ∩Ng*r-int(B) →(1).
Again, Let x∈Ng*r-int (A) ∩Ng*r-int (B).Then x∈Ng*r-int (A) and x∈Ng*r-int (B). Hence, x is a Ng*r-interior point of each sets A and B. It follows that Aand B is Ng*r-neighborhood of x, So that their intersection A ∩B is also Ng*r-neighbourhood of x. Hence x∈Ng*r- int (A∩B).Therefore Ng*r-int (A) ∩Ng*r-int (B) ⊂Ng*r-int (A ∩ B)→(2). From (1) & (2) we get Ng*r-int (A ∩ B) = Ng*r-int (A) ∩Ng*r-int (B).

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