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A Non Newtonian mathematical model on Two-Phase hepatic mean blood flow through arterioles with special reference of hepatitis C

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Abstract
In this paper we observe the two phase mean hepatic blood flow in arterioles keeping in the nature of hepatic circulatory system in human body. We know that a human being have one of red blood cells and other are plasma. P.N. Pandey and V. Upadhyay have considered that blood flow in two phase. They have also applied the Herschel Bulkley Non- Newtonian Model in bio fluid observation. We have collected a clinical data related to hepatitis C for hematocrit versus blood pressure clinical observation and graphical presentation both value is very close. All mathematical presentation is tensorial form using analytical as well as numerical technique. In this hepatic dieses of hepatitis C the role of hematocrit is explicit in the determination of blood pressure.

Keywords: Hepatitis C, Structure of liver, Circulatory system, Herschel Bulkley Non-Newtonian Model, Hematocrit, Blood flow.

1. Introduction (Description of Bio-Physical Problem)

The liver is the largest organ in the body [1]. It is located below the diaphragm in the right upper quadrant of the abdominal cavity. An adult's liver weighs approximately 3 pounds cage and it is separated into a right and left lobe. The right lobe is much larger than the left lobe [2]. It is big organ with many big jobs. The working cells of the liver are known as hepatocytes [3]. Its important work is digestive process because it produces bile. Bile is yellowish-green fluid that aids in the emulsification of fats. Hepatocytes have a unique capacity to reproduce in response to liver injury. [4] The liver is a metabolically active organ responsible for many vital life functions [5]. The primary functions of the liver are bile production and excretion, cholesterol, hormones, and drugs, metabolism of fats, proteins, and carbohydrates [6].

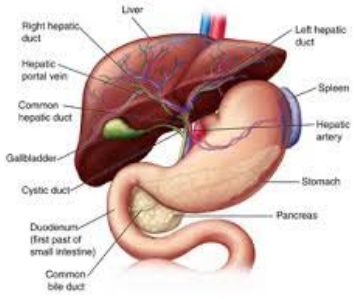

Fig 1: Structure of Liver

There are some common liver diseases include hepatitis infection, fatty liver disease, and cancer, which damage from alcohol [7]. Hepatitis liver dieses are like hepatitis A, B, C. Hepatitis C is a disease caused by a virus that infects the liver. Many people do not know that have hepatitis C until they already have some liver damage [8]. This can take many years. Some people who get hepatitis C have it for short time and then get better. This is called acute hepatitis C. But most people who are infected with the virus go on to develop long-term, or chronic, hepatitis C. [9] Hepatitis C is a blood-borne virus.

Today, most people become infected with the Hepatitis C virus by sharing needles or other equipment to inject Chronic Hepatitis C is a serious disease that results can long-term health problems, even death.

The majority of infected persons might not be aware of their infection because they are not clinically ill. There is no vaccine for Hepatitis C. The best way to prevent Hepatitis C is by avoiding behaviors that can spread the disease, especially injecting drugs [11]. An arterioles is a small diameter blood vessel in the microcirculation that extends and branches out from an artery and leads to capillaries [12]. Arterioles are very small arterioles that deliver blood to capillaries. As arterioles branch off an artery, they have smooth muscle and a few elastic fibers in the tunica media [13]. These gradually taper away as the arterioles becomes smaller, leaving mostly the endothelium and few smooth muscle fibers by the time the arterioles connects to the capillaries arterioles play a key role in regulating blood flow into capillaries. Vasoconstriction of arterioles decreases blood flow into capillaries; vasodilatation increases flow.

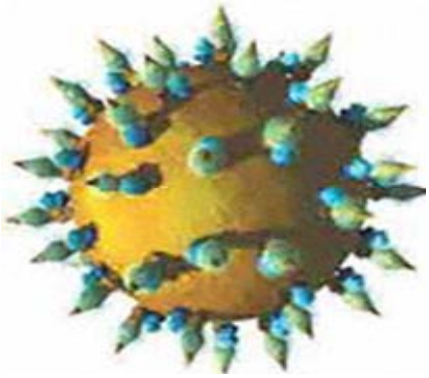

Fig 2: Model of Human Hepatitis C Virus

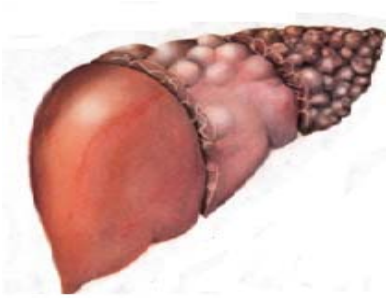

Fig 3: Hepatitis C virus Damage Liver

A change in the diameter of a large number of arterioles at once will also after blood pressure. This work is important for human health. There are several researches, who examined the blood flow in the artery and veins. This work will focus on two phase hepatic blood flow in arterioles with special reference to hepatitis C. The mean arterioles pressure (MAP) is a term used in medicine to describe an average blood pressure in an individual [14]. At normal resting heart rates MAP can be approximated using the more easily measured systolic and diastolic pressure SP and DP [15].

$$MAP \simeq DP + \frac{1}{3}(SP\text{-}DP)$$

$$MAP \simeq \frac{2}{3}(DP) + \frac{1}{3}(SP)$$

At high heart rates is more closely approximated by the arithmetic mean of systolic and diastolic pressures because of the change in shape of the arterioles pressure pulse.

A lot of work is available, but P.N. Pandey and V. Upadhyay (2001) discussed some phenomena in two phase blood flow gave an idea on the two phase hepatic blood flow in arterioles with a liver disease hepatitis C. The work of P.N. Pandey and V. Upadhyay in whole circulatory system and hepatic circulatory system is a sub system of whole circulatory system. This work, applied the Herschel Bulkley Non-Newtonian model. We present an improvement on the previous work in the field and this is discussed separately below. The ultimate use of this model to predict normal reference levels of two phase blood flow in arterioles for individuals patients undergoing Hepatitis C disease.

2. Real Model

We have to select a frame of reference for mathematical modeling of the state of moving blood keeping in view the difficulty and generality of the problem of blood flow, we select generalized three dimensional orthogonal curvilinear co-ordinate system, briefly prescribed as E^3 called as 3-dim Euclidean space, we interpret the quantities related to blood flow in tensorial form which is more comparatively more realistic. The bio physical laws thus expressed fully hold good in any co-ordinate system, which is compulsion for the truth fullness of the law (1990) [16] Now let the coordinate axis be OX^i where O is origin and superscript i = 1, 2, 3 let X^i be the coordinate of any point P in space. The mathematical description of the state if a moving blood id affected by means of functions which gives the distribution of blood velocity $v^k=v^k(X^i,t)$, k = 1,2,3 and of any two thermodynamic quantities pertaining to the blood for instance the pressure p = $p(X^i,t)$ and the density $\rho = \rho(X^i,t)$,as is well known, all the thermodynamic quantities are determined by the value of any two of them, together with equate of state, Hence, if we are given five quantities, namely three components of velocity v^k, the pressure p and densityρ, the state of moving blood is completely determined. All these quantities are, in general, function of the co-ordinate X^i, 1,2,3 and of the time t, We emphasize that $V^k(X^i,t)$ is the velocity of the blood at given point X^i in space and at a given t, i.e. it refers to a point in space and not to fixed particle of blood; in the course of time, the letter move about in space, the same remarks apply to p and ρblood is mixed fluid, Mainly there are two phase in blood, the first phase is plasma, while the other phase is that of blood cells. The blood cells are enclosed with a semi permeable membrane whose density is greater than, that of plasma, Thus blood can be considered as a homogeneous mixture of two-phases [17].

(A) Equation of Continuity for Two Phase Blood Flow

The blood flow is affected by the presence of blood cells. This effect is directly proportional to the volume occupied by blood cells. Let the volume portion covered by blood cells in unit volume be X , this X is replaced by $H/100$, where H is the hematocrit the volume percentage of blood cells [18]. Then the volume portion covered by the plasma will be $(1-X)$. The mass ratio of blood cells to plasma is given below:

$$r = \frac{X\rho_c}{(1-X)\rho_p} \tag{2.1}$$

Where ρ_c and ρ_p are densities of blood cells and blood plasma respectively. Usually this mass ratio is not constant, even then it may be supposed to constant in present context [Singh and Upadhayay (1986)] [19].

The both phase of blood, i.e. blood cells and plasma move with the common velocity. Campbell and Pitcher have presented a model for two phase of blood separately (1958). The equation of continuity for two phases according to the principle of conservation of mass as follow [Kapur and Gupta (1963)] [20].

$$\frac{\partial(X\rho_c)}{\partial t} + (X\rho_c v^i), i = 0 \tag{2.2}$$

and $$\frac{\partial(1-X)\rho_p}{\partial t} + ((1-X)\rho_p v^i), i = 0 \tag{2.3}$$

Where v is the common velocity of two phase blood cells and plasma. If we define the uniform density of the blood ρ_m as follow

$$\frac{1+r}{\rho_m} = \frac{r}{\rho_c} + \frac{1}{\rho_p} \tag{2.4}$$ [21]

From equation (2.3) and (2.4) we can written as,

$$\frac{\partial\rho_m}{\partial t} + (\rho_m v^i), i = 0 \tag{2.5}$$

(B) Equation of Motion for Two Phase Blood Flow

According to [Ruch and Patten (1973)] the hydro dynamical pressure p between the two phases of blood can be supposed to be uniform because the both phases i.e. blood cells and plasma are always in equilibrium state in blood (1973)[21]. Taking viscosity coefficient of blood cells to be η_c and applying the principle of conservation of momentum, we get the equation of motion for the phase of blood cells as follows:

$$X\rho_c \frac{\partial v^j}{\partial t} + (X\rho_c v^j) v^j_{,j} = -Xp_{,j}g^{ij} + X\eta_c (g^{jk} v^j_{,k})_{,j} \tag{2.6}$$

The equation of motion for plasma will be as follows:

$$(1-X)\rho_p \frac{\partial v^i}{\partial t} + \{(1-X)\rho_p v^i\} v^i_{,j} = -(1-X)\, p_{,j} g^{ij} + (1-X)\eta_c (g^{jk} v^i_{,k})_{,j} \tag{2.7}$$

Now adding an equation (2.6), (2.7) and using the relation (2.4), the equation of motion for blood flow with the both phases will be as follows:

$$\rho_m \frac{\partial v^i}{\partial t} + (\rho_m v^j) v^i_{,j} = -p_{,j} + \eta_m (g^{jk} v^i_{,k})_{,j}. \tag{2.8}$$

where $\eta_m = X\eta_c + (1-X)\eta_p$ is the viscosity coefficient of blood as a mixture of two phases.

3. Mathematical Modeling

As the velocity of blood flow decreases, the viscosity of blood increases. The velocity of blood decreases successively. The Herschel Bulkley law hold good on the two phase blood flow through veins arterioles, veinules and whose constitutive equation is as follows:

$T' = \eta_m e^n + T_p (T' \geq T_p)$ And $e = 0 (T' < T_p)$ where, T_p is the yield stress.

When strain $e = 0 (T' < T_p)$ a core region is formed which is flows just like a plug. Let the radius of the plug be r_p .The stress acting on the surface of plug will be T_p . Equating the forces acting on the plug, we get,

$$P\pi r_p^2 = T_p 2\pi r_p \Rightarrow r_p = 2\frac{T_p}{P} \tag{2.9}$$

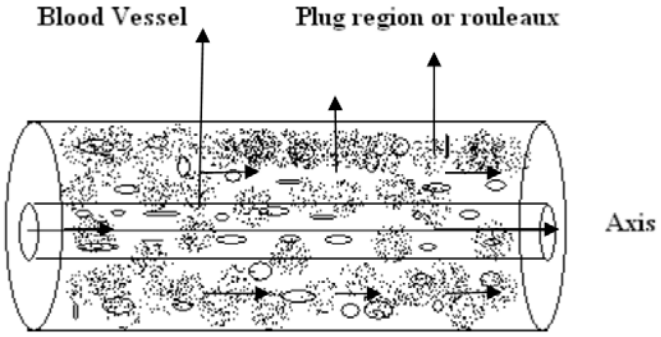

Fig 4

Gometrical Structure of Herschel Bulkley Blood flow Model

The Constitutive equation for test part of the blood vessel is given by where, T_e = effective stress, whose

$T^{'} = \eta_m e^n + T_p \; or \; T^{'} - T_p = \eta_m e^n = T_e$ generalized Form are given below $T^{ij} = -Pg^{ij} + T_e^{ij}$ w $T_e^{ij} = \eta_m \left(e^{ij}\right)^n$

while $e^{ij} = g^{jk} v_k^i$

Now we describe the basic equations for Herschel Bulkley blood flow as follows:

(A) Equation of Continuity

$$\frac{1}{\sqrt{g\sqrt{\left(gV^i\right)}_{,i}}} = 0 \tag{3.0}$$

(B) Equation of Motion Is

$$\rho_m \frac{\partial v^i}{\partial t} + \rho_m V^j V^i_{,j} = -T^{ij}_{e,j} \tag{3.1}$$

where all the symbols have their usual meanings.

4. Analysis (Solution)

The blood vessels are cylindrical; the above governing equations have to be transformed into cylindrical co-ordinates. As we know, $X^1 = r, X^2 = \theta, X^3 = z,$ Matrix of metric tensor in cylindrical co-ordinates is $\left[g_{ij}\right]$ and matrix of a conjugate metric tensor is $\left[g^{ij}\right]$ whereas the Christoffel's symbols of 2nd kind are as follows:

$\begin{Bmatrix} 1 \\ 2 \; 2 \end{Bmatrix} = -r, \begin{Bmatrix} 1 \\ 2 \; 2 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 2 \; 2 \end{Bmatrix} = \frac{1}{r}$ Remaining others is zero.

The governing tensorial equations can be transformed into cylindrical forms which are as follows:

The equation of continuity are given below $\frac{\partial v}{\partial z} = 0$

The equation of motion- r-component $-\frac{\partial p}{\partial z} = 0, \Theta$ -component = 0 and

Z-component $0 = -\frac{\partial p}{\partial z} + \frac{\eta_m}{r}\left[r\left(\frac{\partial v_z}{\partial r}\right)^n\right]$ (3.2)

Here, this fact has been taken in view that the blood flow in axially symmetric in arteries concerned, i.e. $v_\theta = 0$ and v_r, v_z and p do not depend upon θ.

We get $v_z = v(r)$ and $p = p(z)$ and $0 = -\frac{dp}{dz} + \frac{\eta_m}{r}\left[r\left(\frac{dv}{dz}\right)^n\right]$ (3.3)

And the pressure gradient $-\frac{dp}{dz} = p$

$r\left(\frac{dv}{dz}\right)^n = -\frac{pr^2}{2\eta_m} + A,$ We apply boundary condition: at $r = 0. V = V_0$ then $A = 0$

$$-\frac{dv}{dr}=-\left(\frac{p_r}{2\eta_m}\right)^{\frac{1}{n}}$$ Replace r from $r-r_p$

Or ,

$$-\frac{dv}{dr}=\left(\frac{\frac{1}{2}pr-\frac{1}{2}pr_p}{\eta_m}\right)^{\frac{1}{n}}\Rightarrow\frac{dv}{dr}=-\left(\frac{p}{2\eta_m}\right)^{\frac{1}{n}}\left(r-r_p\right)^{\frac{1}{n}} \quad (3.4)$$

Integrating above equation (13) under the no slip boundary condition: v = 0 at r = R we get:

$$V=\left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}}\frac{n}{n+1}\left[\left(R-r_p\right)^{\frac{1}{n}+1}-\left(r-r_p\right)^{\frac{1}{n}+1}\right] \quad (3.5)$$

This is the formula for velocity of blood flow in arterioles, veinules and veins. Putting $r=r_p$ to get the velocity v_p of plug flow are given by,

$$V_p=\frac{n}{n+1}\left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}}\left(R-r_p\right)^{\frac{1}{n}+1} \quad (3.6)$$

Where the value of r_p is taken from equation (2.7), we have

5. Result & Discussion (Bio-Physical Interpretation)

Observations: Hematocrit Vs blood pressure is taken from **Jeevan Jyoti hospital** Allahabad

Patient Name: Mr. Anjani Singh Age-27Years

Diagnosis: Dr. R.K. Sharma

S.N	Date	Hemoglobin(gm)	B.P. Systolic/Distolic	Hematocrite	B.P(Systolic)	B.P(Distolic)
1	24.02.13	3.5	90/70	10.5	11999	9332.57
2	26.02.13	4	100/80	12	13332.24	10665.79
3	27.02.13	4.5	110/85	13.5	14665.46	11332.40
4	25.02.13	5.6	100/74	16.8	13332.24	9865.86
5	28.02.13	6.2	110/90	1.6	14665.46	11999

According to Berkow, Robert, The hematocrit (expressed as percentage points) is normally about three times the hemoglobin concentration (report as per deciliter) [22].

The flux of two phased blood flow in arterioles, veinules and veins is given below:

$$Q=\int_0^{r_p}2\pi rV_p dr+\int_{r_p}^{R}2\pi rVdr$$

$$=\int_0^{r_p}2\pi r\frac{n}{n+1}\left(\frac{p}{2\eta_m}\right)^{\frac{1}{n}}\left(R-r_p\right)^{\frac{1}{n}+1}dr+$$

$$\int_0^{r_p}2\pi r\frac{n}{n+1}\left(\frac{p}{2\eta_m}\right)^{\frac{1}{n}}\left[\left(R-r_p\right)^{\frac{1}{n}+1}-\left(r-r_p\right)^{\frac{1}{n}+1}\right]dr$$ Using (3.4) and (3.6), we get

$$Q=\frac{2\pi n}{(n+1)}\left(\frac{p}{2\eta_m}\right)^{\frac{1}{n}}\left(R-r_p\right)^{\frac{1}{n}+1}\left[\frac{r^2}{2}\right]_0^{r_p}+\frac{2\pi n}{(n+1)}\left(\frac{p}{2\eta_m}\right)^{\frac{1}{n}}\left[\frac{r^2}{2}\left(R-r_p\right)^{\frac{1}{n}+1}-\frac{r\left(r-r_p\right)^{\frac{1}{n}}}{\frac{1}{n}+2}+\frac{\left(r-r_p\right)^{\frac{1}{n}+3}}{\left(\frac{1}{n}+2\right)\left(\frac{1}{n}+3\right)}\right]_{r_p}^{R} \quad (3.7)$$

Q=1000m.l/min = 0.01666llit/sec, R=1,r_p= 1/3 [23]

According to Gustofson,Daniel R(1980).η_p = 0.0015(Pascal-Sec)[24]

η_m= 0.035(Pascal-Sec)

H = 10.5, P =10,665.785 Pascal

Terminal Hepatic length =50μ_m, z_f-z_i= 0.00005meter [25]

By using relation

$\eta_m = \eta_{cX} + \eta_p(1-X)$ Where X=H/100

η_c = 0.32055, and using same above relation, we get

η_m = 0.0.003191H+0.0015

Substituting the value of R, r_p in equation (3.7), we get

$$Q=\frac{2\pi}{27}\left(\frac{p_f-p_i}{3\eta_m}\right)^{\frac{1}{n}}\left[\frac{26n^3+33n^2+9n}{6n^3+11n^2+6n+1}\right]$$

$$\frac{Q\times 27}{2\pi}=\left(\frac{P}{3\eta_m}\right)^{\frac{1}{n}}\left[\frac{26n^3+33n^2+9n}{6n^3+11n^2+6n+1}\right]$$

$$0.071627=(5637630476.6)^{\frac{1}{n}}\left[\frac{26n^3+33n^2+9n}{6n^3+11n^2+6n+1}\right]$$

Solved by numerical method, we get n = -4.7825

$$p_i-p_f=3\eta_m\,(z_f-z_i)\left(\frac{Q\times 27}{2\pi}\right)^n\times\left(\frac{6n^3+11n^2+6n+1}{26n^3+33n^2+9n}\right)^n$$

Substituting the value of Q and n we get

$p_i - p_f$ = (0.02877H+0.013526)

Relation between Blood Pressure and Hematocrit

S.N	Hematocrite	Mean Blood Pressure	Weighted Mean Blood Pressure	Blood Pressure Drop
1	10.5	10,665.785	15,332.07	0.315611
2	12	11,999.015	17,331.91	0.358766
3	13.5	12,998.93	18,665.13	0.401921
4	16.8	11,559.05	16,531.98	0.496862
5	18.6	13,332.23	19,331.73	0.548648

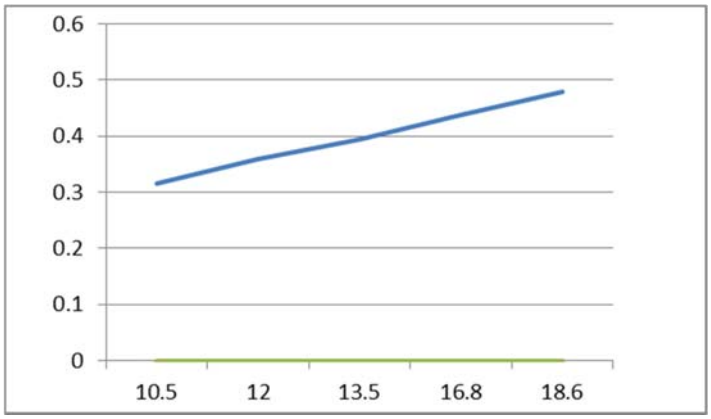

Fig 5: P.D Hematocrit

6. Conclusion

A simple survey of the graph between blood pressure drop and hematocrit in Fig 5 hepatitis C patient shows that when hematocrit increased then blood pressure is also increased. That is the hematocrit is proportional to blood pressure drop.

7. Acknowledgement

I give my sincere thanks to Dr R.K. Sharma physician of Jeevan Jyoti Hospital Allahabad (U.P).

Remark

If this would have been possible to get blood pressure on the particular tissue (liver) during operation of hepatitis C patient then the relation blood pressure and hemoglobin has been measured more accurately.

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