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Matrix representation of Sierpiński graphs

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Abstract
In this Paper, we construct the circuit matrix and the incidence matrix of Sierpiński Graphs. Also we generalize the result for S (n, k) graphs. Finally, the number of edges and number of vertices were also found.

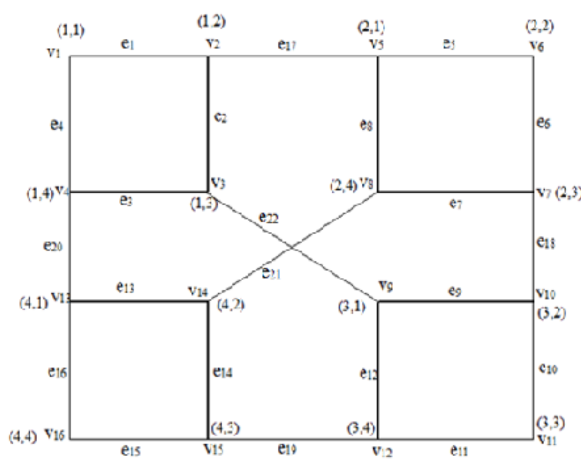
Keywords: Sierpiński Graphs, Circuit matrix, Incidence matrix, Clique.
AMS subject classification (2000): 05C15, 05C69

1. Introduction

Sierpiński Graphs $S(n, k)$ have been introduced in [3] and named as Sierpiński graphs in [4]. The Sierpiński Graph $S(n, k)$ ($n \geq 2$, $k \geq 3$) is defined on the vertex set $\{0, 1, 2, \ldots\ldots, k\text{-}1\}^n$ where two vertices $(i_1, i_2, \ldots\ldots, i_n)$ and $(j_1, j_2, \ldots\ldots, j_n)$ are adjacent if and only if there exists an index h in $\{1, 2, \ldots\ldots, n\}$ such that

(i) $i_t = j_t$ for t=1, 2,. . . . ., h-1
(ii) $i_h \neq j_h$ and
(iii) $i_t = j_h$ and $j_t = i_h$ for t = h + 1, ... , n.

A vertex of the form $(i, i, \ldots\ldots, i)$ of $S(n, k)$ is called an extreme vertex, all other vertices of $S(n, k)$ are called inner vertices. The extreme vertices of $S(n, k)$ are of degree k-1 while the degree of the inner vertices is k. Note that there are exactly k extreme vertices in $S(n, k)$ and that $S(n, k)$ has k vertices. Let $u = (i_1, i_2, \ldots\ldots, i_n)$ be an arbitrary vertex of $S(n, k)$. Let K(u) be the k-clique induced by vertices of the form $(i_1, i_2, \ldots\ldots, i_{n-1}, j)$, $1 \leq j \leq k$. The neighbourhood of an extreme vertex u is K(u) \ {u} and the inner vertex u has only one neighbour that does not belong to K(u) which is of the form $(i_1, i_2, \ldots\ldots, i_{n-2}, i_n, i_{n-1})$. The set of edges {$uv$ edge of $S(n, k)$ | K(u) ≠ K(v) } is a matching.



Graph 1

In the above Sierpiński Graph $S(2,4)$ we can observe the extreme vertices of $S(2,4)$ are (1,1), (2,2), (3,3) and (4,4). Note that each inner vertex u has exactly one neighbour outside the k-clique K(u) ant the set of edges{uv edge of $S(n, k)$ | K(u) ≠ K(v)} is a matching.

In general, Sierpiński graph $S(n, k)$ can be constructed as follows:

We start the construction with a k-clique that can't be the Sierpiński graph $S(1, k)$. To construct $S(2, k)$ we make k copies of the k-clique which is to be connected to each other with an edge set in one-to-one correspondence with the edges of a k-clique. By repeating this procedure we can recursively construct $S(n, k)$ by connecting k copies of $S(n\text{-}1, k)$ with a set of $\frac{k(k-1)}{2}$ edges.

2. Circuit Matrix

In a Circuit Matrix the edges are distinct. In $S(2,4)$, we have Seven circuits. Among these seven circuits the first four circuits are the four cliques in the graph. The fifth circuit is one which covers all the four cliques and the sixth and seventh circuits which includes the cycle that consists of all the non-adjacent vertices.

In Graph 1, the circuits are

$\{e_1, e_2, e_3, e_4\}$, $\{e_5, e_6, e_7, e_8\}$, $\{e_9, e_{10}, e_{11}, e_{12}\}$, $\{e_{13}, e_{14}, e_{15}, e_{16}\}$, $\{e_1, e_{17}, e_5, e_6, e_{18}, e_{10}, e_{11}, e_{19}, e_{15}, e_{16}, e_{20}, e_4\}$, $\{e_4, e_3, e_{22}, e_9, e_{18}, e_6, e_5, e_{17}, e_1\}$, $\{e_6, e_7, e_{21}, e_{13}, e_{20}, e_4, e_1, e_{17}, e_5\}$.

Let the number of different circuits in a graph G be q and the number of edges in G be e. Then a *Circuit Matrix* $A = [a_{ij}]$ of G is a q by e, (0,1)-matrix defined as follows :

$a_{ij} = 1$, if i^{th} circuit includes j^{th} edge, and

$a_{ij} = 0$, otherwise.

The Circuit Matrix of graph 1 is

A =

	e1	e2	e3	e4	e5	e6	e7	e8	e9	e10	e11	e12	e13	e14	e15	e16	e17	e18	e19	e20	e21	e22
1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0
5	1	0	0	1	1	1	0	0	0	1	1	0	0	0	1	1	1	1	1	1	0	0
6	1	0	1	1	1	1	0	0	1	0	0	0	0	0	0	0	1	1	0	0	0	1
7	1	0	0	1	1	1	1	0	0	0	0	0	1	0	0	0	1	0	0	1	1	0

The following observations can be made from the above circuit matrix of the graph 1:

1. Each row of A is a circuit vector.
2. The number of 1's in a row is equal to the number of edges in the corresponding circuit.

3. Incidence Matrix

Let G be a graph with n vertices, e edges and no self-loops. Define an n by e matrix $B = [b_{ij}]$, whose n rows correspond to the n vertices and the e columns correspond to the e edges, as follows:

$b_{ij} = 1$, if j^{th} edge e_j is incident on i^{th} vertex v_i, and

$b_{ij} = 0$, otherwise.

Such a matrix B is called *vertex-edge incidence matrix* (or) simply *incidence matrix*.

The incident matrix of the graph 1 is

B =

	e1	e2	e3	e4	e5	e6	e7	e8	e9	e10	e11	e12	e13	e14	e15	e16	e17	e18	e19	e20	e21	e22
v1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
v2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
v3	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
v4	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
v5	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0
v6	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
v7	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
v8	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0
v9	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	1
v10	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	1	0	0	0	0
v11	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0
v12	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	1	0	0	0
v13	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	1	0	0
v14	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	1	0
v15	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	1	0	0	0
v16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0

The following observations can be made from the above incident matrix of the graph1:

i) Since every edge is incident on exactly two vertices, each column of B has exactly two 1's.
ii) The number of 1's in each row equals the degree of the corresponding vertex.

Next, we find the Transpose matrix of both the circuit and incidence matrices. From the transpose matrix we can conclude the following theorem.

Theorem

Let A and B be the circuit and incidence matrix whose columns are arranged using the same order of edges. Then we can prove that

$A.B^T = B.A^T = 0 \pmod 2$.

Where superscript T denotes the transposed matrix.

Proof:

Consider a vertex v and a circuit C in a graph G. Either v is in C or it is not. If v is not in C then, there is no edge in the circuit C that is incident on v. On the other hand, if v is in C then, the number of those edges in the circuit C that are incident on v is exactly two.

Consider the i^{th} row in B and the j^{th} row in A. Since the edges are arranged in the same order, the nonzero entries in the corresponding positions occur if the particular edge is incident on the i^{th} vertex and is also in the j^{th} circuit.

If the i^{th} vertex is not in the j^{th} circuit, there is no such nonzero entry and the dot product of the two rows in zero. If the i^{th} vertex is in the j^{th} circuit, there will be exactly two 1's in the sum of the products of individual entries. Since 1 + 1 = 0 (mod 2), the dot product of the two arbitrary rows one from A and other from B is zero.

Hence the proof.

Hence,

$$AB^T = \begin{array}{c|cccccccccccccccc|} & v1 & v2 & v3 & v4 & v5 & v6 & v7 & v8 & v9 & v10 & v11 & v12 & v13 & v14 & v15 & v16 \\ 1 & 2 & 2 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 2 & 2 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 2 & 2 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 2 & 2 \\ 5 & 2 & 2 & 0 & 2 & 2 & 2 & 2 & 0 & 0 & 2 & 2 & 2 & 2 & 0 & 0 & 2 \\ 6 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 7 & 2 & 2 & 0 & 2 & 2 & 2 & 2 & 2 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & 0 \end{array} = 0 \pmod 2$$

and

$$BA^T = \begin{array}{c|ccccccc|} & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ v1 & 2 & 0 & 0 & 0 & 2 & 2 & 2 \\ v2 & 2 & 0 & 0 & 0 & 2 & 2 & 2 \\ v3 & 2 & 0 & 0 & 0 & 0 & 2 & 0 \\ v4 & 2 & 0 & 0 & 0 & 2 & 2 & 2 \\ v5 & 0 & 2 & 0 & 0 & 2 & 2 & 2 \\ v6 & 0 & 2 & 0 & 0 & 2 & 2 & 2 \\ v7 & 0 & 2 & 0 & 0 & 2 & 2 & 2 \\ v8 & 0 & 2 & 0 & 0 & 0 & 0 & 2 \\ v9 & 0 & 0 & 2 & 0 & 0 & 2 & 0 \\ v10 & 0 & 0 & 2 & 0 & 2 & 2 & 0 \\ v11 & 0 & 0 & 2 & 0 & 2 & 0 & 0 \\ v12 & 0 & 0 & 2 & 0 & 2 & 0 & 0 \\ v13 & 0 & 0 & 0 & 2 & 2 & 0 & 2 \\ v14 & 0 & 0 & 0 & 2 & 0 & 0 & 2 \\ v15 & 0 & 0 & 0 & 2 & 2 & 0 & 0 \\ v16 & 0 & 0 & 0 & 2 & 2 & 0 & 0 \end{array} = 0 \pmod 2$$

Therefore,

$$AB^T . BA^T = \begin{bmatrix} 16 & 0 & 0 & 0 & 12 & 16 & 12 \\ 0 & 16 & 0 & 0 & 12 & 12 & 16 \\ 0 & 0 & 16 & 0 & 12 & 8 & 0 \\ 0 & 0 & 0 & 16 & 12 & 0 & 8 \\ 12 & 12 & 12 & 12 & 48 & 28 & 28 \\ 16 & 12 & 8 & 0 & 28 & 36 & 24 \\ 12 & 16 & 0 & 8 & 28 & 24 & 36 \end{bmatrix} = 0 \pmod 2$$

From the above matrix we consider the first 4×4 matrix

$$\begin{bmatrix} 16 & 0 & 0 & 0 \\ 0 & 16 & 0 & 0 \\ 0 & 0 & 16 & 0 \\ 0 & 0 & 0 & 16 \end{bmatrix}$$

$$= 16 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

= 16 I

= 4nI. Here n = 4.

Hence this Sierpiński graph $S(2,4)$ has

$$\mathbf{4nI} = 16 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Similarly, we can find the same result for the Sierpiński graphs $S(2,5)$, $S(2,6)$,…………., $S(n,k)$.

Conclusion

Therefore, we can generalize the Sierpiński graphs $S(2,4)$, $S(2,5)$,………, $S(n,k)$ has n^2 vertices and $\frac{n(3n-1)}{2}$ edges.

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