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Group action on G-invariant fuzzy sub lattice with thresholds of complemented lattices

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Abstract
In this paper, we first introduce the concepts of group action on fuzzy sub lattice with thresh hold of a lattice L and listed some properties of them. We define the group action on fuzzy subset of a lattice L and introduce the notion of fuzzy sub lattice with threshold of complemented lattices. Also the notion of G-invariant fuzzy sub lattice with thresh hold of L and homomorphic image, pre-image of fuzzy sub lattice with threshold of L are also studied.

Keywords: Poset, finite group, fuzzy sub lattice, group action, complemented lattice, G-invariant, lattice homomorphism, pre- image

Introduction

The concept of fuzzy sets was first introduced by Zadeh [20] in 1965 and the fuzzy sets have been used in the reconsideration of classical mathematics. Recently, yuan [19] introduced the concept of fuzzy subgroup with thresholds. A fuzzy subgroups with thresholds λ and μ is also called a (λ,μ) –fuzzy subgroups. Yao [16] continued to research (λ,μ)-fuzzy normal subgroups, (λ,μ)- fuzzy quotient subgroups and (λ,μ)- fuzzy subrings in [17]. L.A. Zadeh [20] in 1965 introduced the concept of fuzzy sets to describe vagueness mathematically in its very abstractness. The notion of L-fuzzy sets in lattice theory and introduced the concepts of fuzzy sub lattices and fuzzy ideals in [15]. In particular N. Ajmal and K.V. Thomas [2-4] systematically developed the theory of fuzzy sub lattice. Here we give some basic definitions and results related to fuzzy sub lattice which is taken from their work. The concept of group actions in various algebraic structures in [9, 13]. Let X be a non-empty set. A mapping $\mu: X \to [0, 1]$ is called a fuzzy subset of X. Rosenfeld [12] applied the concept of fuzzy sets to the theory of groups and defined the concept of fuzzy subgroups of a group. Since then, many papers concerning various fuzzy algebraic structures have appeared in the literature [1, 5-7, 11, 18]. In this paper, we first introduce the concepts of fuzzy sub lattice with thresh hold of a lattice L and listed some properties of them. We define the group action on fuzzy subset of a lattice L and introduce the notion of fuzzy sub lattice with threshold of complemented lattices. Many properties of fuzzy sub lattice with threshold of complemented lattice L and G-invariant fuzzy sub lattice with thresh hold of L are studied. The homomorphic image and pre-image of fuzzy sub lattice with threshold of L are also established.

2. Preliminaries

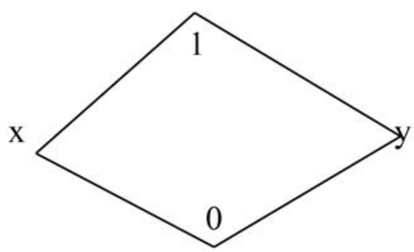
In this section we always use L to stand for a lattice.

Definition 2.1 [5]: A nonempty set L together with two binary operations $\vee$ and $\wedge$ on L is called a lattice if it satisfies the following identities: L_1: (a) $x \vee y \approx y \vee x$ (b) $x \wedge y \approx y \wedge x$, L_2: (a) $x \vee (y \vee z) \approx (x \vee y) \vee z$ (b) $x \wedge (y \wedge z) \approx (x \wedge y) \wedge z$, L_3: (a) $x \vee x \approx x$ (b) $x \wedge x \approx x$, L_4: (a) $x \approx x \vee (x \wedge y)$ (b) $x \approx x \wedge (x \vee y)$. The operation $\wedge$ is called meet and the operation $\vee$ is called join. Let L be the set of propositions and let $\vee$ denote the connective ``or'' and $\wedge$ denote the connective ``and''. Then, L_1 to L_4 are well-known properties from propositional logic

Definition 2.2 [2]: A poset L is a lattice if and only if for every a,b in L both sup{a,b} and in{a,b} exist in L. One can show that the both definitions of a lattice are equivalent. If L is a lattice by the first condition, then define ≤ on L by a ≤ b if and only if a = a ∧ b, and so b = a ∨ b.If L is a lattice by the second condition, then define the operations ∧ and ∨ by a ∨ b = sup{a,b} anda ∧ b = inf{a,b}.
Throughout this paper L = ⟨L,∨,∧⟩ denotes a lattice and ⟨[0,1],∨,∧⟩ is a complete lattice, where [0,1] is the set of real's between 0 and 1 and x ∨ y = max{x, y}, x ∧ y = min{x, y}.
An action of G on X is a map G×X → X denoted by (g,x) ├ (gh)x for all x ϵ X and g,h ϵ G. Throughout these notes, G denotes a group and X, Y denote sets. We use symbols g,g_1, g^1 …… to denote elements of G. Similar x,x_1,x^1 ….. to denote element of X and y,y_1,y^1,….to denote elements of Y.

Definition 2.3: A fuzzy subset A of a lattice L is said to be fuzzy sub lattice with thresh hold of L if, for all x,y ε L(FSL1) A(x∧ y) ∨ α≥ (A(x) ∧A (y)) ∧βand (FSL2) A (x∨ y) ∨α≥(A(x) ∧A (y)) ∧ βor some time the above definition is recalled as A(x ∧ y)∧ A (x ∨ y) ∨ α≥ (A(x) ∧A (y)) ∧ βfor all x,y ε L and α, β ε[0,1] ans α≤ β. Let L be a lattice and $\chi_i$is a characteristic function of a subset I of L. Then χ_i is a fuzzy lattice if and only if I is sub lattice.

Example: Suppose L is the following Boolean algebra we define fuzzy subsets A of L as follows



A(1) = 0, A(x) = 0, A(y) = 1/3, A(0) = 1.Therefore A is fuzzy sub lattice of L
Let L be the lattice and G a finite group which acts in L (that is for all g ε G, x ε L, x^g= gxg^{-1}. The identity of G is denoted by e.

Definition 2.4: A group action of G on a fuzzy set A of a lattice L is denoted by A^g and it is defined by A^g (x) = A (x^g), g ϵG. From the definition of group action G on a fuzzy set, following results are easy to verify.

Lemma 2.1: Let A and B be two fuzzy sets of L and G a finite group which acts on L. Then
(i) If A is subset of B, then A^g is also subset of B^g, for all gε G.
(ii) $(AUB)^g$= A^gU B^g;
(iii) $(A∩B)^g$ = A^g ∩ B^g;
(iv) $(A×B)^g$ = A^g × B^g;
(v) $A(g_1)^{g2}$ = Ag_1g_2;for all g_1, g_2 ε G.
(vi) $(Ag)^{g-1}$ = A^e = A, for all g ε G.

Lemma 2.2: Let G be a finite group which acts on al lattice L. Then for every x,y εL, g ε G, we have
(i) $(x∨y)^g$= $x^g∨y^g$
(ii) $(x∧y)^g$= $x^g∧y^g$
Proof; (i) since $(x∨y)^g$ = g $(x∨y)g^{-1}$
= (gxg^{-1}) ∨(gyg^{-1})= $x^g∨y^g$.
(ii) $(x∧y)^g$= g $(x∧y)g^{-1}$ = (gxg^{-1}) ∧(gyg^{-1}) = $x^g∧y^g$.

3. Homomorphism of G-invariant Fuzzy Sub lattice with thresh hold

In this section, we study the image and pre-image of fuzzy sub lattice with thresh hold under the lattice homomorphism's.

Definition 3.1 [13]: Let A be fuzzy sub lattice with thresh hold of a lattice L and G be a finite group which acts on L. Then A is said to be G-invariant fuzzy sub lattice with thresh hold of L if and only if $A^g(x)$ = A(x^g) ≥ A(x) for all x ϵ L and g ϵ G.

Lemma 3.1: Let f : L_1→ L_2 be two lattices and let A be fuzzy sub lattice with thresh hold of L_1 and G a finite group which acts on L_1 and L_2. Let f: L_1→ L_2 be a mapping defined by f(x^g) = $(f(x))^g$, for all x ε L, g ε G. Then f is lattice homomorphism. We call the mapf as G-lattice homomorphism.
Proof: Let x,yε L,gε G. Then we have
f($x^g∨y^g$) = $(f(x ∨ y))^g$= $(f(x) ∨ f(y))^g$= $(f(x))^g$∨$(f(y))^g$= f(x^g)∨ f(y^g)
f($x^g∧y^g$)= $(f(x ∧ y))^g$= $(f(x) ∧ f(y))^g$= $(f(x))^g$∧$(f(y))^g$= f(x^g) ∧f(y^g).
Hence f is G-lattice homorphism.

Theorem 3.1: Let A be a fuzzy subset of L and G a finite group which acts on L. Then the following are equivalent;
(i) A^g is a fuzzy sub lattice with thresh hold of L;
(ii) A^g is a sub lattice of L for any p ε [α, β] where $A^g{}_{P\neq}$Φ.
Proof; "(i) implies (ii)"
Let A be a fuzzy sub lattice with thresh hold of L and G a finite group which acts on L.
For anypε[α, β], such that $A_p{}^g$≠ Φ.
We need to show that $x^g∧y^g$ε $A_p{}^g$ and $x^g∨y^g$ ε $A_p{}^g$ for all x^g,y^g ε$A_p{}^g$.
From x^g ε $A_p{}^g$, we know that $A^g(x)$ = A(x^g) ≥ p. And similarly we obtain that A(y^g) ≥ p. Thus
A^g(x ∧y) ∨α= A$(x ∧ y)^g$∨α = A($x^g∧y^g$) ∨ α≥ A((x^g) ∧ A(y^g)) ∧β≥ ($A^g(x)$∧$A^g(x)$) ∧β ≥ p. we also know thatα ≤ p. Then we conclude that A^g(x∧ y) ≥p. so $x^g∧y^g$ε$A_p{}^g$.
Simliarly, we can be proved $x^g∨y^g$ε $A_p{}^g$.
"(ii) implies (i)"
Conversly, Let $A_p{}^g$be a sub lattice of L. If there existsx_o,y_o ε Lsuch that A^g($x_o∧y_o$) ∨α< p = A^g(x_o)∧ A^g(y_o) ∧ β < A($x_o{}^g$) ∧ A($y_o{}^g$) ∧ β, then p ε [α, β]. A($x_o{}^g$) ∧ A($y_o{}^g$) ≥ p so $x_o{}^g$ε $A_p{}^g$ and $y_o{}^g$ ε$A_p{}^g$
But A^g($x_o∧y_o$) < p, that is $x_o{}^g∧y_o{}^g$does not belong to$A_p{}^g$. This is a contradiction with that$A_p{}^g$is a sub lattice of L. Thus A^g(x ∧y)∨ α≥ A((x^g) ∧ A(y^g)) ∧βholds for x^g, y^gε L. Similarly we can prove A^g(x∨ y) ∨ α≥ A((x^g)∧A(y^g))∧ β.

Theorem 3.2: A fuzzy subset A of alattice L, then the following are equivalent
For any x^g, y^gL (i) A^g(x∧y) ∨ α≥ ($A^g(x)$∨ $A^g(y)$) ∧β(ii) x^g ≤y^gimplies$A^g(x)$ ∨α≥ $A^g(y)$∧ β.
Proof: "(i) implies (ii)"
If x^g ≤ y^g, then $x^g∧y^g$ = x^g. Thus$A^g(x)$ ∨ α = A^g(x ∧ y) ∨ α≥$A^g(x)$ ∨$A^g(y)$ ∧β ≥ $A^g(y)$ ∧β."(ii) implies (i)"
From $x^g∧y^g$ ≤ α, we know that A^g(x ∧y)∨α≥ $A^g(x)$ ∧β and from $x^g∧y^g$ ≤ y.
We conclude that A^g(x ∧ y) ∨α≥ $A^g(y)$ ∧β. Thus A^g(x∧y) ∨α≥ ($A^g(x)$ ∧β)∨($A^g(y)$ ∧β) = ($A^g(x)$ ∨$A^g(y)$) ∧ β.

Definition 3.2: A fuzzy subset A of a lattice L is said to be fuzzy ideal with thresh hold of L if, for all x,y ε L(FIL_1) A(x∧ y) ∨ α≥ (A(x) ∧A (y))∧ βand (FIL_2) A (x∨ y)∨ α≥(A(x) ∨A (y)) ∧β.

Theorem 3.3: Let A be a fuzzy sub latticeof L and G a finite group which acts on L. Then the following are equivalent;

(i) A^g is a fuzzy ideal with thresh hold of L;

(ii) A^g isfuzzy ideal of L for any p ε [α, β] where $A_p^g \neq \Phi$.

Proof: "(i)implies (ii)"

Let A be a fuzzy ideal with thresh hold of L and G a finite group which acts on L.

For anypε[α, β], such that $A_p^g \neq \Phi$.

We need to show that $x^g \wedge y^g \varepsilon A_p^g$for all x^g, $\varepsilon A_p^g . y^g \varepsilon$ L.

From $A^g(x) = A(x^g) \geq p$. we obtain that$A^g(x \wedge y) \vee \alpha = A(x \wedge y)^g \vee \alpha = A(x^g \wedge y^g) \vee \alpha \geq A((x^g) \wedge A(y^g)) \wedge \beta \geq (A^g(x) \wedge A^g(x)) \wedge \beta$. Then we conclude that $A^g(x \wedge y) \geq p$ since $\alpha < p$.so $x^g \wedge y^g \varepsilon A_p^g$.

"(ii) implies (i)" Conversly, Let A_p^gbe a ideal of L for all pε [α, β],where $A^p g \neq \Phi$.. If there existsx^g_o, y^g_o ε Lsuch that $A^g(x_o \wedge y_o) \vee \alpha < p = (A^g(x_o) \wedge A^g(y_o)) \wedge \beta \geq p$ then p ε [α,β]. $A^g(x_o) \vee A^g(y_o) \geq p$, so $x^g_{o\,\varepsilon} A_p^g$. or$y^g_o \varepsilon A_p^g$. But $A^g(x_o \wedge y_o) < p$, that is $x^g_o \wedge y^g_o \varepsilon A_p^g$.This is a contradiction with that $A_p^g$is a ideal of L. Thus $A(x^g \wedge y^g) \vee \alpha \geq (A^g(x) \vee A^g(y)) \wedge \beta$holds for x^g, $y^g \varepsilon$ L. The proof is ended.

Theorem 3.4: Let f: $L_1 \rightarrow L_2$ be a lattice homomorphism and let A be fuzzy sub lattice with thresh hold of L_1 and G a finite group which acts on L. Then $f(A^g)$ is a fuzzy sub lattice with thresh hold of L_2, where

$f(A^g)(y) = \sup\{ A^g(x) / f(x) = y \}$, for all gεG. x ε L

Proof:

If $f^{-1}(y) = \phi$ or $f^{-1}(y) = \phi$ for any y_1, y_2 ε L_2, then $f(A^g)(y_1 \vee y_2) \vee \alpha \geq (f(A^g)(y_1) \wedge f(A^g)(y_2)) \wedge \beta$.

Suppose that $f^{-1}(y) \neq \phi$ or $f^{-1}(y) \neq \phi$ for any y_1^g, y_2^g ε L_2,

Then $(f(A^g)(y_1 \vee y_2)) \vee \alpha = \sup\{ A^g(t) / f(t) = y_1^g \vee y_2^g \} \vee \alpha$

t ε L

$= \sup \{A^g(t) \vee \alpha / f(t) = y_1^g \vee y_2^g \} \vee \alpha$ t ε L

$\geq \sup\{ A^g(x_1 g \vee x_2 g) / f(x_1 g) = y_1^g, f(x_2 g) = y_2^g \} \vee \alpha x_1^g, x_2^g \varepsilon$ L

$= \sup \{ A^g(x_1^g) \wedge Ag(x_2^g) \wedge \beta / f(x_1^g) = y_1^g, f(x_2^g) = y_2^g \} \vee \alpha x_1^g, x_2^g \varepsilon$ L

$= \sup \{A^g(x_1^g) / f(x_1^g) = y_1^g\} \wedge \sup \{A^g(x_2^g) / f(x_2^g) = y_1^g\} \wedge \beta$

$x^g \varepsilon L_1$ $x^g \varepsilon L_1$

$=(f(A^g)(y_1) \wedge f(A^g(y^2))) \wedge \beta$.Similarly, we have $(f(A^g)(y_1 \wedge y_2) \vee \alpha \geq (f(A^g)(y_1) \wedge f(A^g(y_2)) \wedge \beta$. So $f(A^g)$ is fuzzy sub lattice with thresh hold of L_2.

Theorem 3.5: Let f : $L_1 \rightarrow L_2$ be a surjective homomorphism and let A be fuzzy ideal with thresh hold of L_1 and G a finite group which acts on L. Then $f(A^g)$ is a fuzzy ideal with thresh hold of L_2.

Proof: For any y_1^g, y_2^g ε L_2,$f^{-1}(y_1)$ and $f^{-1}(y_2)$ are non empty. We have

Then $(f(A^g)(y_1 \wedge y_2)) \vee \alpha = \sup \{A^g(t) / f(t) = y_1^g \wedge y_2^g \} \vee \alpha$.

t ε L

$= \sup\{A^g(t) \vee \alpha / f(t) = y_1^g \wedge y_2^g\} \vee \alpha$.

t ε L

$\geq \sup\{A^g(x_1^g \vee x_2^g) / f(x_1^g) = y_1^g, f(x_2^g) = y_2^g\} \vee \alpha\ x_1^g, x_2^g \varepsilon$ L

$= \sup \{(A^g(x_1^g) \vee A^g(x_2^g)) \wedge \beta / f(x_1^g) = y_1^g, f(x_2^g) = y_2^g\} x_1^g, x_2^g \varepsilon$ L

$= \sup \{A^g(x_1^g) / f(x_1^g) = y_1^g\} \vee \sup \{A^g(x_2 g) / f(x_2 g) = y_1^g\} \wedge \beta$

$x^g \varepsilon L_1$ $x^g \varepsilon L_1$

$= (f(A^g)(y_1) \vee f(A^g(y_2))) \wedge \beta$.

Theorem 3.6: Let f: $L_1 \rightarrow L_2$ be lattice homomorphism and let Bbe a fuzzy sub lattice with thresh hold of L_2 and G a finite group which acts on L. Then $f^{-1}(B^g)$ is a fuzzy sub lattice with thresh hold of L_1, where$f^{-1}(B^g)(x) = B(f(x)^g) = B(f(x^g))$, for all x ε L1.

Proof: For any x_1^g, $x_2^g \varepsilon$L.

$f^{-1}(B^g)(x_1 \vee x_2) \vee \alpha = B (f(x_1 \wedge x_2)^g) \vee \alpha$

$= B(f(x_1)^g \wedge f(x_2)^g) \vee \alpha$(since f is lattice homomorphism)

$\geq (B(f(x_1)^g) \vee (Bg (f(x_2)^g) \wedge \beta$

$= (f^{-1}(B^g)(x_1) \wedge f^{-1}(B^g)(x_2)) \wedge \beta$

Similarly, we have

$f^{-1}(B^g)(x_1 \wedge x_2) \vee \alpha \geq (f^{-1}(B^g)(x_1)) \wedge f^{-1}(B^g)(x_2))) \wedge \beta$, so $f^{-1}(B)$ is a fuzzy sub lattice with thresh hold of L_1.

Theorem 3.7: Let f: $L_1 \rightarrow L_2$ be a lattice homomorphism and let B be fuzzy ideal with thresh hold of L_2 and G a finite group which acts on L. Then $f^{-1}(B)$ is a fuzzy ideal with thresh hold of L_1.

Proof: For any x_1^g, $x_2^g \varepsilon L_1$.

$f^{-1}(B^g)(x_1 \wedge x_2) \vee \alpha = B (f(x_1 \wedge x_2)^g) \vee \alpha$

$= B(f(x_1)^g \wedge f(x_2)^g) \vee \alpha$

$\geq (B(f(x_1)^g) \vee B (f(x_2)^g) \wedge \beta$(since f is lattice homomorphism)

$= (f^{-1}(B^g)(x_1)) \vee f^{-1}(B^g)(x_2))) \wedge \beta$. so $f^{-1}(B)$ is a fuzzy ideal of L_1.

4. Fuzzy sub lattice with thresh hold of Complemented Lattices

Let L be a lattice with greatest element 1 and the least element 0. Let x∈ L. By a complement of x in L is meant an element y in L such that $x \wedge y = 0$ and $x \vee y = 1$.

Definition 4.1 [4]: A lattice L with the greatest element 1 and the least element 0 is called complemented if all its element have unique complements.

Definition 4.2: A fuzzy subset A of a complemented lattice L is said to be fuzzy sub lattice with thresh hold of L if, for all x,y ε L (FSCL1)$A(x \wedge y) \vee \alpha \geq (A(x) \wedge A(y)) \wedge \beta$and (FSCL2) $A(x \vee y) \vee \alpha \geq (A(x) \wedge A(y)) \wedge \beta$and $A(c^1) \vee \alpha \geq A(c) \wedge \beta$ where c^1 is the complement of c.

Theorem 4.1: Let A be a fuzzy sub latticeofa complemented lattice L and g a finite group which acts on L. Then the following are equivalent;

(i) A^g is a fuzzy sub lattice with thresh hold of L;

(ii) A^g is a sub lattice of L for any p ε [α, β] where $A_p^g \neq \Phi$.

Proposition 4.1: If A be a fuzzy sub lattice of a complemented lattice L and G a finite group which acts on L, then $A^g(0) \vee \alpha \geq A^g(x) \wedge \beta$and$A^g(1) \vee \alpha \geq A^g(x) \wedge \beta$for xL.

Proof: For xε L and let x^1 be the complement of x. Then $A^g(1) \vee \alpha = A^g(x \vee x^1) \vee \alpha = (A^g(x \vee x^1) \vee \alpha) \vee \alpha \geq ((A^g(x) \wedge A^g(x^1))\beta) \vee \alpha = (A^g(x) \vee \alpha) \wedge (A^g(x^1) \vee \alpha)(\beta \vee \alpha) \geq A^g(x)(A^g(x) \wedge \beta) \wedge \beta = A^g(x) \wedge \beta$. Again, $A^g(0) \vee \alpha = (A^g(x \wedge x^1) \vee \alpha) \vee \alpha \geq ((A^g(x) \wedge A^g(x^1)) \wedge \beta) \vee \alpha = (A^g(x) \vee \alpha) \wedge (A^g(x^1) \vee \alpha) \wedge (\beta \vee \alpha) \geq A^g(x) \wedge (A^g(x) \wedge \beta) \wedge \beta = A^g(x) \wedge \beta$.

Proposition 4.2: If A be a fuzzy sub lattice of a complemented lattice L and G a finite group which acts on L. Suppose that $A^g(0) \neq A^g(1)$, then either $A^g(0) \vee A^g(1) \leq \alpha$holds on $A^g(0) \wedge A^g(1) \geq \beta$ holds.

Proof: Suppose $A^g(0) < A^g(1)$, If $A^g(0) \vee A^g(1) > \alpha$ and $A^g(0) \wedge A^g(1) < \beta$.

Four cases are possible.

Case (i) : If $A^g(0) > \alpha$ and $A^g(0) < \beta$, then $A^g(0) \vee \alpha = A^g(0) < A^g(1)$. Note that $A^g(0) < \beta$, we obtain that $A^g(0) <$

$A^g(1) \wedge \beta$.that is $Ag(0) \vee \alpha < Ag(1) \wedge \beta$. This is a contradiction with the previous proposition.

Case(ii): : If $A^g(0) > \alpha$ and $A^g(1) < \beta$, then $A^g(0) \vee \alpha = A^g(0) < A^g(1) = A^g(1) \wedge \beta$. This is a contradiction with the previous proposition.

Case(iii): : If $A^g(1) > \alpha$ and $A^g(0) < \beta$, then from $A^g(0) < \beta$ and $A^g(0) < A^g(1)$. we obtain that $A^g(0) < A^g(1) \wedge \beta$. From $\alpha < A^g(1)$ and $\alpha < \beta$.we conclude that $\alpha < A^g(1) \wedge \beta$.So $A^g(0) \vee \alpha < A^g(1) \wedge \beta$.This is a contradiction with the previous proposition.

Caes(iv): : If $A^g(1) > \alpha$ and $A^g(1) < \beta$, then from $A^g(0) < A^g(1)$andα $< A^g(1)$.we obtain that $A^g(0) \vee \alpha < A^g(1) = A^g(1) \wedge \beta$. This is a contradiction with the proposition 4.1.If $A^g(1) < A^g(0)$, we can prove the results dually.

Definition 4.3: A fuzzy subset A of a complemented lattice L is said to be fuzzy ideal with thresh hold of L if, for all x,yε L ($FSCI_1$)$A(x \wedge y) \vee \alpha \geq (A(x) \wedge A(y)) \wedge \beta$and ($FSCI_2$) $A(x \vee y) \vee \alpha \geq (A(x) \vee A(y)) \wedge \beta$and ($FSCI_3$) $A(c^1) \vee \alpha \geq A(c) \wedge \beta$ where c^1 is the complement of c.

Theorem 4.2: Let A be a fuzzy subset of L and G a finite group which acts on a complemented lattice L. Then the following are equivalent;

(i) A^g is a fuzzy ideal with thresh hold of L;

(ii) A^g is a sub lattice of L for any p ε [α, β] where $A_p{}^g \neq \Phi$.

Theorem 4.3: Let f: $L_1 \rightarrow L_2$ be a surjective homomorphism where L_1 and $L_2$are complemented lattices and let A be fuzzy sub lattice with thresh hold of L_1 and G a finite group which acts on L. Then $f(A^g)$ is a fuzzyideal with thresh hold of L_2.

Theorem 4.4: :Let f: $L_1 \rightarrow L_2$ be lattice homomorphism where L_1 and $L_2$are complemented lattices and let Bbe a fuzzy sub lattice with thresh hold of L_2 and G a finite groupwhich acts on L. Then $f^{-1}(B^g)$ is a fuzzy sub lattice with thresh hold of L^1, where $f^{-1}(B^g)(x) = B(f(x))$, for allx ϵG.

Theorem 4.5: Let L be a complemented lattice on which G acts on L and let *f* be a G- lattice homomorphism from L into L′. If B be a G-invariant fuzzy sub lattice with thresh hold of L′, then $f^{-1}(B)$ is a G-invariant fuzzy sub lattice with thresh hold of L.

Proof: Since B is G-invariant fuzzy sub lattice with thresh hold of L^1.Therefore, $B^g = B$, for all g ∈ G. Now,$(f^{-1}(B))^g \vee \alpha = f(B^g) \vee \alpha \geq f(B) \wedge \beta$, for all g εG.Therefore,$f^{-1}(B)$is G-invariant fuzzy sub lattice with thresh hold of L.

Theorem 4.6: Let L be a complemented lattice on which G acts on L and let *f* be a bijective G-lattice homomorphism from L into L′. If A is a G-invariant fuzzy sub lattice of L, then *f* (A) is a G-invariant fuzzy sub lattice of L′.

Proof: Since A is G-invariant fuzzy sub lattice with thresh hold of L^1.Therefore, $A^g = A$, for all g∈G Now, $(f(A))^g \vee \alpha = f(A^g) \vee \alpha \geq f(A) \wedge \beta$, for all g εG. Therefore, f(A)is G-invariant fuzzy sub lattice with thresh hold of L.

Conclusion: In this paper, we define the group action on fuzzy sub lattice lattices and introduce the notion fuzzy sub lattice and fuzzy ideals with thresh hold of complemented lattices. We also study the concept of G-invariant sub lattice of L. The homomorphic image and pre-image of fuzzy sub lattice with thresh hold of L also established. A number of associated results will be obtained. One can obtain the results in the field of bipolar fuzzy structures and soft substructures.

References

1. Ali T, Ray AK. On product of fuzzy sub lattices, The Journal of Fuzzy mathematice. 2007; 15(2):375-381.
2. Ajmal N, Thomas KV. Fuzzy Lattice-I, the Journal of Fuzzy Mathematics. 2012; 10(2):255-274.
3. Ajmal N, Thomas KV. Fuzzy Lattice-II, the Journal of Fuzzy Mathematics.2012; 10(2):275-296.
4. Ajmal N, Thomas KV. Fuzzy Lattices, Infor Sciences 1994; 79:271-291.
5. Bhakat SK, Das P. Fuzzy subrings and ideals redefined, Fuzzy Sets Syst1996; 81:383-393.
6. Birkhoff G. Lattice theory, American Mathematical Society, Providence, RI, 1984.
7. Bo Y, Wu Wang Ming. Fuzzy ideals on a distributive lattice, Fuzzy Sets Syst1990; 35:231-240.
8. Goguen JA. L Fuzzy Sets, J Math Anal Appl. 1967; 18:145-174.
9. Han JC. Group Actions in a Regular Ring. Bulletin of the Korean Mathematical Society.2005; 42:807-815.
10. Koguep BBN, Nkuimi C, Lele C. On fuzy prime ideals of lattices, SAMSA Journal of Pure and Applied Mathematics.2008; 3:1-11.
11. Liu WJ. Fuzzy invariant subgroups and fuzzy ideals, Fuzzy Sets Syst1982; 8:133-139.
12. Rosenfeld: Fuzzy groups, J Math Anal Appl. 1971; 35:512-517.
13. Sharma RP, Sharma S. Group Action on Fuzzy Ideals. Communications in A, 1998.
14. Satya Saibaba GSV. Fuzzy lattice ordered groups, South East Asian Bulletin of Mathematics 32,749-766 2008lgebra, 26:4207-4220.
15. Tepavacevic A, Trajkovski G.L-fuzzy lattices: an introduction, Fuzzy Sets Syst2001; 123:209-216.
16. Yao B. (λ, μ)-fuzzy normal subgroup and (λ, μ) –fuzzy quotient subgroups, Journal of fuzzy mathematics. 2005, 695-705.
17. Yao B. (λ,μ)- fuzzy subrings and (λ,μ)- fuzzy ideals, The Journal of fuzzy mathematics.2007; 15(4):981-987.
18. Yao B. Fuzzy theory on group and ring, Science and Technolgy Press, Beijing (In Chinese), 2008.
19. YuanX, Zhang CC, Ren Y. Generlized fuzzy groups and many-valued implications, Fuzzy sets and systems2003; 138:205-211.
20. Zadeh LA. Fuzzy sets, Inform and Control 1965; 8:338-353

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