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A comparative study of fuzzy multiple regression model and least square method

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Abstract
The objectives of this study is to formulate a multiple fuzzy linear regression model using crisp input/output to investigate the relationship between explanatory and response variables to estimate the model parameters. For present study, fuzzy linear regression model proposed by Zadeh'sis used which is based on fuzzy linear function. Comparative study of fuzzy multiple regression model and conventional multiple regression model is done on the basis of coefficient of determination which is used as goodness of fit for both the models. Finally, a numerical example is provided for demonstration of the results. It is observed that the fuzzy multiple regression model is more suitable than the conventional multiple regression model resulting in higher coefficient of determination.

Keywords: Fuzzy linear regression, SST, SSR, Coefficient of determination, least square method.

1. Introduction

Regression analysis has a widespread application in various fields, such as business, engineering, agriculture, and economics, to explain the statistical relationship between explanatory and response variables. The use of statistical linear regression is bounded by some assumption about the given data that is error terms are mutually independent and identical distributed. Statistical regression model can be applied only if the given data are distributed according to statistical model, and the relation between explanatory and response variables is crisp. However, in real life situations decision-making processes the data will be fuzzy in nature. For example, the observations are represented in linguistic terms, vagueness, such kinds of data the fuzzy regression model is suitable to construct the relationship between input and output variables in the fuzzy environment.

The fuzzy linear regression model was first introduced by Tanaka *et al*. [1], by using linear programming problem to determine the regression coefficient as a fuzzy numbers. Later Tanaka [2], Tanaka and Watada [5], and Tanaka *et al*. [3] made some improvements. As pointed out by Redden and Woodall [5], their method is very sensitive to outliers. Moreover, the spread of the estimated response becomes wider as more observations are included in the model. The second approach developed by Diamond [6], which minimizes the total error some of square of the output is called the fuzzy least square method. On the basis of a possibilistic regression model.

2. Literature Review

Diamond [6] introduced the fuzzy regression model to minimize the sum of squares of differences for the centre of fuzzy of fuzzy numbers and the sum of squares of differences for spreads of fuzzy numbers. PierpaoloD'Urso, *et al*. [11]. Introduced new approach of fuzzy linear regression analysis. They had developed doubly linear adaptive fuzzy regression model, based on two linear models such as centre regression model and a spread regression model. The first one was explained the centre of the fuzzy observations, while the second one was for their spreads. They had observed that doubly linear adaptive fuzzy regression analysis had alternative methods for fuzzy linear regression analysis. Volker Krätschmer [12] developed new fuzzy linear regression models. He had generalized the type of single

Ordinary equation in linear regression models by incorporated the physical vagueness of the involved items in the form of fuzzy data for the variables. He had suggested that ordinary least-squares method was greater flexibility for modelling and estimation Yun-His, *et al.* [13], Developed hybrid fuzzy least-squares regression. They were used weighted fuzzy-arithmetic mean and least-squares fitting criterion. They had compared hybrid regression with the ordinary regression and other fuzzy regression methods. They suggested that hybrid fuzzy regression model satisfied a limiting behaviour that the fuzziness decreases, the equations were similar to the results of the ordinary regression. Chiang Kao *et al*. [14], proposed two-stage fuzzy linear regression model. In first stage, the fuzzy observations was de-fuzzified so that the traditional least-squares method was applied to find a crisp regression line showed the general trend of the data. In the second stage, the error term of the fuzzy regression model was represents the fuzziness of the data andalso determined to given the regression model was best explanatory power for the data. Finally they suggested that two-stage method had better performance than the Kim–Bishu's and Diamond fuzzy linear regression model. Hsien-Chung Wu [18] proposed fuzzy estimates of regression parameters with the help of "Resolution Identity". He said that the fuzzy estimates was constructed from the alpha level least-squares estimates used the alpha level real-valued data. Finally he had developed two computational procedures to solve the optimization problems.

Kyung. Kim, *et al*. [17]. Proposed fuzzy least absolute deviation method to construct fuzzy linear regression model with fuzzy input and fuzzy output. They had suggested that the fuzzy least absolute deviation method was more effective than the least square method used the in the fuzzy regression analysis. Rajan Alex [16] introduced fuzzy regression and fuzzy inference. He was applied the two kinds of information resources, quantitative and qualitative information and also used simultaneously in practical prediction. Finally he suggested that fuzzy regression and fuzzy inference had better performance than pure regression or pure inference model.

Hye-Youngung, *et al*. [15], proposed rank transformation method. They investigated a method to obtain a predicted output with respect to a specific target value. They suggested that the rank transformation method in fuzzy regression model was better performs than the Chen and Hsuehand Diamond fuzzy regression models. Furkan Baseret *et al*. [20] applied hybrid fuzzy least squares regression analysis to predict future claim costs by used the concept of London Chain Ladder (LCL) method. They had suggested that the hybrid fuzzy least-squares regression model was taken both randomness and fuzziness type of uncertainty into a regression model.

A.B Ubale, S.L Sananse [21] introduced brief research trends in application of fuzzy regression analysis in different field.

3. Materials and Methods

A regression Model that involves more than one explanatory or independent variables, called as multiple linear regression model. This model generalizes the simple linear regression in two ways. It allows the mean function *E(y)* to depend on more than one explanatory variables.

Let *y* denotes the dependent (or study) variable that is linearly related to *k* independent (or explanatory) variables $X_1, X_2, \ldots, X_n$ through the parameters $\beta_1, \beta_2, \ldots, \beta_k$ and we write

$$y = X_1\beta_1 + X_2\beta_2 + \cdots + X_k\beta_k + \varepsilon \quad (1)$$

This is called as the multiple linear regression model. The parameters $\beta_1, \beta_2, \ldots, \beta_k$ are the regression coefficients associated with $X_1, X_2, \ldots, X_n$ respectively and ε is the random error component reflecting the difference between the observed and fitted linear relationship.

These *n* equations can be written as

$$\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1k} \\ x_{21} & x_{22} & \cdots & x_{2k} \\ \vdots & \vdots & \vdots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nk} \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_k \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{pmatrix}$$

Or

$$y = X\beta + \varepsilon \quad (2)$$

where $y = (y_1, y_2, \ldots, y_n)'$ is a *n* ×1 vector of *n* observation on study variable,

$$X = \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1k} \\ x_{21} & x_{22} & \cdots & x_{2k} \\ \vdots & \vdots & \vdots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nk} \end{pmatrix}$$

is a *n* x *k* matrix of *n* observations on each of the *k* explanatory variables, $\beta = (\beta_1, \beta_2, \ldots, \beta_n)'$ is a *k* x 1 vector of regression coefficients and $\varepsilon = (\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n)'$ is a *n* x 1 vector of random error components or disturbance term.

If intercept term is present, take first column of *X* to be (1,1,…,1)'. So that

$$X = \begin{pmatrix} 1 & x_{11} & x_{12} & \cdots & x_{1k} \\ 1 & x_{21} & x_{22} & \cdots & x_{2k} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_{n1} & x_{n2} & \cdots & x_{nk} \end{pmatrix}$$

In this case, there are (*k* – 1) explanatory variables and one intercept term.

3.1 Least Squares Methods (LS)

A general procedure for the estimation of regression coefficient vector is to minimize

$$\sum_{i=1}^{n} M(\varepsilon_i) = \sum_{i=1}^{n} M(y_i - x_{i1}\beta_1 - x_{i2}\beta_2 - \cdots - x_{ik}\beta_k) \quad (3)$$

for a suitably chosen function *M*.

Let *B* be the set of all possible vectors *β*. If there is no further information, then *B* is *k*-dimensional real Euclidean space. The object is to find a vector $b' = (b_1, b_2, \ldots, b_k)$ from *B* that minimizes the sum of squared deviations of ε_i '*s*, i.e.,

$$S(\beta) = \sum_{i=1}^{n} \varepsilon_i^{\,2} = \varepsilon'\varepsilon = (y - X\beta)'(y - X\beta) \quad (4)$$

for given *y* and *X*. A minimum will always exist as *S*(*β*) is a real valued, convex and differentiable function. Write

$$S(\beta). = y'y + \beta'X'X\beta - 2\beta'X'y.$$

Differentiate (4) with respect to *β*

$$\frac{\partial S(\beta)}{\partial \beta} = 2X'X\beta - 2X'y$$

$$\frac{\partial^2 S(\beta)}{\partial \beta \partial \beta'} = 2X'X$$

The normal equation is

$$\frac{\partial S(\beta)}{\partial \beta} = 0$$

$$= X'X\beta = X'y \quad (5)$$

Equation (5) is the least-squares normal equations. To solve the normal equations, multiply both sides of equation (5) by the inverse of $X'X$. Thus, the least squares estimator of β is

$$\beta = (X'X)^{-1}X'y \quad (6)$$

3.2Coefficient of determination (R^2)

Let *R* be the multiple correlation coefficient between *y*and$X_1, X_2, \ldots, X_k$, then square of multiple correlation coefficient, (R^2) is called as coefficient of determination. The value of R^2commonly describes that how well the simple regression line fits to the observed data. This is also treated as a measure of goodness of fit of the model.

Then

$$R^2 = 1 - \frac{SS_{res}}{SS_T} \quad (7)$$

Where

SS_{res} : sum of squares due to residuals,

SS_T: total sum of squares,

R^2measure the explanatory power of the model which in turn reflects the goodness of fit of the model.

3.3 Estimate the parameters of Multiple Fuzzy regression

Fuzzy linear regression analysis is first proposed by H Tanaka [1], and applied for many researches by Hsiao-Fan Wang *et al*. [4] the model is given below.

$$\tilde{Y} = \tilde{A}_0 X_0 + \tilde{A}_1 X_1 + \cdots + \tilde{A}_N X_N = \tilde{A}X \quad (8)$$

where$X = [X_0, X_1, \dots . X_N]^T$ is a vector of independent variables, $\tilde{A} = [\tilde{A}_0, \tilde{A}_1, \dots . \tilde{A}_N]^T$ is a vector of fuzzy coefficient presented in the form of triangular fuzzy numbers denoted by $\tilde{A}_j = (\alpha_j, c_j)$with its membership functiondescribed as

$$\mu_{\tilde{A}_j}(a_j) = \begin{cases} 1 - \frac{|\alpha_j - a_j|}{c_j} & , \alpha_j - c_j \le a_j \le \alpha_j + c_j, \forall j = 1, 2, \dots, N. \\ 0, otherwise \end{cases} \quad (9)$$

Where $\alpha_j$the central value and c_j is the spread value.

Therefore the equation (8) can be written as

$$\tilde{Y}_i = (\alpha_0, c_0) + (\alpha_1, c_1)X_1 + (\alpha_2, c_2)X_2 + \cdots + (\alpha_N, c_N)X_N \quad (10)$$

Equation (10) is fuzzy regression model for crisp input and crisp output data. By applying the Extension Principle [5], it construct the membership function of fuzzy number $\tilde{Y}_i$ as shown in (11) and each value of dependent variable can be estimated as a fuzzy number is introduced in [9] as follows:

$\tilde{Y}_i = (Y_i^{\,L}, Y_i^{\,h=1}, Y_i^{\,U})$ $i = 1, 2, \dots, M$ where the

Lower bound of $\tilde{Y}_i$ is $Y_i^{\,L} = \sum_{j=0}^{N}(\alpha_j - c_j)\, X_{ij}$,

The central value of $\tilde{Y}_i$ is $Y_i^{\,h=1} = \sum_{j=0}^{N} \alpha_j\, X_{ij}$ and

The upper bound of $\tilde{Y}_i$ is $Y_i^{\,U} = \sum_{j=0}^{N}(\alpha_j + c_j)\, X_{ij}$.

$$\mu(Y_i) = \begin{cases} 1 - \frac{|Y_i - X^t\alpha|}{c^t|X|} \; X \ne 0, \\ 1 \; X = 0, Y \ne 0 \; \forall i = 1, 2, \dots, M. \\ 0, X = 0, Y = 0 \end{cases} \quad (11)$$

To determine the fuzzy parameters while minimizing the total sum of the spreads of the estimated values for a certain *h* level, using a linear programming problem in [12] called Min problem, as follows:

$$Min \sum_{i=1}^{m} \sum_{j=0}^{N} (c_j |X_{ij}|)$$

s. t.

$$\sum_{j=0}^{N} \alpha_j X_{ij} + (1 - h) \sum_{j=0}^{N} c_j |X_{ij}| \ge Y_i i = 1, 2, \dots, M$$

$$\sum_{j=0}^{N} \alpha_j X_{ij} - (1 - h) \sum_{j=0}^{N} c_j |X_{ij}| \le Y_i i = 1, 2, \dots, M \quad (12)$$

$$c_j \ge 0, a \in R, X_{i0} = 1, (0 \le h \le 1)$$

In fuzzy linear regression, values of the vector independent variablesX_i have its corresponding fuzzy numbers $\tilde{Y}_i$ in which, without any other information, the probabilities of occurrence of all points in $\tilde{Y}_i$ are assumed to be equal the membership functions of the fuzzy parameters in a fuzzy linear regression model are symmetric, then the values of $Y_i^{\,h=1}$is equal to $\bar{Y}_i$is estimated by introduced in [9].as follows:

Then the $\tilde{Y}_i$ we have $Y_i^{\,L} = Y_i^{\,h=1} - c^t|X|andY_i^{\,U} = Y_i^{\,h=1} + c^t|X|$

Therefore

$$\bar{Y}_i = \frac{Y_i^{\,L} + Y_i^{\,U}}{2} = [(Y_i^{\,h=1} - c^t|X|) + (Y_i^{\,h=1} + c^t|X|)]/2 = Y_i^{\,h=1} \quad (13)$$

We consider the M data points the $Y_i^{\,L} \le Y_i \le Y_i^{\,U}$,

$Y_i = Y_i^{\,L} + a_i\Delta$,(14)

Where Δ is the difference between lower bound and upper bound.

$a_i$is thecentervalues

$$Y_i^{\,h=1} = Y_i^{\,L} + \frac{n_i}{2\Delta}. \quad (15)$$

$\forall i = 1, 2, \dots, M$. Since the difference between $Y_i^{\,L}$ and $Y_i^{\,U}$

Where n_i is the number of observation?

$$(Y_i - Y_i^{\,L}) + (Y_i^{\,U} - Y_i) = n_i\Delta = (Y_i^{\,h=1} - Y_i^{\,L}) + (Y_i^{\,U} - Y_i^{\,h=1}),$$

$$\sum_{i=1}^{M}[(Y_i - Y_i^{\,L}) + (Y_i^{\,U} - Y_i)]^2 = \sum_{i=1}^{M}[(Y_i^{\,h=1} - Y_i^{\,L}) + (Y_i^{\,U} - Y_i^{\,h=1})]^2.$$

$\Rightarrow$

$$\sum_{i=1}^{M}\left(Y_i - Y_i^{\,L}\right)^2 + \sum_{i=1}^{M}\left(Y_i^{\,U} - Y_i\right)^2 + 2\sum_{i=1}^{M}[(Y_i - Y_i^{\,L}).(Y_i^{\,U} - Y_i)] \qquad (16)$$

$$= \sum_{i=1}^{M}(Y_i^{\,h=1} - Y_i^{\,L})^2 \sum_{i=1}^{M}(Y_i^{\,U} - Y_i^{\,h=1})^2 + 2\sum_{i=1}^{M}\left[(Y_i^{\,h=1} - Y_i^{\,L}).(Y_i^{\,U} - Y_i)\right].$$

Putting the values equation (14),(15) in equation (17) can be written as

$$\sum_{i=1}^{M}\left(Y_i - Y_i^{\,L}\right)^2 + \sum_{i=1}^{M}\left(Y_i^{\,U} - Y_i\right)^2 + 2\sum_{i=1}^{M}[(a_i.\Delta).(n_i.\Delta - a_i.\Delta)]$$

$$= \sum_{i=1}^{M}(Y_i^{\,h=1} - Y_i^{\,L})^2 \sum_{i=1}^{M}(Y_i^{\,U} - Y_i^{\,h=1})^2 + 2\sum_{i=1}^{M}\left[(n_i.{}^{\Delta}/_{2}).(n_i.{}^{\Delta}/_{2})\right].$$

$\Rightarrow$

$$2\sum_{i=1}^{M}\left[(n_i.{}^{\Delta}/_{2}).(n_i.{}^{\Delta}/_{2})\right]. - 2\sum_{i=1}^{M}[(a_i.\Delta).(n_i.\Delta - a_i.\Delta)]$$

$$= 2\sum_{i=1}^{M}\left[(n_i.{}^{\Delta}/_{2})^2. - a_i n_i \Delta^2 + a_i^{\,2}\Delta^2\right] = 2\sum_{i=1}^{M}\left(\frac{n_i.\Delta}{2} - a_i.\Delta\right)^2$$

$$= 2\sum_{i=1}^{M}(Y_i^{\,L} + \frac{n_i.\Delta}{2} - (Y_i^{\,L} + a_i.\Delta))^2 = 2\sum_{i=1}^{M}(Y_i^{\,h=1} - Y_i)^2. \qquad (17)$$

By using equation (16) and (17),

$$\sum_{i=1}^{M}(Y_i - Y_i^{\,L})^2 + \sum_{i=1}^{M}(Y_i^{\,U} - Y_i)^2 = \sum_{i=1}^{M}(Y_i^{\,h=1} - Y_i^{\,L})^2 + \sum_{i=1}^{M}(Y_i^{\,U} - Y_i^{\,h=1})^2 + 2\sum_{i=1}^{M}(Y_i^{\,h=1} - Y_i)^2. \qquad (18)$$

Let $\sum_{i=1}^{M}(Y_i - Y_i^{\,L})^2 + \sum_{i=1}^{M}(Y_i^{\,U} - Y_i)^2$ be the total sum of squares (SST) of fuzzy regression interval, $\sum_{i=1}^{M}(Y_i^{\,h=1} - Y_i^{\,L})^2 + \sum_{i=1}^{M}(Y_i^{\,U} - Y_i^{\,h=1})^2$ be the regression sum of squares (SSR) and $2\sum_{i=1}^{M}(Y_i^{\,h=1} - Y_i)^2$. be the error sum of squares (SSE), then by using Equation (18) total sum of squares is as follows:

$$SST = SSR + SSE. \qquad (19)$$

Here the total sum of squares (SST) measures the total variation of Y_i between lower and upper bounds. The error sum of squares (SSE) estimates the difference when we use $Y_i^{\,h=1}$ to estimate Y_i whereas regressionsum of squares (SSR) represents the variation of $Y_i^{\,h=1}$ with respect to lower and upper bounds.

The measure of the degree of interpretation let us define an index of confidence by

$$IC = \frac{SSR}{SST} = 1 - \frac{SSE}{SST} \qquad (20)$$

which is similar to the coefficient of determinant, R^2, in statistics. Since SST is a measure of the interval variation between $Y_i^{\,L}$ and, $Y_i^{\,U}$ and SSR represents the variation in $\widetilde{Y}_i$ with respect to the centerregression line $Y_i^{\,h=1}$ so IC measures the degree of the variation of Y between Y^{L} and Y^{U} that can be explained by the centre regression line $Y^{h=1}$. Because $0 \le SSE \le SST$, it follows that $0 \le IC \le 1$. So, it means that the higher the IC, the better is the Y h=1 i used to represent Y_i.

3.4 Numerical example

Clerical employees of a large financial organization included questions related to employee satisfaction with their supervisor. There was a question designed to measure the overall performance of a supervisor, as well as questions that were related to specific activities involving interaction between supervisor and employee. An exploratory study has undertaken to try to explain the relationship between specific supervisor characteristics and overall satisfaction with supervisors as perceived by the employees. The response variable is overall rating of job being done by supervisor(Y), and explanatory variables are handles employee complaints(X_1), Does not allow special privileges(X_2), opportunity to learn new things(X_3), Too critical of poor performances(X_4), Rate of advancing to better jobs (X_5). The data set is taken from [19], given below.

Table 1: Supervisor Performance Data

Y	X_1	X_2	X_3	X_4	X_5
81	90	50	72	54	36
74	85	64	69	79	63
65	60	65	75	80	60
65	70	46	57	85	46
50	58	68	54	78	52
50	40	33	34	64	33
64	61	52	62	80	41
53	66	52	50	80	37
40	37	42	58	57	49
63	54	42	48	75	33
66	77	66	63	76	72
78	75	58	74	78	49
48	57	44	45	83	38
85	85	71	71	74	55
82	82	39	59	78	39

By using the equation (6) the parameters are estimated using MATLAB software as and multiple linear regression equation is obtained as follows:

$$Y = 8.71 + 0.640X_1 - 0.211X_2 + 0.459X_3 + 0.056X_4 - 0.156X_5 \quad (21)$$

By using equation (7), to determine coefficient of determination of multiple regression model is $R^2 = 83.8\%$ which indicate that these five variables causes 83.8% variability in the Overall rating of job being done by supervisor.

By using equation (12) the parameters are estimated using MATLAB software considering h=0.6 as minimum fuzziness criteria and the multiple regression equation is obtained as follows:

$$Y = (6.3969, 89.3259) + (0.6269, 0.0987)X_1 + (-0.1632, 0.6580)X_2 + (0.6790, -0.6170)X_3 + (-0.0066, -0.7288)X_4 + (-0.3133, -0.5573)X_5 \quad (24)$$

And also by using equation (20), to determine coefficient of determination of multiple fuzzy regression model is $R^2 = 88.08\%$ which indicate that these five variables causes 88.08% variability in the Overall rating of job being done by supervisor.

By using equation (13), the parameters are estimated as follows:

Table 2: Lowe bound, upper bound, central values, spread, of Supervisor Performance Data

Center	Spread	Lower bound	Upper bound	$\bar{Y}$	$Y_i^{h=1}$
91.9107	27.2669	64.6438	119.1776	91.9107	91.9107
75.8303	4.5693	71.261	80.3996	75.8303	75.8303
65.0019	0.0009	65.001	65.0028	65.0019	65.0019
66.5029	3.7501	62.7528	70.253	66.5029	66.5029
51.5191	20.6505	30.8686	72.1696	51.5191	51.5191
38.412	28.9758	9.4362	67.3878	38.412	38.412
64.8761	10.1553	54.7208	75.0314	64.8761	64.8761
61.1158	20.282	40.8338	81.3978	61.1158	61.1158
46.3919	15.9785	30.4134	62.3704	46.3919	46.3919
55.1532	19.6248	35.5284	74.778	55.1532	55.1532
63.6148	5.9684	57.6464	69.5832	63.6148	63.6148
78.3283	5.0803	73.248	83.4086	78.3283	78.3283
53.0512	14.471	38.5802	67.5222	53.0512	53.0512
78.5853	16.0437	62.5416	94.629	78.5853	78.5853
78.7654	8.0972	70.6682	86.8626	78.7654	78.7654

4. Conclusion

In this paper least square regression analysis and multiple fuzzy regression model are compared using coefficient of determination for goodness of fit. The problem under study shows that the fuzzy multiple regression model is performing better than the least square method. Therefore, multiple fuzzy regression models can produce better prediction as compared to least square method.

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