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A note on intuitionistic fuzzy volterra spaces

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Abstract

In this paper, the concepts of intuitionistic fuzzy ε_r -Volterra spaces and intuitionistic fuzzy ε_p -Volterra spaces are introduced and studied. We will discuss several characterizations of those spaces.

Keywords: Intuitionistic fuzzy topology, Intuitionistic fuzzy Volterra spaces, Intuitionistic fuzzy weakly Volterra spaces, Intuitionistic fuzzy ε_r -Volterra spaces, Intuitionistic fuzzy ε_p -Volterra spaces.

AMS Classification: 54A40, 03E72.

1. Introduction

The concept of fuzzy set was introduced by Zadeh in his classic paper ^[14]. The theory of fuzzy topological spaces was introduced and developed by C.L.Chang ^[4]. Since Atannasov ^[1, 2, 3] introduced the notion of intuitionistic fuzzy sets, Çoker ^[5] defined the intuitionistic fuzzy topological spaces. The concept of Volterra spaces have been studied extensively in classical topology ^[6, 7, 8]. The concepts of fuzzy Volterra spaces, fuzzy weakly Volterra spaces and generalized fuzzy Volterra spaces in fuzzy topological spaces are introduced and studied by the authors in ^[12, 13]. The concept of intuitionistic fuzzy weakly Volterra space was introduced and studied by the authors in ^[10, 11]. In this paper we study the characterizations of intuitionistic generalized fuzzy Volterra spaces.

2. Preliminaries

Definition 2.1. ^[1] Let X be a non-empty fixed set and I the closed interval $[0,1]$. An intuitionistic fuzzy set (IFS) A is an object of following form

$$A = \{ \langle x, \mu_A(x), \vartheta_A(x) \rangle \mid x \in X \}$$

where the mappings $\mu_A: X \rightarrow I$ and $\vartheta_A: X \rightarrow I$ denote the degree of membership (namely) $\mu_A(x)$ and the degree of non-membership (namely) $\vartheta_A(x)$ for each element $x \in X$ to the set A respectively, and $0 \leq \mu_A(x) + \vartheta_A(x) \leq 1$ for each $x \in X$.

Definition 2.2. ^[1] Let A and B be IFS'S of the form $A = \{ \langle x, \mu_A(x), \vartheta_A(x) \rangle \mid x \in X \}$ and $B = \{ \langle x, \mu_B(x), \vartheta_B(x) \rangle \mid x \in X \}$. Then

- (i) $A \subseteq B$ iff $\mu_A(x) \leq \mu_B(x)$ and $\vartheta_A(x) \geq \vartheta_B(x)$;
- (ii) $\bar{A} = \{ \langle x, \vartheta_A(x), \mu_A(x) \rangle \mid x \in X \}$;
- (iii) $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \vartheta_A(x) \vee \vartheta_B(x) \rangle \mid x \in X \}$;
- (iv) $A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \vartheta_A(x) \wedge \vartheta_B(x) \rangle \mid x \in X \}$.

Definition 2.3. ^[1] $0_{\sim} = \{ \langle x, 0, 1 \rangle \mid x \in X \}$ and $1_{\sim} = \{ \langle x, 1, 0 \rangle \mid x \in X \}$.

Definition 2.4. ^[5] An intuitionistic fuzzy topology (IFT for short) in Çoker's sense on a nonempty set X is a family τ of intuitionistic fuzzy sets in X satisfying the following axioms:

- (T₁) $0_{\sim}, 1_{\sim} \in \tau$,
- (T₂) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,
- (T₃) $\bigcup_{i \in I} G_i \in \tau$ for any arbitrary family $\{G_i: i \in I\} \subseteq \tau$.

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In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS for short) and each IFS in τ is known as an intuitionistic fuzzy open set (IFOS for short) in X .

Definition 2.5. [5] The complement \bar{A} of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS for short) in X .

Definition 2.6. [5] Let (X, τ) be an IFTS and $A = \{ \langle x, \mu_A(x), \vartheta_A(x) \rangle : x \in X \}$ be an IFS in X . Then the fuzzy interior and fuzzy closure of A are defined by $IFcl(A) = \cap \{K : K \text{ is an IFCS in } X \text{ and } A \subseteq K\}$ and $IFint(A) = \cup \{G : G \text{ is an IFOS in } X \text{ and } G \subseteq A\}$.

Proposition 2.7. [5] Let (X, τ) be an IFTS and A be IFS'S in X . Then the following properties hold:

- (i) $1_{\sim} - IFcl(A) = IFint(1_{\sim} - A)$,
- (ii) $1_{\sim} - IFint(A) = IFcl(1_{\sim} - A)$.

Definition 2.8. [9] An IFS A in an IFTS X is called an intuitionistic fuzzy preopen set (IFPOS for short) if $A \subseteq int(cl(A))$. The complement \bar{A} of an IFPOS in X is called an intuitionistic fuzzy preclosed set (IFPCS for short) in X .

Definition 2.9 [9] Let $A = \{ \langle x, \mu_A(x), \vartheta_A(x) \rangle : x \in X \}$ be an IFS in IFTS X . Then $IFpreint(A) = \cup \{G : G \text{ is an IFPOS in } X \text{ and } G \subseteq A\}$ is called an intuitionistic fuzzy preinterior of A ; $IFprecl(A) = \cap \{K : K \text{ is an IFPCS in } X \text{ and } A \subseteq K\}$ is an intuitionistic fuzzy preclosure of A .

Definition 2.10. [11] An IFS A in an IFTS X is called an intuitionistic fuzzy dense if there exists no intuitionistic fuzzy closed set B in (X, τ) such that $A \subseteq B \subseteq 1_{\sim}$.

Definition 2.11. [11] An intuitionistic fuzzy set A in an intuitionistic fuzzy topological space (X, τ) is called an intuitionistic fuzzy nowhere dense set if there exists no intuitionistic fuzzy open set U in (X, τ) such that $U \subseteq IFcl(A)$. That is, $IFint IFcl(A) = 0_{\sim}$.

Theorem 2.12. [11] If A is an intuitionistic fuzzy nowhere dense set in IFTS (X, τ) , then $1_{\sim} - A$ is an intuitionistic fuzzy dense set in (X, τ) .

Definition 2.13 [11] An intuitionistic fuzzy set A in an intuitionistic fuzzy topological space (X, τ) is called an intuitionistic fuzzy G_{δ} set in (X, τ) if $A = \cap_{i=1}^{\infty} A_i$ where $A_i \in \tau$, for each i .

Definition 2.14. [11] An intuitionistic fuzzy set A in an intuitionistic fuzzy topological space (X, τ) is called an intuitionistic fuzzy F_{σ} set in (X, τ) if $A = \cup_{i=1}^{\infty} A_i$ where $A_i \in \tau$, for each i .

Definition 2.15. [10] Let (X, τ) be an IFTS. Then (X, τ) is called an intuitionistic fuzzy Volterra space if $IFcl(\cap_{i=1}^n A_i) = 1_{\sim}$, where A_i 's are intuitionistic fuzzy dense and intuitionistic fuzzy G_{δ} sets in (X, τ) .

Definition 2.16. [11] Let (X, τ) be an IFTS. Then (X, τ) is called an intuitionistic fuzzy weakly Volterra space if $IFcl(\cap_{i=1}^{\infty} A_i) \neq 0_{\sim}$, where A_i 's are intuitionistic fuzzy dense and intuitionistic fuzzy G_{δ} sets in (X, τ) .

Definition 2.17. [11] An intuitionistic fuzzy set A in an intuitionistic fuzzy topological space (X, τ) is called an intuitionistic fuzzy first category set if $A = \cup_{i=1}^{\infty} A_i$, where A_i 's are intuitionistic fuzzy nowhere dense sets in (X, τ) . Any other intuitionistic fuzzy set in (X, τ) is said to be of second category.

Definition 2.18. [11] An IFTS (X, τ) is called an intuitionistic fuzzy first category space set if $1_{\sim} = \cup_{i=1}^{\infty} A_i$, where A_i 's are intuitionistic fuzzy nowhere dense sets in (X, τ) . (X, τ) is called an intuitionistic fuzzy second category space if it is not an intuitionistic fuzzy first category space.

Definition 2.19. [11] An IFTS (X, τ) is called an intuitionistic fuzzy P -space if countable intersection of intuitionistic fuzzy open sets in (X, τ) is intuitionistic fuzzy open in (X, τ) .

Definition 2.20. [11] An IFTS (X, τ) is called an intuitionistic fuzzy submaximal space if for each intuitionistic fuzzy set A in (X, τ) such that $IFcl(A) = 1_{\sim}$, then $A \in \tau$.

Definition 2.21. [11] An IFTS (X, τ) is called an intuitionistic fuzzy D -Baire space if every intuitionistic fuzzy first category set in (X, τ) is an intuitionistic fuzzy nowhere dense in (X, τ) .

3. Intuitionistic fuzzy Volterra space

Definition 3.1. Let A be an intuitionistic fuzzy first category set in intuitionistic fuzzy topological space (X, τ) . Then $1_{\sim} - A$ is called an intuitionistic fuzzy residual set in (X, τ) .

Definition 3.2. An IFTS (X, τ) is called an intuitionistic fuzzy ε_r -Volterra space if $IFcl(\cap_{i=1}^n A_i) = 1_{\sim}$, where A_i 's are intuitionistic fuzzy dense and intuitionistic fuzzy residual sets in (X, τ) .

Proposition 3.3. If the IFTS (X, τ) is an intuitionistic fuzzy ε_r -Volterra space, then

$IFint(\cup_{i=1}^n A_i) = 0_{\sim}$, where the IFS, A_i 's are intuitionistic fuzzy first category sets such that $IFint(A_i) = 0_{\sim}$ in (X, τ) .

Proof. Let A_i 's ($i=1, 2, \dots, n$) be intuitionistic fuzzy first category sets such that $IFint(A_i) = 0_{\sim}$ in (X, τ) . Then $(1_{\sim} - A_i)$'s are intuitionistic fuzzy residual sets such that $IFcl(1_{\sim} - A_i) = 1_{\sim}$ in (X, τ) . Since (X, τ) is an intuitionistic fuzzy ε_r -Volterra space, $IFcl(\cap_{i=1}^n (1_{\sim} - A_i)) = 1_{\sim}$. Then $IFcl(1_{\sim} - \cup_{i=1}^n A_i) = 1_{\sim}$ and hence $1_{\sim} - IFint(\cup_{i=1}^n A_i) = 1_{\sim}$. Therefore, we have $IFint(\cup_{i=1}^n A_i) = 0_{\sim}$, where A_i 's are intuitionistic fuzzy first category sets such that $IFint(A_i) = 0_{\sim}$ in (X, τ) .

Proposition 3.4. Let (X, τ) be an intuitionistic fuzzy ε_r -Volterra space. Then (X, τ) is an intuitionistic fuzzy Volterra space.

Proof. Let A_i 's ($i=1, 2, \dots, n$) be intuitionistic fuzzy dense and intuitionistic fuzzy G_{δ} sets in (X, τ) . Then, A_i 's are intuitionistic fuzzy residual sets in (X, τ) . This implies that A_i 's are intuitionistic fuzzy dense and intuitionistic fuzzy residual sets in (X, τ) . Since (X, τ) is an intuitionistic

fuzzy ε_r -Volterra space, $\text{IFcl}(\bigcap_{i=1}^n A_i) = 1_-$. Hence (X, τ) is an intuitionistic fuzzy Volterra space.

Proposition 3.5. If each intuitionistic fuzzy nowhere dense set is an intuitionistic fuzzy closed set in an intuitionistic fuzzy Volterra space (X, τ) , then (X, τ) is an intuitionistic fuzzy ε_r -Volterra space.

Proof. Let A_i 's ($i=1,2,\dots,n$) be intuitionistic fuzzy dense and intuitionistic fuzzy residual sets in (X, τ) . Since A_i 's are intuitionistic fuzzy residual sets, $(1_- A_i)$'s are intuitionistic fuzzy first category sets in (X, τ) . Now $1_- A_i = \bigcup_{i=1}^\infty B_{ij}$, where B_{ij} 's are intuitionistic fuzzy nowhere dense sets in (X, τ) . From the hypothesis, the intuitionistic fuzzy nowhere dense sets B_{ij} 's are intuitionistic fuzzy closed sets and hence $(1_- A_i)$'s are intuitionistic fuzzy F_σ sets in (X, τ) . Therefore A_i 's are intuitionistic fuzzy G_δ sets in (X, τ) . Since (X, τ) is an intuitionistic fuzzy Volterra space, $\text{IFcl}(\bigcap_{i=1}^n A_i) = 1_-$. Hence (X, τ) is an intuitionistic fuzzy ε_r -Volterra space.

Definition 3.6. An IFTS (X, τ) is called an intuitionistic fuzzy nodec space if every non-zero intuitionistic fuzzy nowhere dense set A is intuitionistic fuzzy closed in (X, τ) . That is, if A is an intuitionistic fuzzy nowhere dense set in (X, τ) , then $1_- A \in \tau$.

Proposition 3.7. If the IFTS (X, τ) is an intuitionistic fuzzy Volterra space and intuitionistic fuzzy nodec space, then IFTS (X, τ) is an intuitionistic fuzzy ε_r -Volterra space.

Proof. Let (X, τ) be an intuitionistic fuzzy Volterra space and intuitionistic fuzzy nodec space and A_i 's ($i=1,2,\dots,n$) be intuitionistic fuzzy dense and intuitionistic fuzzy residual sets in (X, τ) . Since A_i 's are intuitionistic fuzzy residual sets, $(1_- A_i)$'s are intuitionistic fuzzy first category sets in (X, τ) . Now $1_- A_i = \bigcup_{i=1}^\infty B_{ij}$, where B_{ij} 's are intuitionistic fuzzy nowhere dense sets in (X, τ) . Since (X, τ) is an intuitionistic fuzzy nodec space, intuitionistic fuzzy nowhere dense sets B_{ij} 's are intuitionistic fuzzy closed in (X, τ) . By the Proposition 3.5, (X, τ) is an intuitionistic fuzzy ε_r -Volterra space.

Proposition 3.8. If the IFTS (X, τ) is an intuitionistic fuzzy Volterra space and intuitionistic fuzzy submaximal space, then (X, τ) is an intuitionistic fuzzy ε_r -Volterra space.

Proof. The proof is similar to Proposition 3.7.

Proposition 3.9. If the IFTS (X, τ) is an intuitionistic fuzzy ε_r -Volterra space and intuitionistic fuzzy D-Baire space, then $\text{IFint}(\bigcup_{i=1}^n A_i) = 0_-$, where the IFS, A_i 's are intuitionistic fuzzy first category sets in (X, τ) .

Proof. Let (X, τ) be an intuitionistic fuzzy ε_r -Volterra space and intuitionistic fuzzy D-Baire space and A_i 's ($i=1,2,3,\dots,n$) are intuitionistic fuzzy first category sets in (X, τ) . Since (X, τ) is an intuitionistic fuzzy D-Baire space, the intuitionistic fuzzy first category sets A_i 's are intuitionistic fuzzy nowhere dense sets in (X, τ) and hence $(1_- A_i)$'s are intuitionistic fuzzy dense sets in (X, τ) . Then $(1_- A_i)$'s are intuitionistic fuzzy dense and intuitionistic fuzzy residual sets in (X, τ) . By the hypothesis, $\text{IFcl}(\bigcap_{i=1}^n (1_- A_i)) = 1_-$. This implies that $\text{IFcl}(1_- \bigcup_{i=1}^n A_i) = 1_- - \text{IFint}(\bigcup_{i=1}^n A_i) = 1_-$. Therefore $\text{IFint}(\bigcup_{i=1}^n A_i) = 0_-$, where the IFS, A_i 's are intuitionistic fuzzy first category sets in (X, τ) .

Definition 3.10. Let (X, τ) be an IFTS. Then (X, τ) is called an intuitionistic fuzzy Baire space if $\text{IFint}(\bigcup_{i=1}^\infty A_i) = 0_-$, where A_i 's are intuitionistic fuzzy nowhere dense sets in (X, τ) .

Proposition 3.11. If $\bigcup_{i=1}^\infty A_i$, where the IFS, A_i 's are intuitionistic fuzzy nowhere dense sets, is an intuitionistic fuzzy nowhere dense set in an intuitionistic fuzzy Baire space (X, τ) , then (X, τ) is an intuitionistic fuzzy ε_r -Volterra space.

Proof. The proof is similar to Proposition 3.7.

Proposition 3.12. If each intuitionistic fuzzy first category set is an intuitionistic fuzzy closed set in an intuitionistic fuzzy Baire space (X, τ) , then (X, τ) is an intuitionistic fuzzy ε_r -Volterra space.

Proof. Let (X, τ) be an intuitionistic fuzzy Baire space and A_i 's ($i=1,2,3,\dots,n$) are intuitionistic fuzzy dense and intuitionistic fuzzy residual sets in (X, τ) . Since A_i 's are intuitionistic fuzzy residual sets, $(1_- A_i)$'s are intuitionistic fuzzy first category sets in (X, τ) . By the hypothesis, the intuitionistic fuzzy first category sets $(1_- A_i)$'s are intuitionistic fuzzy closed sets in (X, τ) and hence A_i 's are intuitionistic fuzzy open sets in (X, τ) . Since A_i 's are intuitionistic fuzzy dense and intuitionistic fuzzy open sets in (X, τ) , $(1_- A_i)$'s are intuitionistic fuzzy nowhere dense sets in (X, τ) . Since (X, τ) is an intuitionistic fuzzy Baire space, $\text{IFint}(\bigcup_{i=1}^n (1_- A_i)) \subseteq \text{IFint}(\bigcup_{i=1}^\infty (1_- A_i)) = 0_-$. That is, $\text{IFint}(\bigcup_{i=1}^n (1_- A_i)) = 0_-$. Hence $\text{IFcl}(\bigcap_{i=1}^n A_i) = 1_-$, where A_i 's are intuitionistic fuzzy dense and intuitionistic fuzzy residual sets in (X, τ) . Therefore (X, τ) is an intuitionistic fuzzy ε_r -Volterra space.

Definition 3.13. An IFTS (X, τ) is called an intuitionistic fuzzy ε_p -Volterra space if $\text{IFcl}(\bigcap_{i=1}^n A_i) = 1_-$, where the IFS, A_i 's are intuitionistic fuzzy pre-open and intuitionistic fuzzy G_δ sets in (X, τ) .

Proposition 3.14. If the IFTS (X, τ) is an intuitionistic fuzzy ε_p -Volterra space, then $\text{IFint}(\bigcup_{i=1}^n A_i) = 0_-$, where the IFS, A_i 's are intuitionistic fuzzy pre-closed and an intuitionistic fuzzy F_σ sets in (X, τ) .

Proof. Let A_i 's ($i=1,2,3,\dots,n$) be intuitionistic fuzzy pre-closed and intuitionistic fuzzy F_σ sets in (X, τ) . Then $(1_- A_i)$'s are intuitionistic fuzzy pre-open and intuitionistic fuzzy G_δ sets in (X, τ) . By the hypothesis, $\text{IFcl}(\bigcap_{i=1}^n (1_- A_i)) = 1_-$. Then $\text{IFcl}(1_- \bigcup_{i=1}^n A_i) = 1_- - \text{IFint}(\bigcup_{i=1}^n A_i) = 1_-$. Therefore, we have $\text{IFint}(\bigcup_{i=1}^n A_i) = 0_-$ where A_i 's are intuitionistic fuzzy pre-closed and intuitionistic fuzzy F_σ sets in (X, τ) .

Proposition 3.15. If the IFTS (X, τ) is an intuitionistic fuzzy ε_p -Volterra space, then (X, τ) is an intuitionistic fuzzy Volterra space.

Proof. Let A_i 's ($i=1,2,3,\dots,n$) be intuitionistic fuzzy dense and intuitionistic fuzzy G_δ sets in (X, τ) . Since A_i 's are intuitionistic fuzzy dense sets, $\text{IFcl}(A_i) = 1_-$. Now $\text{IFint}(\text{IFcl}(A_i)) = 1_-$. Then $A_i \subseteq \text{IFint}(\text{IFcl}(A_i))$. Since (X, τ) is an intuitionistic fuzzy ε_p -Volterra space and A_i 's are intuitionistic fuzzy pre-open and intuitionistic fuzzy G_δ sets in (X, τ) , $\text{IFcl}(\bigcap_{i=1}^n A_i) = 1_-$, where A_i 's are intuitionistic fuzzy dense and intuitionistic fuzzy G_δ sets in (X, τ) . Hence (X, τ) is an intuitionistic fuzzy Volterra space.

Proposition 3.16. If $IFpreint(\bigcup_{i=1}^n A_i) = 0_*$, where the IFS, A_i 's are intuitionistic fuzzy pre-closed sets in an intuitionistic fuzzy topological space (X, τ) , then (X, τ) is an intuitionistic fuzzy ε_p -Volterra space.

Proof. Let A_i 's ($i=1,2,3,\dots,n$) be intuitionistic fuzzy pre-open and intuitionistic fuzzy G_δ sets in (X, τ) . Then $(1_+ - A_i)$'s are intuitionistic fuzzy pre-closed in (X, τ) . By the hypothesis, $IFpreint(\bigcup_{i=1}^n (1_+ - A_i)) = 0_*$.

This implies that $1_+ - IFprecl(\bigcap_{i=1}^n A_i) = 0_*$ and hence $IFprecl(\bigcap_{i=1}^n A_i) = 1_+$. Since $IFprecl(\bigcap_{i=1}^n A_i) \subseteq IFcl(\bigcap_{i=1}^n A_i)$, we have $IFcl(\bigcap_{i=1}^n A_i) = 1_+$. Therefore $IFcl(\bigcap_{i=1}^n A_i) = 1_+$ where A_i 's are intuitionistic fuzzy pre-open and intuitionistic fuzzy G_δ sets in (X, τ) . Hence (X, τ) is an intuitionistic fuzzy ε_p -Volterra space.

Proposition 3.17. If $IFprecl(\bigcap_{i=1}^n A_i) = 1_+$, where the IFS, A_i 's are intuitionistic fuzzy pre-open sets in an intuitionistic fuzzy topological space (X, τ) , then (X, τ) is an intuitionistic fuzzy ε_p -Volterra space.

Proof. The proof is similar to Proposition 3.16.

Proposition 3.18. If $A = \bigcup_{i=1}^n A_i$, where A_i 's are intuitionistic fuzzy pre-closed sets, is an intuitionistic fuzzy pre-nowhere dense set in an intuitionistic fuzzy topological space (X, τ) , then (X, τ) is an intuitionistic fuzzy ε_p -Volterra space.

Proof. Suppose that $A = \bigcup_{i=1}^n A_i$, where A_i 's are intuitionistic fuzzy pre-closed sets and A is an intuitionistic fuzzy nowhere dense sets in (X, τ) . Then $IFpreint(IFprecl(A)) = 0_*$.

Since $IFpreint(A) \subseteq IFpreint(IFprecl(A))$, we have $IFpreint(A) = 0_*$. Then $IFpreint(\bigcup_{i=1}^n A_i) = 0_*$, where A_i 's are intuitionistic fuzzy pre-closed sets in (X, τ) . Hence by proposition 3.6, (X, τ) is an intuitionistic fuzzy ε_p -Volterra space.

Proposition 3.19. If each intuitionistic fuzzy pre-closed set is an intuitionistic fuzzy nowhere dense sets in intuitionistic fuzzy ε_r -Volterra space (X, τ) , then (X, τ) is an intuitionistic fuzzy ε_p -Volterra space.

Proof. It is clear from the Definition 3.2 and Definition 3.13.

Proposition 3.20. If an intuitionistic fuzzy ε_p -Volterra space (X, τ) is an intuitionistic fuzzy submaximal space, then (X, τ) is an intuitionistic fuzzy ε_r -Volterra space.

Proof. Immediate from the definitions.

Proposition 3.21. If each intuitionistic fuzzy nowhere dense set is an intuitionistic fuzzy closed set in an intuitionistic fuzzy ε_p -Volterra space (X, τ) , then (X, τ) is an intuitionistic fuzzy ε_r -Volterra space.

Proof. It is clear from the definitions.

Proposition 3.22. If an intuitionistic fuzzy ε_p -Volterra space (X, τ) is an intuitionistic fuzzy nodec space, then (X, τ) is an intuitionistic fuzzy ε_r -Volterra space.

Proof. Let (X, τ) be an intuitionistic fuzzy ε_p -Volterra space and an intuitionistic fuzzy nodec space. Since (X, τ) is an intuitionistic fuzzy nodec space, each intuitionistic fuzzy nowhere dense set is an intuitionistic fuzzy closed set in (X, τ) . By Proposition 3.21, (X, τ) is an intuitionistic fuzzy ε_r -Volterra space.

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