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Bipolar vague soft topological space

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Abstract
The focus of this paper is to explore the notion of soft vague set in bipolar setting. Further we characterize the properties of this new set namely bipolar vague soft sets.

Keywords: Soft set, vague set, vague soft set, bipolar vague set, bipolar vague soft set and bipolar vague soft topological space

1. Introduction

In dealing with uncertainties many theories have been recently developed, including the theory of probability, theory of fuzzy sets, theory of intuitionistic fuzzy sets and theory of rough sets and so on. Although many new techniques have been developed as a result of these theories, yet difficulties are still there. The major difficulties arise due to inadequacy of parameters.

In 1999, Molodtsov [13], initiated the novel concept of soft set theory, which was a completely new approach for modelling uncertainty and had a rich potential for application in several directions. This so called soft set theory is free from the difficulties affecting existing methods. The fuzzy set was introduced by Zadeh [18] in 1965 where each element had a degree of membership. Roy and Maji [10] provided some results on an application of fuzzy soft sets in decision making problem. The theory of vague sets was first proposed by Gau and Buehrer [8]. A vague set is defined by a truth-membership functiontv and false membership function f_v where $t_v(x)$ is a lower bound on the grade of membership of x derived from the evidence for x. And $f_v(x)$ is a lower bound and the negation of x derived from the evidence against x. The value of $t_v(x)$ and $f_v(x)$ are both defined on the closed interval $[0, 1]$ with each point in a basic X, where $t_v(x) + f_v(x) \le 1$. Vague set theory is actually an extension of fuzzy set theory and vague sets are regarded as a special case of context-dependent fuzzy sets. Bipolar valued fuzzy set is another extension of fuzzy set. In 2000, Lee [9] initiated an extension of fuzzy set named bipolar valued fuzzy set. He gave two kinds of representation of the notion of bipolar valued fuzzy sets. In case of bipolar valued fuzzy sets membership degree ranges enlarge from the interval $[1, 0]$ to $[0, 1]$. Muhammad Aslam [15] had combined the fuzzy soft set with bipolar and introduced a new mathematical model 'bipolar fuzzy soft sets'

In the present study, we combine the concept of vague soft set and soft set. We introduce the notion of bipolar vague soft set and study their fundamental properties. We study basic operation of bipolar vague soft set and derive some of their topological properties. Here we recall the definitions that are prerequisite for this paper.

2. Preliminaries

Vague soft sets

Let U be a universe, E a set of parameters, V(U) the power set of vague set on U, and $A \subseteq E$. The concept of a vague soft set is given by the following definition.

Definition 2.1 [17]: A pair $(\hat{F}, A)$ is called a vague soft set over U, where $\hat{F}$ is a mapping given by

$\hat{F}: A \to V(U)$.

In other words, a vague soft set over U is a parametrized family of vague set of the universe U. For $\varepsilon \in A$, $\mu_{\hat{F}(\varepsilon)}: U \to [0,1]$ is regarded as the set of ε - approximate elements of the vague soft sets .

Definition 2.2 [17]**:** If $(\hat{F}, A)$ and $(\hat{G}, B)$ are two vague soft sets over U, "$(\hat{F}, A)$ and $(\hat{G}, B)$" denoted by $(\hat{F}, A) \wedge (\hat{G}, B)$ isdefined by

$$(\hat{F}, A) \wedge (\hat{G}, B) = (\hat{H}, A \times B), where\ t_{\hat{H}(\alpha,\beta)}(x) = \min\{t_{\hat{F}(\alpha)}(x), t_{\hat{G}(\beta)}(x)\},\ 1 - f_{\hat{H}(\alpha,\beta)}(x)$$
$$= \min\{1 - f_{\hat{F}(\alpha)}(x), 1 - f_{\hat{G}(\beta)}(x)\}, \forall (\alpha, \beta) \in A \times B, x \in U.$$

Definition 2.3 [6]**:** If $(\hat{F}, A)$ and $(\hat{G}, B)$ are two vague soft sets over U, "$(\hat{F}, A)$ and $(\hat{G}, B)$" denoted by $(\hat{F}, A) \vee (\hat{G}, B)$ is defined by

$$(\hat{F}, A) \vee (\hat{G}, B) = (\hat{O}, A \times B), where\ t_{\hat{O}(\alpha,\beta)}(x) = \max\{t_{\hat{F}(\alpha)}(x), t_{\hat{G}(\beta)}(x)\},\ 1 - f_{\hat{O}(\alpha,\beta)}(x)$$
$$= \max\{1 - f_{\hat{F}(\alpha)}(x), 1 - f_{\hat{G}(\beta)}(x)\}, \forall (\alpha, \beta) \in A \times B, x \in U.$$

Definition 2.4 [6]**:** The union of two vague soft sets of $(\hat{F}, A)$ and $(\hat{G}, B)$ over the universe U is a vague soft set $(\hat{H}, C)$, where $C = A \cup B$ and $\forall\ e \in C$

$$t_{\hat{H}(e)} = \begin{cases} t_{\hat{F}(e)}(x)\ if\ \ e \in A - B, x \in U \\ t_{\hat{G}(e)}(x)\ if\ \ e \in B - A, x \in U \\ \max(t_{\hat{F}(e)}(x), t_{\hat{G}(e)}(x))\ if\ e \in A \cap B, x \in U \end{cases}$$

$$1 - f_{\hat{H}(e)} = \begin{cases} 1 - f_{\hat{F}(e)}(x)\ if\ \ e \in A - B, x \in U \\ 1 - f_{\hat{G}(e)}(x)\ if\ \ e \in B - A, x \in U \\ \max(1 - f_{\hat{F}(e)}(x), 1 - f_{\hat{G}(e)}(x))\ if\ e \in A \cap B, x \in U \end{cases}$$

Definition 2.5 [6]**:** The intersection of two vague soft sets of $(\hat{F}, A)$ and $(\hat{G}, B)$ over the universe U is a vague soft set $(\hat{H}, C)$, where $C = A \cap B$ and $\forall\ e \in C$

$$t_{\hat{H}(e)} = \begin{cases} t_{\hat{F}(e)}(x)\ if\ \ e \in A - B, x \in U \\ t_{\hat{G}(e)}(x)\ if\ \ e \in B - A, x \in U \\ \min(t_{\hat{F}(e)}(x), t_{\hat{G}(e)}(x))\ if\ e \in A \cap B, x \in U \end{cases}$$

$$1 - f_{\hat{H}(e)} = \begin{cases} 1 - f_{\hat{F}(e)}(x)\ if\ \ e \in A - B, x \in U \\ 1 - f_{\hat{G}(e)}(x)\ if\ \ e \in B - A, x \in U \\ \min(1 - f_{\hat{F}(e)}(x), 1 - f_{\hat{G}(e)}(x))\ if\ e \in A \cap B, x \in U \end{cases}$$

3. Bipolar vague soft sets

Definition 3.1: A bipolar vague set A in a universe U is an object having the form, $A = \{(x, v_A^+(x), v_A^-(x)): x \in U\}$ where $v_A^+(x) = [t_A^+(x)\ \ 1 - f_A^+(x)], v_A^-(x) = [t_A^-(x)\ \ -1 - f_A^-(x)]$ and $v_A^+(x): U \to [0,1], v_A^-(x): U \to [-1,0]$. So v_A^+ denote for positive information and v_A^- denote for negative information.

Definition 3.2: Let U be an universe, E a set of parameters and A$\subset E$. Define $F: A \to BV^U$, $where\ BV^U$ is the collection of all bipolar vague subsets of U. Then (F, A) is said to be a bipolar vague soft set over a universe U. It is defined by

$(F,A)=\{(x,v_e^+(x),v_e^-(x)): for\ all\ x\in U\ and\ e\in A\}$

Example 3.3: Let $U=\{c_1,c_2,c_3,c_4\}$ be a set of four cars under consideration and $E=\{e_1=\text{costly}, e_2=Beautiful, e_3=fuel\,efficient, e_4=mileage\}$ be the set of parameters and $A=\{e_1,e_2,e_3\}\subseteq E.$
Then,

$$(F,A)=\left\{\begin{array}{l} F(e_1)=\left\{\begin{array}{l}(c_1,[0.2,0.8][-0.2,-0.4]),(c_2,[0.3,0.5][-0.4,-0.6]),\\ (c_3,[0.4,0.6][-0.3,-0.6],(c_4,[0.5,0.5][-0.4,-0.5])\end{array}\right\},\\ F(e_2)=\left\{\begin{array}{l}(c_1,[0.3,0.7][-0.1,-0.5]),(c_2,[0.3,0.7][-0.3,-0.8]),\\ (c_3,[0.6,0.7][-0.2,-0.5],(c_4,[0.6,0.8][-0.4,-0.9])\end{array}\right\},\\ F(e_3)=\left\{\begin{array}{l}(c_1,[0.1,0.9][-0.5,-0.7]),(c_2,[0.2,0.8][-0.4,-0.9]),\\ (c_3,[0.5,0.5][-0.3,-0.7],(c_4,[0.5,0.5][-0.2,-0.5])\end{array}\right\}\end{array}\right\}$$

Definition 3.4: A Bipolar vague soft set A is contained in another bipolar vague soft set B, (i.e) A$\subseteq$B if $\forall x\in X, v_A^+(x)\le v_B^+(x),\ v_A^-(x)\ge v_B^-(x)$ that is
$t_A^+(x)\le t_B^+(x), 1-f_A^+(x)\le 1-f_B^+(x)\ and\ t_A^-(x)\ge t_B^+(x), -1-f_A^-(x)\ge -1-f_B^-(x)$.

Definition 3.5: Let X be a non-empty set, and $A=\{(x,v_A^+(x),v_A^-(x))\},\ B=\{(x,v_B^+(x),v_B^-(x))\}$ are bipolar vague soft sets. Then

$$A\,\tilde{\cup}\,B=(x,\max(v_A^+(x),v_B^+(x)),\min(v_A^-(x),v_B^-(x)))$$

$$A\,\tilde{\cap}\,B=(x,\min(v_A^+(x),v_B^+(x)),\max(v_A^-(x),v_B^-(x)))\ where$$

$$\max(v_A^+(x),v_B^+(x))=\{[\max([t_A^+(x),t_B^+(x)]\max([1-f_A^+(x),1-f_B^+(x)])\min([t_A^-(x),t_B^-(x)],$$
$$\min([-1-f_A^-(x,-1-f_B^-(x)])]\}$$

Definition 3.6: A bipolar vague soft set (F,A) over the universe U is said to be empty bipolar vague soft set with respect to the parameters A if $v_A^+(x)=[0,0], v_A^-(x)=[0,0]$, (i,e)
$t_A^+(x)=0,\ 1-f_A^+(x)=0, and\ t_A^-(x)=0,\ -1-f_A^-(x)=0\ \ \forall x\in U,\ \forall e\in A.$ It is denote by $\tilde{0}_v$.

Definition 3.7: A bipolar vague soft set (F,A) over the universe U is said to be universe bipolar vague soft set with respect to the parameters A if $v_A^+(x)=[1,1], v_A^-(x)=[-1,-1]$, (i,e)
$t_A^+(x)=1,\ 1-f_A^+(x)=1, and\ t_A^-(x)=1,\ -1-f_A^-(x)=1\ \ \forall x\in U,\ \forall e\in A.$ It is denoted by $\tilde{1}_v$.

Definition 3.8

(i) F_E is called absolute bipolar vague soft set over U if $F(e)=\tilde{1}_v\ \ for\ any\ e\in E.$ We denote it by U_E.

(ii) F_E is called relative null bipolar vague soft set over U if $F(e)=\tilde{0}_v\ \ for\ any\ e\in E.$ We denote it by ϕ_E.

Obviously $\phi_E=U_E^C$ and $U_E=\phi_E^C$.

Definition 3.9: Let (F,A) and (G,A) be two bipolar vague soft sets over a common universe U. Then,

(1) $(F,A)\,\tilde{\wedge}\,(G,B)$ is a bipolar vague soft set defined by
$(F,A)\,\tilde{\wedge}\,(G,B)=(H,A\times B)\ \ where\ H(a,b)=F(a)\cap G(b)$ for all $(a,b)\in C=A\times B,$ where $\cap$ is the intersection operation of sets.

(2) $(F,A)\,\tilde{\vee}\,(G,B)$ is a bipolar vague soft set defined by
$(F,A)\,\tilde{\vee}\,(G,B)=(H,A\times B)\ \ where\ H(a,b)=F(a)\cup G(b)$ for all $(a,b)\in C=A\times B,$ where $\cup$ is the union operation of sets.

Proposition 3.10: Let (F,A) and (G,A) be BVSS in BVSS(U)$_A$. Then the following are true.

$i)\ (F,A)\tilde{\subseteq}(G,A)\ \ iff\ \ (F,A)\tilde{\cap}(G,A)=(F,A)$

$ii)\ (F,A)\tilde{\subseteq}(G,A)\ \ iff\ \ (F,A)\tilde{\cup}(G,A)=(G,A)$

Proof:(i)

Suppose that $(F,A)\tilde{\subseteq}(G,A)$ .Then $F(e)\subseteq G(e)$ for all $e\in A.$ Let $(F,A)\tilde{\cap}(G,A)=(H,A)$. Since $H(e)=F(e)\cap G(e)=F(e)$ for all $e\in A,$ by definition $(H,A)=(F,A).$ Supposethat $(F,A)\tilde{\cap}(G,A)=(F,A).$ Let $(F,A)\tilde{\cap}(G,A)=(H,A).$

Since $H(e)=F(e)\cap G(e)=F(e)$ for all $e\in A,$ we know that $F(e)\subseteq G(e)$ for all e$\in A$. Hence $(F,A)\tilde{\subseteq}(G,A).$

(ii) The proof is similar to (i)

Proposition 3.11: Let $(F,A),(G,A),(H,A),(S,A)\in BVSS(U)_A.$ Then the following are true

(i) If $(F,A)\tilde{\cap}(G,A)=\phi_A$ *then* $(F,A)\tilde{\subseteq}(G,A)^C$

(ii) If $(F,A)\tilde{\subseteq}(G,A)$ *and* $(G,A)\tilde{\subseteq}(H,A)$ *then* $(F,A)\tilde{\subseteq}(H,A)$

(iii) If $(F,A)\tilde{\subseteq}(G,A)$ *and* $(H,A)\tilde{\subseteq}(S,A)$ *then* $(F,A)\tilde{\cap}(H,A)\tilde{\subseteq}(G,A)\tilde{\cap}(S,A)$

(iv) $(F,A)\tilde{\subseteq}(G,A)$ *iff* $(G,A)^C\tilde{\subseteq}(F,A)^C$.

Proof:(i) Suppose that $(F,A)\tilde{\cap}(G,A)=\phi_A$. Then $F(e)\cap G(e)=\phi.$ So $F(e)\subseteq U\setminus G(e)=G^C(e)$ for all $e\in A.$

Therefore we have $(F,A)\tilde{\subseteq}(G,A)^C.$

Proof of (ii) and (iii) are obvious.

(iv) $(F,A)\tilde{\subseteq}(G,A)\Leftrightarrow F(e)\tilde{\subseteq}G(e)$ for all $e\in A$

$\Leftrightarrow (G(e))^C\subseteq(F(e))^C$ for all $e\in A$

$\Leftrightarrow G^C(e)\subseteq F^C(e)$ for all $e\in A$

$\Leftrightarrow (G,A)^C\tilde{\subseteq}(F,A)^C$

Definition 3.12

Let I be an arbitrary indexed set and $\{(F_i,A)\}_{i\in I}$ be a subfamily of BVSS$(U)_A$.

(i)The union of these BVSS is the BVSS(H,A) where $H(e)=\bigcup_{i\in I}F_i(e)$ for each $e\in A$. We write $\tilde{\bigcup}_{i\in I}(F_i,A)=(H,A)$

(ii)The intersection of these BVSS is the BVSS(M,A) where M(e)=$\bigcap_{i\in I}F_i(e)$ for each $e\in A$. We write $\tilde{\bigcap}_{i\in I}(F_i,A)=(M,A).$

Proposition 3.13

Let I be an arbitrary indexed set and $\{(F_i,A)\}_{i\in I}$ be a subfamily of BVSS$(U)_A$.

Then (i) $[\tilde{\bigcup}_{i\in I}(F_i,A)]^C=\tilde{\bigcap}_{i\in I}(F_i,A)^C$

(ii) $[\tilde{\bigcap}_{i\in I}(F_i,A)]^C=\tilde{\bigcup}_{i\in I}(F_i,A)^C$

Proof

(i) $[\tilde{\bigcup}_{i\in I}(F_i,A)]^C=(H,A)^C$, By definition $H^C(e)=U_E\setminus H(e)=U_E\setminus\bigcup_{i\in i}F_i(e)=\bigcap_{i\in I}(U_E\setminus F_i(e))$ for all $e\in A$. On the other hand, $[\tilde{\bigcap}_{i\in I}(F_i,A)]^C=(K,A)$. By definition, $K(e)=\bigcap_{i\in I}F_i^C(e)=\bigcap_{i\in i}(U-F_i(e))$ for all $e\in A$.

(ii) $[\tilde{\bigcap}_{i\in I}(F_i,A)]^C=(H,A)^C$, By definition, $H^C(e)=U_E\setminus H(e)=U_E\setminus\bigcap_{i\in I}F_i(e)=\bigcup_{i\in I}(U_E\setminus F_i(e))$ for all $e\in A$. On the other hand, $[\tilde{\bigcup}_{i\in I}(F_i,A)]^C=(K,A)$. By definition, $K(e)=\bigcup_{i\in I}F_i^C(e)=\bigcup_{i\in I}(U_E-F_i(e))$ for all $e\in A$.

Proposition 3.14

(i) $(F,A)\tilde{\bigcap}(\phi,B)=(\phi,A\bigcap B)$

(ii) $(F,A)\tilde{\bigcap}(U,B)=(F,A\bigcup B)$

Proof

(i) We have for (F,A)

$F(e) = \{(x, v_{F(e)}{}^{+}(x), v_{F(e)}{}^{-}(x)) : x \in U\} \forall e \in A$

Let $(\phi, B) = (G, B)$ Then, $G(e) = \{(x,[0,0],[0,0]) : x \in U\} \forall e \in B$

Let $(F, A) \tilde{\bigcap} (\phi, B) = (F, A) \tilde{\bigcap} (G, B) = (H, C)$ where $C = A \bigcap B and \forall e \in C$

$H(e) = \{(x, \min\{v_{F(e)}{}^{+}(x), v_{G(e)}{}^{+}\}, \max\{v_{F(e)}{}^{-}, v_{G(e)}{}^{-}(x)\}) : x \in U\}$

$= \{(x, \min\{v_{F(e)}{}^{+}(x), [0,0]\}, \max\{v_{F(e)}{}^{-}, [0,0]\}) : x \in U\}$

$= \{(x,[0,0],[0,0]) : x \in U\}$

$= (G, B) = (\phi, B)$

Thus $(F, A) \tilde{\bigcap} (\phi, B) = (\phi, B)$

(ii) Proof is similar to (i)

Proposition 3.15

a. $(\phi, A)^{C} = (U, A)$

b. $(U, A)^{c} = (\phi, A)$

Proof

i)Let $(\phi, A) = (F, A)$**,** then for all $e \in A$,

$F(e) = \{(x,[t^{+}_{F(e)}(x), 1 - f^{+}_{F(e)}(x)], [t^{-}_{F(e)}(x), -1 - f^{-}_{F(e)}(x)]) : x \in U\}$

$\{(x,[0,0],[0,0]) : x \in U\}$

$(\phi, A)^{C} = (F, A)^{C}$ then for all $e \in A$,

$(F(e))^{C} = \{(x,[t^{+}_{F(e)}(x), 1 - f^{+}_{F(e)}(x)], [t^{-}_{F(e)}(x), -1 - f^{-}_{F(e)}(x)]) : x \in U\}^{C}$

$= \{(x,[f^{+}_{F(e)}(x), 1 - t^{+}_{F(e)}(x)], [f^{-}_{F(e)}(x), -1 - t^{-}_{F(e)}(x)]) : x \in U\}$

$= \{(x,[1,1],[-1,-1]) : x \in U\} = U$

Thus

$(\phi, A)^{C} = (U, A)$.

ii) Proof is similar to (i).

Proposition 3.16: Let (F, A) be a bipolar vague soft set over a common universe U. Then,

$i) (F, A) \tilde{\bigcup} (\phi, A) = (F, A)$

$ii) (F, A) \tilde{\bigcup} (U, A) = (U, A)$

$iii) (F, A) \tilde{\bigcup} (F, A) = (F, A)$

Proof

i) $(F, A) = \{e, (x, v^{+}_{F(e)}(x), v^{-}_{F(e)}(x)) : x \in U\} \forall e \in A$

$(\phi, A) = \{e, (x,[0,0],[0,0]) : x \in U\} \forall e \in A$

$(F, A) \tilde{\bigcup} (\phi, A) = \{e, (x, \max(v^{+}_{F(e)}(x), [0,0]), \min(v^{-}_{F(e)}(x), [0,0])) : x \in U\} \forall e \in A$

$= \{e, (x, v^{+}_{F(e)}(x), v^{-}_{F(e)}(x)) : x \in U\} \forall e \in A$

$= (F, A)$

Proof (ii) and (iii) is similar to (i).

Proposition 3.17

i) $(F, A) \tilde{\bigcap} (\phi, A) = (\phi, A)$

ii) $\left(F,A\right)\tilde{\cap}\left(U,A\right)=\left(F,A\right)$

Proof

$$(F,A)=\left\{e,(x,v_A{}^{+}(x),v_A{}^{-}(x)):x\in U\right\}\forall e\in A.$$

$$(\phi,A)=\{e,(x,[0,0],[0,0]):x\in U\}\forall e\in A$$

$$\left(F,A\right)\tilde{\cap}\left(\phi,A\right)=\{e,(x,\min(v_A{}^{+}(x),[0,0]),\max(v_A{}^{-}(x),[0,0])$$

$$=\{e,(x,[0,0],[0,0]):x\in U\}\forall e\in A$$

$$=(\phi,A)$$

Thus,

$$(F,A)\tilde{\cap}(\phi,A)=(\phi,A).$$

(ii) Proof is similar to (i)

Proposition 3.18

(i) $(F,A)\tilde{\cup}(\phi,B)=(F,A)\ \ iff\ \ \ B\tilde{\subseteq}A$

(ii) $(F,A)\tilde{\cup}(U,B)=(U,A)\ \ iff\ \ \ A\tilde{\subseteq}B$

Proof

i)We have for (F,A)

$$F(e)=\{e,(x,v^{+}_{F(e)}(x),v^{-}_{F(e)}(x)):x\in U\}\ \forall e\in A$$

Also let $(\phi,B)=(G,B)$ then $G(e)=\{(x,[0,0],[0,0]):x\in U\}\forall\, e\in B$

Let $(F,A)\tilde{\cup}(\phi,B)=(F,A)\tilde{\cup}(G,B)=(H,C)\quad where\ \ C=A\cup B\ \ and\ \ \ \forall e\in C$

$$H(e)=\begin{cases}\{(x,\ v^{+}_{F(e)}(x),v^{-}_{F(e)}(x)):x\in U\}\ if\ \ \ e\in A-B\\ \{(x,\ v^{+}_{G(e)}(x),v^{-}_{G(e)}(x)):x\in U\}\ if\ \ \ e\in B-A\\ \{(x,\ \max(v^{+}_{F(e)}(x),v^{+}_{G(e)}(x)),\min(v^{-}_{F(e)}(x),v^{-}_{G(e)}(x))):x\in U\}\,if\ e\in A\cap B\end{cases}$$

$$=\begin{cases}\{(x,\ v^{+}_{F(e)}(x),v^{-}_{F(e)}(x)):x\in U\}\ if\ \ \ e\in A-B\\ \{(x,\ [0,0][0,0]):x\in U\}\ if\ \ \ e\in B-A\\ \{(x,\ \max(v^{+}_{F(e)}(x),[0,0]),\min(v^{-}_{F(e)}(x),[0,0])):x\in U\}\,if\ e\in A\cap B\end{cases}$$

$$=\begin{cases}\{(x,\ v^{+}_{F(e)}(x),v^{-}_{F(e)}(x)):x\in U\}\ if\ \ \ e\in A-B\\ \{(x,\ [0,0],[0,0]):x\in U\}\ if\ \ \ e\in B-A\\ \{(x,\ v^{+}_{F(e)}(x),v^{-}_{F(e)}(x)):x\in U\}\,if\ e\in A\cap B\end{cases}$$

Let $B\subseteq A$

Then H(e) $=\begin{cases}\{(x,\ v^{+}_{F(e)}(x),v^{-}_{F(e)}(x)):x\in U\}\ if\ \ \ e\in A-B\\ \{(x,\ v^{+}_{F(e)}(x),v^{-}_{F(e)}(x)):x\in U\}\,if\ e\in A\cap B\end{cases}$

$=F(e)\ \ \forall\ e\in A$

Conversely Let $(F,A)\tilde{\cup}(\phi,B)=(F,A)$

Then $A=A\cup B\Rightarrow B\subseteq A$.

(ii) Proof is similar to (i)

Proposition 3.19

$$i)((F,A)\tilde{\cup}(G,B))^{C}\ \tilde{\subseteq}\,(F,A)^{C}\ \tilde{\cup}\,(G,B)^{C}$$

$$ii)(F,A)^{C}\ \tilde{\cap}\,(G,B)^{C}\ \tilde{\subseteq}((F,A)\tilde{\cap}(G,B))^{C}$$

Proof:(i) Let $(F,A)\tilde{\cup}(G,B)=(H,C)$ *where* $C=A\cup B$ *and* $\forall e\in C$

$$H(e)=\begin{cases}\{(x,[t^{+}_{F(e)}(x),1-f^{+}_{F(e)}(x)],[t^{-}_{F(e)}(x),-1-f^{-}_{F(e)}(x)]):x\in U\}\ if\ \ e\in A-B\\ \{(x,[t^{+}_{G(e)}(x),1-f^{+}_{G(e)}(x)],[t^{-}_{G(e)}(x),-1-f^{-}_{G(e)}(x)]):x\in U\ if\ \ e\in B-A\\ \{(x,\ \max([t^{+}_{F(e)}(x),t^{+}_{G(e)}(x)]),\max([1-f^{+}_{F(e)}(x),1-f^{+}_{G(e)}(x)]),\min([t^{-}_{F(e)}(x),t^{-}_{G(e)}(x)])\\ \quad\min([-1-f^{-}_{F(e)}(x),-1-f^{-}_{G(e)}(x)])):x\in U\}\ \ if\ e\in A\cap B\end{cases}$$

Thus $((F,A)\tilde{\cup}(G,B))^{C}=(H,C)^{C}$ *where* $C=A\cup B$ *and* $\forall e\in C$

$$(H(e))^{C}=\begin{cases}(F(e))^{C}\ if\ \ e\in A-B\\ (G(e))^{C}\ if\ \ e\in B-A\\ ((F(e)\cup G(e))^{C}\ if\ e\in A\cap B\end{cases}$$

$$H(e)^{C}=\begin{cases}\{(x,[f^{+}_{F(e)}(x),1-t^{+}_{F(e)}(x)],[f^{-}_{F(e)}(x),-1-t^{-}_{F(e)}(x)]\}\ if\ \ e\in A-B\\ \{(x,[f^{+}_{G(e)}(x),1-t^{+}_{G(e)}(x)],[f^{-}_{G(e)}(x),-1-t^{-}_{G(e)}(x)])\}\ if\ \ e\in B-A\\ \{(x,\ \min([f^{+}_{F(e)}(x),f^{+}_{G(e)}(x)])\min([1-t^{+}_{F(e)}(x),1-t^{+}_{G(e)}(x)]),\max([f^{-}_{F(e)}(x),f^{-}_{G(e)}(x)])\\ \quad\max([-1-t^{-}_{F(e)}(x),-1-t^{-}_{G(e)}(x)]):x\in U\}\ if\ e\in A\cap B\end{cases}$$

Again $(F,A)^{C}\tilde{\cup}(G,B)^{C}=(I,J)$ where $J=A\cup B$ and $\forall e\in J$

$$I(e)=\begin{cases}(F(e))^{C}\ if\ \ e\in A-B\\ (G(e))^{C}\ if\ \ e\in B-A\\ ((F(e)\cup G(e))^{C}\ if\ e\in A\cap B\end{cases}$$

$$=\begin{cases}\{(x,[f^{+}_{F(e)}(x),1-t^{+}_{F(e)}(x)],[f^{-}_{F(e)}(x),-1-t^{-}_{F(e)}(x)]):x\in U\}\ if\ \ e\in A-B\\ \{(x,[f^{+}_{G(e)}(x),1-t^{+}_{G(e)}(x)],[f^{-}_{G(e)}(x),-1-t^{-}_{G(e)}(x)]):x\in U\ if\ \ e\in B-A\\ \{(x,\ \max([f^{+}_{F(e)}(x),f^{+}_{G(e)}(x)]),\max([1-t^{+}_{F(e)}(x),1-t^{+}_{G(e)}(x)]),\min([f^{-}_{F(e)}(x),f^{-}_{G(e)}(x)])\\ \quad\min([-1-t^{-}_{F(e)}(x),-1-t^{-}_{G(e)}(x)])):x\in U\}\ \ if\ e\in A\cap B\end{cases}$$

$C\subseteq J\ \ \forall\ e\in J,\ \ (H(e))^{C}\subseteq I(e)$

Thus $((F,A)\tilde{\cup}(G,B))^{C}\tilde{\subseteq}(F,A)^{C}\tilde{\cup}(G,B)^{C}$.

ii) Let $(F,A)\tilde{\cap}(G,B)=(H,C)$ where $C=A\cup B$ and $\forall e\in C$

$H(e)=F(e)\cap G(e)$

$$=\begin{Bmatrix}(x,\ \min([t^{+}_{F(e)}(x),t^{+}_{G(e)}(x)]),\min([1-f^{+}_{F(e)}(x),1-f^{+}_{G(e)}(x)]),\max([t^{-}_{F(e)}(x),t^{-}_{G(e)}(x)])\\ \max([-1-f^{-}_{F(e)}(x),-1-f^{-}_{G(e)}(x)]):x\in U\end{Bmatrix}$$

Thus $((F,A)\tilde{\cap}(G,B))^{C}=(H,C)^{C}$ where $C=A\cup B$ and $\forall e\in C$

$$(H(e))^{C}=\begin{Bmatrix}(x,\ \min([t^{+}_{F(e)}(x),t^{+}_{G(e)}(x)]),\min([1-f^{+}_{F(e)}(x),1-f^{+}_{G(e)}(x)]),\max([t^{-}_{F(e)}(x),t^{-}_{G(e)}(x)])\\ \max([-1-f^{-}_{F(e)}(x),-1-f^{-}_{G(e)}(x)]):x\in U\end{Bmatrix}^{C}$$

$$=\begin{Bmatrix}(x,\ \max([f^{+}_{F(e)}(x),f^{+}_{G(e)}(x)]),\max([1-t^{+}_{F(e)}(x),1-t^{+}_{G(e)}(x)]),\min([f^{-}_{F(e)}(x),f^{-}_{G(e)}(x)])\\ \min([-1-t^{-}_{F(e)}(x),-1-t^{-}_{G(e)}(x)])):x\in U\end{Bmatrix}$$

Again $(F,A)^{C}\tilde{\cap}(G,A)^{C}=(I,J)$ say where $J=A\cup B\ \forall e\in J$

$I(e)=(F(e))^{C}\tilde{\cap}(G(e))^{C}$

$$= \left\{ \begin{array}{l} (x,\ \min([\, f^{+}_{F(e)}(x),\ f^{+}_{G(e)}(x)]), \min[1 - t^{+}_{F(e)}(x), 1 - t^{+}_{G(e)}(x)]), \max([\, f^{-}_{F(e)}(x),\ f^{-}_{G(e)}(x)]) \\ \quad \max([-1 - t^{-}_{F(e)}(x), -1 - t^{-}_{G(e)}(x)])) : x \in U \end{array} \right\}$$

We see that $C = J$ and $\forall e \in J$, $I(e) \subseteq (H(e))^{C}$ e

Thus $(F,A)^{C} \tilde{\cap} (G,B)^{C} \tilde{\subseteq} ((F,A) \tilde{\cap} (G,B))^{C}$.

Proposition 3.20: De Morgan's law for bipolar vague soft sets (F,A) and (G,B)

(i) $((F,A) \tilde{\cup} (G,B))^{C} = (F,A)^{C} \tilde{\cap} (G,B)^{C}$

(ii) $((F,A) \tilde{\cap} (G,B))^{C} = (F,A)^{C} \tilde{\cup} (G,B)^{C}$

Proof: (i) Let $(F,A) \tilde{\cup} (G,A) = (H,A)$ where $\forall\ e \in A$

$$H(e) = F(e) \cup G(e)$$

$$= \{(x,\ \max([\, f^{+}_{F(e)}(x),\ f^{+}_{G(e)}(x)]), \max([1 - t^{+}_{F(e)}(x), 1 - t^{+}_{G(e)}(x)]), \min([\, f^{-}_{F(e)}(x),\ f^{-}_{G(e)}(x)])$$
$$\min([-1 - t^{-}_{F(e)}(x), -1 - t^{-}_{G(e)}(x)])) : x \in U\}$$

Thus $((F,A) \tilde{\cup} (G,A))^{C} = (H,A)^{C}$ where $\forall\ e \in A$

$$H(e)^{C} = (x,\ \min([\, f^{+}_{F(e)}(x),\ f^{+}_{G(e)}(x)]) \min([1 - t^{+}_{F(e)}(x), 1 - t^{+}_{G(e)}(x)]), \max([\, f^{-}_{F(e)}(x),\ f^{-}_{G(e)}(x)])$$
$$\max([-1 - t^{-}_{F(e)}(x), -1 - t^{-}_{G(e)}(x)]) : x \in U)$$

Again $(F,A)^{C} \tilde{\cap} (G,A)^{C} = (I,A) \quad where \quad \forall e \in A$

$$I(e) = (F(e))^{C} \tilde{\cap} (G(e))^{C}$$

$$= \left\{ \begin{array}{l} (x,\ \min([\, f^{+}_{F(e)}(x),\ f^{+}_{G(e)}(x)]), \min[1 - t^{+}_{F(e)}(x), 1 - t^{+}_{G(e)}(x)]), \max([\, f^{-}_{F(e)}(x),\ f^{-}_{G(e)}(x)]) \\ \quad \max([-1 - t^{-}_{F(e)}(x), -1 - t^{-}_{G(e)}(x)])) : x \in U \end{array} \right\}$$

Thus $((F,A) \tilde{\cup} (G,A))^{C} = (F,A)^{C} \tilde{\cap} (G,A)^{C}$.

Proof (ii) is similar to (i)

Proposition 3.21: Idempotent property. If (F,A), (G,B) are two bipolar vague soft sets over U. Then,

(1) $(F,A) \tilde{\wedge} (F,A) = (F,A)$

(2) $(F,A) \tilde{\vee} (F,A) = (F,A)$

Proof: (1) $(F,A) \tilde{\wedge} (F,A) = (F,A)$

Suppose that $(F,A) \tilde{\wedge} (F,A) = (H,C), \quad where \quad C = A \times A$

$H(a,a) = F(a) \cap F(a)$,Since $F(a) \cap F(a) = F(a)$

$= F(a)$

$(H,C) = (F,A)$

$(F,A) \tilde{\wedge} (F,A) = (F,A)$

Proof (2) is similar to (1)

4. Bipolar vague soft topological spaces

Definition 4.1: Let (F_A, E) be a BVS set on (U,E) and τ be a collection of bipolar vague soft subsets of (F_A, E),

$(i)\ \phi_E, U_E \in \tau$

$(ii)\ F_E,\ G_E \in \tau\ implies\ F_E \tilde{\cap} G_E \in \tau$

$(iii)\ \{(F_\alpha)_E : \alpha \in \Gamma\} \subseteq \tau\ implies\ \tilde{\cup}\{(F_\alpha) : \alpha \in \Gamma\} \in \tau$

The triplet (U, τ, E) is called a bipolar vague soft topological space (BVSTS) over U.

Every member of τ is called an bipolar vague soft open set in U.

F_E is called a bipolar vague soft closed set in U if $F_E \in \tau^C$ where $\tau^C = \{F_E^C : F_E \in \tau\}$

Example 4.2: Let $U = \{a_1, a_2, a_3\}$ and $E = \{e_1, e_2\}$. Let F_E, G_E, H_E be bipolar vague soft set where

$F(e_1) = \{< a_1,[0.4,0.6][-0.2,-0.3] >, < a_2,[0.2,0.4][-0.1,-0.4] >, < a_3,[0.1,0.4][-0.2,-0.4] >\}$

$F(e_2) = \{< a_1,[0.2,0.4][-0.3,-0.4] >, < a_2,[0.4,0.6][-0.2,-0.5] >, < a_3,[0.6,0.7][-0.4,-0.5] >\}$

$G(e_1) = \{< a_1,[0.2,0.3][-0.4,-0.6] >, < a_2,[0.1,0.4][-0.2,-0.4] >, < a_3,[0.2,0.6][-0.2,-0.3] >\}$

$G(e_2) = \{< a_1,[0.4,0.6][-0.2,-0.4] >, < a_2,[0.4,0.7][-0.4,-0.6] >, < a_3,[0.4,0.6][-0.2,-0.4] >\}$

$H(e_1) = \{< a_1,[0.4,0.6][-0.4,-0.6] >, < a_2,[0.2,0.4][-0.2,-0.4] >, < a_3,[0.2,0.6][-0.2,-0.4] >\}$

$H(e_2) = \{< a_1,[0.4,0.6][-0.3,-0.4] >, < a_2,[0.4,0.7][-0.4,-0.6] >, < a_3,[0.6,0.7][-0.4,-0.5] >\}$

$L(e_1) = \{< a_1,[0.2,0.3][-0.2,-0.3] >, < a_2,[0.1,0.4][-0.1,-0.4] >, < a_3,[0.2,0.6][-0.2,-0.4] >\}$

$L(e_2) = \{< a_1,[0.2,0.4][-0.2,-0.4] >, < a_2,[0.4,0.6][-0.2,-0.5] >, < a_3,[0.4,0.6][-0.2,-0.4] >\}$

$\tau = \{F_E, G_E, H_E, L_E, \phi_E, U_E\}$ is a Bipolar vague soft topology on U.

Proposition 4.3: Let (U, τ_1, E) and (U, τ_2, E) be two bipolar vague soft topological spaces. Define $\tau_1 \cap \tau_2 = \{F_E : F_E \in \tau_1 \;\; and \;\; F_E \in \tau_2\}$. Then $\tau_1 \cap \tau_2$ is an BVST on U.

Proof: Obviously $\phi_E, U_E \in \tau_1 \cap \tau_2$. Let $F_E, G_E \in \tau_1 \cap \tau_2$. Then $F_E, G_E \in \tau_1$ and $F_E, G_E \in \tau_2$. τ_1 and τ_2 are two BVST's on U. Then $F_E \cap G_E \in \tau_1$ and $F_E \cap G_E \in \tau_2$. Hence $F_E \cap G_E \in \tau_1 \cap \tau_2$. Let $\{(F_\alpha)_E : \alpha \in \tau\} \subseteq \tau_1 \cap \tau_2$. Then $(F_\alpha)_E \in \tau_1$ and $(F_\alpha)_E \in \tau_2$ for any $\alpha \in \Gamma$. Since τ_1 and τ_2 are two BVST's on U, $\tilde{\cup}\{(F_\alpha)_E : \alpha \in \Gamma\} \in \tau_1$ and $\tilde{\cup}\{(F_\alpha)_E : \alpha \in \Gamma\} \in \tau_2$. Thus $\tilde{\cup}\{(F_\alpha)_E : \alpha \in \Gamma\} \in \tau_1 \cap \tau_2$.

5 properties of bipolar vague soft topological spaces

Definition 5.1: Let (U, τ, E) be a BVSTS and let $F_E \in BVSS(U)_E$. Then interior and closure of F_E denoted respectively by BVSInt(F_E) and BVSCl (F_E) are defined as follows.

BVSInt(F_E)= $\tilde{\cup}\{G_E \in \tau : G_E \tilde{\subseteq} F_E\}$.

BVSCl(F_E)= $\tilde{\cup}\{G_E \in \tau^C : F_E \tilde{\subseteq} G_E\}$.

Example 5.2: we consider the BVST given in example 4.2. Let

$M(e_1) = \{< a_1,[0.5,0.7][-0.3,-0.4] >, < a_2,[0.3,0.5][-0.2,-0.6] >, < a_3,[0.2,0.5][-0.3,-0.5] >\}$

$M(e_2) = \{< a_1,[0.4,0.6][-0.4,-0.6] >, < a_2,[0.5,0.6][-0.3,-0.4] >, < a_3,[0.7,0.8][-0.5,-0.6] >\}$

BVSInt(M_E)=F_E.

Theorem 5.3:

Let (U, τ, E) be a BVSS over U. Then the following properties hold.

(i) U_E and ϕ_E are BVS closed set over U.

(ii) The intersection of any number of BVS closed sets is a BVS closed set over U.

(iii) The union of any two BVS closed sets is an BVS closed set over U.

Proof: It is obvious.

Theorem 5.4: Let (U, τ, E) be a BVSS and let $F_E \in BVSS(U)_E$. Then the following properties hold.

(i) $BVSInt(F_E) \tilde{\subseteq} F_E$

(ii) $F_E \tilde{\subseteq} G_E \Rightarrow BVSInt(F_E) \tilde{\subseteq} BVSInt(G_E)$

(iii) $BVSInt(F_E) \in \tau$

(iv) F_E is a BVS open set $\Leftrightarrow$ $BVSInt(F_E) = F_E$

(v) $BVSInt(BVSInt(F_E)) = BVSInt(F_E)$

(vi) $BVSInt(\phi_E) = \phi_E$, $BVSInt(U_E) = U_E$

Proof

(i) and (ii) are obvious.

(iii) Obviously $\tilde{\bigcup}\{G_E \in \tau : G_E \tilde{\subseteq} F_E\} \in \tau$

Note that $\tilde{\bigcup}\{G_E \in \tau : G_E \tilde{\subseteq} F_E\} = BVSInt(F_E) \Rightarrow BVSInt(F_E) \in \tau$

(iv) Necessity: Let F_E be a BVS open set,ie., $F_E \in \tau$ .By (i) and (ii) $BVSInt(F_E) \tilde{\subseteq} F_E$.

Since $F_E \in \tau$ and $F_E \tilde{\subseteq} F_E$

Then $F_E \tilde{\subseteq} \tilde{\bigcup}\{G_E \in \tau : G_E \tilde{\subseteq} F_E\} = BVSInt(F_E)$.

ie., $F_E \tilde{\subseteq} BVSInt(F_E)$. Thus $BVSInt = F_E$

Sufficency: Let $BVSInt(F_E) = F_E$

By (iii) $BVSInt(F_E) \in \tau$, ie., F_E is a BVS open set.

(v)To prove $BVSInt(BVSInt(F_E)) = BVSInt(F_E)$, By (iii) $BVSInt(F_E) \in \tau$

By (iv) $BVSInt(BVSInt(F_E)) = BVSInt(F_E)$

(vi)We know that U_E and $\phi_E \in \tau$

By (iv) $BVSInt(\phi_E) = \phi_E,$ $BVSInt(U_E) = U_E$

Theorem 5.5

Let (U, τ, E) be a BVSS and Let $F_E \in BVSS(U)_E$. Then the following properties hold.

(i) $(F_E) \tilde{\subseteq} BVSCl(F)_E$

(ii) $F_E \tilde{\subseteq} G_E \Rightarrow BVSCl(F)_E \tilde{\subseteq} BVSCl(G_E)$.

(iii) $(BVSCl(F)_E)^C \in \tau$.

(iv) F_E is a BVS Closed set $\Leftrightarrow$ $BVSCl(F)_E = F_E$.

(v) $BVSCl(BVSCl(F_E) = BVSCl(F_E)$.

(vi) $BVSS(\phi_E) = \phi_E, BVSCl(U_E) = U_E$.

Proof

(i) and (ii) are obvious.

(iii) By theorem 5.4. (iii) $BVSInt(F^C{}_E) \in \tau$.

Therefore $[(BVSCl(F_E)]^C = (\bigcap\{G_E \in \tau^C : F_E \tilde{\subseteq} G_E\})^C$

$= \tilde{\bigcup}\{G_E \in \tau : G_E \tilde{\subseteq} F^C{}_E\}$

$= BVSIntF_E{}^C$

Then [$(BVSCl(F_E)]^C \in \tau$.

(iv) Necessity: By theorem 5.5 (i) $F_E \tilde{\subseteq} BVSCl(F_E)$

Let F_E be a BVS closed set ie., $F_E \in \tau^C and F_E \tilde{\subseteq} F_E$.

$BVSCl(F_E) = \tilde{\bigcap}\{G_E \in \tau^C : F_E \tilde{\subseteq} G_E\} \tilde{\subseteq} \{F_E \in \tau^C : F_E \tilde{\subseteq} F_E\}$

Then $BVSCl(F_E) \tilde{\subseteq} F_E$. Thus $F_E = BVSCl(F_E)$.

Sufficiency: This holds by (iii).

(v) and (vi) hold by (iii) and (iv).

Theorem 5.6

Let (U, τ, E) be a BVST and Let $F_E, G_E \in BVSS(U)_E$. Then the following properties hold.

(i) $BVSInt(F_E) \tilde{\bigcap} BVSInt(F_E) = BVSInt(F_E \tilde{\bigcap} G_E)$

(ii) $BVSInt(F_E)\tilde{\bigcup} BVSInt(F_E) \tilde{\subseteq} BVSInt(F_E \tilde{\bigcup} G_E)$

(iii) $BVSCl(F_E)\tilde{\bigcup} BVSCl(G_E) = BVSCl(F_E \tilde{\bigcup} G_E)$

(iv) $BVSCl(F_E \tilde{\bigcup} G_E) \tilde{\subseteq} BVSCl(F_E)\tilde{\bigcap} BVSCl(G_E)$

(v) $(BVSInt(F_E))^C = BVSCl({F_E}^C)$

(vi) $(BVSCl(F_E))^C = BVSInt({F_E}^C)$

Proof

(i)Since $F(e)\tilde{\bigcap} G(e) \tilde{\subseteq} F(e)$ for any $e \in E,$

We have $F_E \tilde{\bigcap} G_E \tilde{\subseteq} F_E$ .By theorem 5.4 (ii) $BVSInt(F_E \tilde{\bigcap} G_E) \tilde{\subseteq} BVSInt(F_E)$

Similarly $BVSInt(F_E \tilde{\bigcap} G_E) \tilde{\subseteq} BVSInt(G_E).$

Thus $BVSInt(F_E \tilde{\bigcap} G_E) \tilde{\subseteq} BVSInt(F_E)\tilde{\bigcap} BVSInt(G_E)$

By theorem 5.4 (i), $BVSInt(F_E \tilde{\subseteq} F_E and BVSInt(G_E) \tilde{\subseteq} (G_E)$

Then $BVSInt(F_E \tilde{\bigcap} G_E) \tilde{\subseteq} F_E \tilde{\bigcap} G_E$

So $BVSInt(F_E)\tilde{\bigcap} BVSInt(F_E) \tilde{\subseteq} BVSInt(F_E \tilde{\bigcap} G_E).$

Similarly we prove (ii),(iii) and (iv).

(v) $(BVSInt(F_E))^C = \tilde{\bigcup}(\{G_E \in \tau : G_E \tilde{\subseteq} F_E\})^C = \tilde{\bigcap}\{G_E \in \tau^C : {F_E}^C \tilde{\subseteq} G_E\}$

$= BVSInt(F_E^C)$

(vi)The proof is similar to (v).

Example 5.7: Let $U = \{b_1, b_2,\}$ and $E = \{e_1, e_2\}$. Let F_E be bipolar vague soft set where

$F(e_1) = \{< b_1,[0.2,0.4][-0.3,-0.6] >, < b_2,[0.6,0.8][-0.2,-0.5] >\}$

$F(e_2) = \{< b_1,[0.3,0.6][-0.2,-0.4] >, < b_2,[0.4,0.5][-0.2,-0.4] >\}$

Obviously $\tau = \{F_E, \phi_E, U_E\}$ is an bipolar vague soft topology on U. G_E, H_E are

$G(e_1) = \{< b_1,[0.1,0.2][-0.2,-0.4] >, < b_2,[0.6,0.8][-0.2,-0.5] >\}$

$G(e_2) = \{< b_1,[0.2,0.6][-0.4,-0.4] >, < b_2,[0.3,0.5][-0.2,-0.4] >\}$

$H(e_1) = \{< b_1,[0.2,0.4][-0.3,-0.6] >, < b_2,[0.4,0.8][-0.2,-0.4] >\}$

$H(e_2) = \{< b_1,[0.3,0.4][-0.2,-0.4] >, < b_2,[0.4,0.4][-0.3,-0.5] >\}$

(i) BVSInt(G_E) $= \phi =$ BVSInt(H_E)

$G_E \tilde{\bigcup} H_E = F_E$

$BVSInt(G_E)\tilde{\bigcup} BVSInt(H_E) = \phi_E \tilde{\bigcup} \phi_E = \phi_E$ and $BVSInt(G_E \tilde{\bigcup} H_E) = BVSInt(F_E) = F_E$

Therefore $BVSInt(G_E)\tilde{\bigcup} BVSInt(H_E) \neq BVSInt(G_E \tilde{\bigcup} H_E)$

6 Conclusion

We have introduced the concept of Bipolar Vague soft sets and studied some of its properties. We have introduced topological structure on Bipolar Vague soft sets and characterized some of its properties. Therefore, this paper gives an idea for the beginning of a new study for approximations of data with uncertainties.

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