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Generalized pre-closed sets in vague topological spaces

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Abstract
This paper is developed to study a new class of sets in vague topological spaces namely vague generalized pre-closed sets. Further we have introduced some new generalized space to analyse their properties. Also some applications of vague generalized pre-closed sets namely vague $_{p}T_{1/2}$ space and vague $_{gp}T_{1/2}$ space are given.

Keywords: Vague topology, vague generalized pre-closed sets, vague $_{p}T_{1/2}$ space and vague $_{gp}T_{1/2}$ space.

1. Introduction

Closed sets are fundamental objects in a topological space. One can define the topology on a set by using either the axioms for the closed sets. In 1970, Levine [7] initiated the study of generalized closed sets. By definition, a subset S of a topological space X is called *generalized closed* if cl(A) $\subseteq U$ whenever $A \subseteq U$ and U is open. This notion has been studied extensively in recent years by many topologists because generalized closed sets are not only natural generalizations of closed sets. More importantly, they also suggest several new properties of topological spaces.

The concept of fuzzy sets was introduced by Zadeh [14] in 1965. The idea of fuzzy set is welcome because it handles uncertainty and vagueness which Cantorian set could not address. In fuzzy set theory, the membership of an element to a fuzzy set is a single value between zero and one. The theory of fuzzy topology was introduced by C.L. Chang [5] in 1967; several researches were conducted on the generalizations of the notions of fuzzy sets and fuzzy topology. In 1986, the concept of intuitionistic fuzzy sets was introduced by Atanassov [2] as a generalization of fuzzy sets.

The theory of vague sets was first proposed by Gau and Buehre [6] as an extension of fuzzy set theory and vague sets are regarded as a special case of context-dependent fuzzy sets. The basic concepts of vague set theory and its extensions defined by [4, 6]. In this paper we introduce the concept of vague generalized pre closed sets and vague generalized pre-open sets and we also obtain their properties and relations with counter examples.

2. Preliminaries

Definition 2.1: [3] A vague set A in the universe of discourse X is characterized by two membership functions given by:

1. A true membership function $t_A : X \to [0,1]$ and
2. A false membership function $f_A : X \to [0,1]$.

where $t_A(x)$ is lower bound on the grade of membership of x derived from the "evidence for x", $f_A(x)$ is a lower bound on the negation of x derived from the "evidence for x" and $t_A(x) + f_A(x) \leq 1$. hus the grade of membership of x in the vague set A is bounded by a subinterval $[t_A(x), 1 - f_A(x)]$ of [0, 1]. T This indicates that if the actual grade of membership $\mu(x)$, then $t_A(x) \leq \mu(x) \leq 1 - f_A(x)$. The vague set A is written as $A = \{\langle x, [t_A(x), 1 - f_A(x)]\rangle / x \in X\}$ where the interval $[t_A(x), 1 - f_A(x)]$ is called the "vague value" of x in A and is denoted by $V_A(x)$.

Definition 2.2: [3] Let A and B be VSs of the form $A=\{\langle x,[t_A(x),1-f_A(x)]\rangle / x\in X\}$ and $B=\{\langle x,[t_B(x),1-f_B(x)]\rangle / x\in X\}$. Then

a) $A\subseteq B$ if and only if $t_A(x)\le t_B(x)$ and $1-f_A(x)\le 1-f_B(x)$ for all $x\in X$.

b) $A=B$ if and only if $A\subseteq B$ and $B\subseteq A$.

c) $A^c=\{\langle x,[f_A(x),1-t_A(x)]\rangle / x\in X\}$.

d) $A\cap B=\{\langle x,[(t_A(x)\wedge t_B(x)),((1-f_A(x))\wedge(1-f_B(x)))]\rangle / x\in X\}$.

e) $A\cup B=\{\langle x,[(t_A(x)\vee t_B(x)),((1-f_A(x))\vee(1-f_B(x)))]\rangle / x\in X\}$.

For the sake of simplicity, we shall use the notion $A=\{\langle x,[t_A(x),1-f_A(x)]\rangle\}$ instead of $A=\{\langle x,[t_A(x),1-f_A(x)]\rangle / x\in X\}$.

Definition 2.3: Let (X,τ) be a topological space. A subset *A* of *X* is called:

i) *semi closed set* (SCS in short)[8] if $\operatorname{int}(cl(A))\subseteq A$,

ii) *pre- closed set* (PCS in short)[11] if $cl(\operatorname{int}(A))\subseteq A$,

iii) α *-closed set* (α CS in short)[12] if $cl(\operatorname{int}(cl(A)))\subseteq A$,

iv) *regular closed set* (RCS in short)[13] if $A=cl(\operatorname{int}(A))$.

Definition 2.4: Let (X,τ) be a topological space. A subset *A* of *X* is called:

i) *generalized closed* (briefly, *g-closed*) [7] if $cl(A)\subseteq U$, whenever $A\subseteq U$ and *U* is open in X .

ii) *generalized semi closed* (briefly, *gs-closed*) [1] if $scl(A)\subseteq U$ whenever $A\subseteq U$ and *U* is open in X .

iii) *α-generalized closed* (briefly, *αg-closed*) [9] if $\alpha\, cl(A)\subseteq U$ whenever $A\subseteq U$ and *U* is open in X .

iv) *generalized pre closed* (briefly, *gp-closed*) [10] if $pcl(A)\subseteq U$ whenever $A\subseteq U$ and *U* is open in X .

3. Vague Topological Space

Definition 3.1: A vague topology (VT in short) on X is a family τ of vague sets (VS in short) in X satisfying the following axioms.

- $0, 1\in\tau$
- $G_1\cap G_2\in\tau$, for any $G_1, G_2\in\tau$
- $\cup G_i\in\tau$ for any family $\{G_i / i\in J\}\subseteq\tau$.

In this case the pair (X,τ) is called a vague topological space (VTS in short) and any VS in τ is known as a vague open set (VOS in short) in X .

The complement A^c of a VOS in a VTS (X,τ) is called a vague closed set (VCS in short) in X .

Definition 3.2: Let (X,τ) be a VTS and $A=\{\langle x,[t_A,1-f_A]\rangle\}$ be a VS in X . Then the vague interior and a vague closure are defined by

- $v\operatorname{int}(A)=\cup\{G / G \text{ is a VOS in } X \text{ and } G\subseteq A\}$,
- $vcl(A)=\cap\{K / K \text{ is a VCS in } X \text{ and } A\subseteq K\}$.

Note that for any VS A in (X,τ), we have $vcl(A^c)=(v\operatorname{int}(A))^c$ and $v\operatorname{int}(A^c)=(vcl(A))^c$.

Definition 3.3: A VS $A=\{\langle x,[t_A,1-f_A]\rangle\}$ in a VTS (X,τ) is said to be a

i) *vague semi closed set* (VSCS in short) if $v\operatorname{int}(vcl(A))\subseteq A$.

ii) *vague semi open set* (VSOS in short) if $A\subseteq vcl(v\operatorname{int}(A))$.

iii) *vague pre- closed set* (VPCS in short) if $vcl(v\operatorname{int}(A))\subseteq A$.

iv) *vague pre-open set* (VPOS in short) if $A\subseteq v\operatorname{int}(vcl(A))$.

v) *vague α -closed set* (Vα CS in short) if $vcl(v\operatorname{int}(vcl(A))) \subseteq A$.

vi) *vague α -open set* (Vα OS in short) if $A \subseteq v\operatorname{int}(vcl(v\operatorname{int}(A)))$.

vii) *vague regular open set* (VROS in short) if $A = v\operatorname{int}(vcl(A))$.

viii) *vague regular closed set* (VRCS in short) if $A = vcl(v\operatorname{int}(A))$.

Definition 3.4: Let A be a VS of a VTS (X,τ). Then the vague semi interior of A ($vs\operatorname{int}(A)$ in short) and vague semi closure of A ($vscl(A)$ in short) are defined by

- $vs\operatorname{int}(A) = \cup\{G / G \text{ is a VSOS in } X \text{ and } G \subseteq A\}$,
- $vscl(A) = \cap\{K / K \text{ is a VSCS in } X \text{ and } A \subseteq K\}$.

Result 3.5: Let A be a VS of a VTS (X,τ), then

i) $vscl(A) = A \cup v\operatorname{int}(vcl(A))$

ii) $vs\operatorname{int}(A) = A \cap vcl(v\operatorname{int}(A))$

Definition 3.6: Let A be a VS of a VTS (X,τ). Then the vague alpha interior of A ($v\alpha\operatorname{int}(A)$ in short) and vague alpha closure of A ($v\alpha cl(A)$ in short) are defined by

- $v\alpha\operatorname{int}(A) = \cup\{G / G \text{ is a V}\alpha\text{OS in } X \text{ and } G \subseteq A\}$,
- $v\alpha cl(A) = \cap\{K / K \text{ is a V}\alpha\text{CS in } X \text{ and } A \subseteq K\}$.

Result 3.7: Let A be a VS of a VTS (X,τ), then

i) $v\alpha cl(A) = A \cup vcl(v\operatorname{int}(vcl(A)))$

ii) $v\alpha\operatorname{int}(A) = A \cap v\operatorname{int}(vcl(v\operatorname{int}(A)))$

Definition 3.8: A VS A of a VTS (X,τ) is said to be a *vague generalized closed set* (VGCS in short) if $vcl(A) \subseteq U$ whenever $A \subseteq U$ and U is a VOS in X.

Definition 3.9: A VS A of a VTS (X,τ) is said to be a *vague generalized semi closed set* (VGSCS in short) if $vscl(A) \subseteq U$ whenever $A \subseteq U$ and U is a VOS in X.

Definition 3.10: A VS A of a VTS (X,τ) is said to be a *vague alpha generalized closed set* (VαGCS in short) if $v\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is a VOS in X.

Definition 3.11: Let (X,τ) be a VTS and $A = \{\langle x, [t_A, 1 - f_A]\rangle\}$ be a VS in X. The vague pre interior of A is denoted by $vp\operatorname{int}(A)$ and is defined by the union of all vague pre-open sets of X which are contained in A. The intersection of all vague pre-closed sets containing A is called the pre-closure of A and is denoted by $vpcl(A)$.

- $vp\operatorname{int}(A) = \cup\{G / G \text{ is a VPOS in } X \text{ and } G \subseteq A\}$,
- $vpcl(A) = \cap\{K / K \text{ is a VPCS in } X \text{ and } A \subseteq K\}$.

Result 3.12: Let A be a VS of a VTS (X,τ), then $vpcl(A) = A \cup vcl(v\operatorname{int}(A))$

4. Vague generalized pre-closed set:

In this section we introduce vague generalized pre-closed set and some of its properties.

Definition 4.1: A VS A is said to be a vague generalized pre-closed set (VGPCS in short) in (X,τ) if $vpcl(A) \subseteq U$ whenever $A \subseteq U$ and U is a VOS in X. The family of all VGPCSs of a VTS (X,τ) is denoted by VGPC(X).

Example 4.2: Let $X = \{a,b\}$ and let $\tau = \{0, G, 1\}$ is a VT on X, where $G = \{\langle x, [0.4,0.7], [0.3,0.5]\rangle\}$. Then the VS $A = \{\langle x, [0.1,0.5], [0.3,0.6]\rangle\}$ is a VGPCS in X.

Theorem 4.3: Every VCS is a VGPCS but not conversely.

Proof: Let A be a VCS in X and let $A \subseteq U$ and U is a VOS in (X, τ). Since $vpcl(A) \subseteq vcl(A)$ and A is a VCS in X, $vpcl(A) \subseteq vcl(A) = A \subseteq U$. Therefore A is a VGPCS in X.

Example 4.4: Let $X = \{a,b\}$ and let $\tau = \{0, G, 1\}$ is a VT on X, where $G = \{\langle x, [0.5,0.6], [0.3,0.5]\rangle\}$. Then the VS $A = \{\langle x, [0.3,0.4], [0.6,0.8]\rangle\}$ is a VGPCS in X but not a VCS in X.

Theorem 4.5: Every Vα CS is a VGPCS but not conversely.

Proof: Let A be a Vα CS in X and let $A \subseteq U$ and U is a VOS in (X, τ). By hypothesis, $vcl(v\operatorname{int}(vcl(A))) \subseteq A$. Since $A \subseteq vcl(A), vcl(v\operatorname{int}(A)) \subseteq vcl(v\operatorname{int}(vcl(A))) \subseteq A$. Hence $vpcl(A) \subseteq A \subseteq U$. Therefore A is a VGPCS in X.

Example 4.6: Let $X = \{a,b\}$ and let $\tau = \{0, G, 1\}$ is a VT on X, where $G = \{\langle x, [0.5,0.6], [0.3,0.7]\rangle\}$. Then the VS $A = \{\langle x, [0.5,0.6], [0.3,0.5]\rangle\}$ is a VGPCS in X but not a Vα CS in X since $vcl(v\operatorname{int}(vcl(A))) = 1 \not\subset A.$

Theorem 4.7: Every VGCS is a VGPCS but not conversely.

Proof: Let A be a VGCS in X and let $A \subseteq U$ and U is a VOS in (X, τ). Since $vpcl(A) \subseteq vcl(A)$ and by hypothesis, $vpcl(A) \subseteq U$. Therefore A is a VGPCS in X.

Example 4.8: Let $X = \{a,b\}$ and let $\tau = \{0, G, 1\}$ is a VT on X, where $G = \{\langle x, [0.5,0.9], [0.6,0.7]\rangle\}$. Then the VS $A = \{\langle x, [0.3,0.4], [0.2,0.4]\rangle\}$ is a VGPCS in X but not a VGCS in X since $vcl(A) = 1 \not\subset G.$

Theorem 4.9: Every VRCS is a VGPCS but not conversely.

Proof: Let A be a VRCS in X. By Definition 3.3, $A = vcl(v\operatorname{int}(A))$. This implies $vcl(A) = vcl(v\operatorname{int}(A))$. Therefore $vcl(A) = A$. That is A is a VCS in X. By Theorem 4.3, A is a VGPCS in X.

Example 4.10: Let $X = \{a,b\}$ and let $\tau = \{0, G, 1\}$ is a VT on X, where $G = \{\langle x, [0.3,0.5], [0.4,0.6]\rangle\}$. Then the VS $A = \{\langle x, [0.5,0.8], [0.4,0.7]\rangle\}$ is a VGPCS in X but not a VRCS in X since $vcl(v\operatorname{int}(A)) = \{\langle x, [0.5,0.7], [0.4,0.6]\rangle\} \neq A.$

Theorem 4.11: Every VPCS is a VGPCS but not conversely.

Proof: Let A be a VPCS in X and let $A \subseteq U$ and U is a VOS in (X, τ). By Definition 3.3, $vcl(v\operatorname{int}(A)) \subseteq A$. This implies $vpcl(A) = A \cup vcl(v\operatorname{int}(A)) \subseteq A$. Therefore $vpcl(A) \subseteq U$. Hence A is a VGPCS in X.

Example 4.12: Let $X = \{a,b\}$ and let $\tau = \{0, G, 1\}$ is a VT on X, where $G = \{\langle x, [0.4,0.7], [0.4,0.8]\rangle\}$. Then the VS $A = \{\langle x, [0.5,0.8], [0.6,0.9]\rangle\}$ is a VGPCS in X but not a VPCS in X since $vcl(v\operatorname{int}(A)) = 1 \not\subset A.$

Theorem 4.13: Every Vα GCS is a VGPCS but not conversely.

Proof: Let A be a Vα GCS in X and let $A \subseteq U$ and U is a VOS in (X, τ). By Result 3.7, $A \cup vcl(v\operatorname{int}(vcl(A))) \subseteq U$. This implies $vcl(v\operatorname{int}(vcl(A))) \subseteq U$ and $vcl(v\operatorname{int}(A)) \subseteq U$. Therefore $vpcl(A) = A \cup vcl(v\operatorname{int}(A)) \subseteq U$. Hence A is a VGPCS in X.

Example 4.14: Let $X = \{a,b\}$ and let $\tau = \{0,G,1\}$ is a VT on X, where $G = \{\langle x,[0.2,0.5],[0.3,0.5]\rangle\}$. Then the VS $A = \{\langle x,[0.1,0.4],[0.2,0.3]\rangle\}$ is a VGPCS in X but not a V α GCS in X since $v\alpha cl(A) = 1 \not\subset G$.

Proposition 4.15: VSCS and VGPCS are independent to each other.

Example 4.16: Let $X = \{a,b\}$ and let $\tau = \{0,G,1\}$ is a VT on X, where $G = \{\langle x,[0.2,0.5],[0.3,0.5]\rangle\}$. Then the VS $A = G$ is a VSCS in X but not a VGPCS in X since $A \subseteq T$ but $vpcl(A) = 1 \not\subset G$.

Example 4.17: Let $X = \{a,b\}$ and let $\tau = \{0,G,1\}$ is a VT on X, where $G = \{\langle x,[0.6,0.7],[0.5,0.8]\rangle\}$. Then the VS $A = \{\langle x,[0.4,0.7],[0.6,0.9]\rangle\}$ is a VGPCS in X but not a VSCS in X since $v\,\mathrm{int}(vcl(A)) = 1 \not\subset A$.

Proposition 4.18: VGSCS and VGPCS are independent to each other.

Example 4.19: Let $X = \{a,b\}$ and let $\tau = \{0,G,1\}$ is a VT on X, where $G = \{\langle x,[0.3,0.4],[0.2,0.5]\rangle\}$. Then the VS $A = T$ is a VGSCS in X but not a VGPCS in X since $A \subseteq G$ but $vpcl(A) = \{\langle x,[0.6,0.7],[0.5,0.8]\rangle\} \not\subset G$.

Example 4.20: Let $X = \{a,b\}$ and let $\tau = \{0,G,1\}$ is a VT on X, where $G = \{\langle x,[0.2,0.4],[0.3,0.4]\rangle\}$. Then the VS $A = \{\langle x,[0.1,0.3],[0.3,0.4]\rangle\}$ is a VGPCS in X but not a VGSCS in X since $vscl(A) = \{\langle x,[0.6,0.8],[0.6,0.7]\rangle\} \not\subset G$.

Summing up the above theorems, we have the following implication.

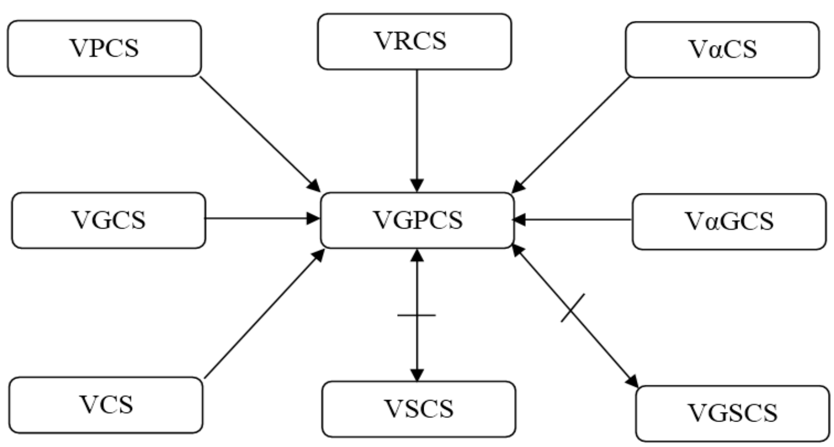

Fig 1

In this diagram by "A ⟶ B" we mean A implies B but not conversely and "A ⟷ B" means A and B are independent of each other. None of them is reversible.

Remark 4.21: The union of any two VGPCSs is not a VGPCS in general as seen in the following example.

Example 4.22: Let $X = \{a,b\}$ and let $G = \{\langle x,[0.3,0.7],[0.3,0.8]\rangle\}$. Then $\tau = \{0,G,1\}$ is a VT on X and the VSs $A = \{\langle x,[0.2,0.8],[0.6,0.7]\rangle\}$, $B = \{\langle x,[0.5,0.8],[0.5,0.8]\rangle\}$ are VGPCSs but $A \cup B$ is not a VGPCS in X.

5. Vague generalized pre-open set

In this section we introduce vague generalized pre-open set and some of its properties.

Definition 5.1: A VS A is said to be a vague generalized pre-open set (VGPOS in short) in (X,τ) if the complement A^c is a VGPCS in X. The family of all VGPOSs of a VTS (X,τ) is denoted by VGPO(X).

Example 5.2: Let $X = \{a,b\}$ and let $\tau = \{0,G,1\}$ is a VT on X, where $G = \{\langle x,[0.5,0.6],[0.4,0.7]\rangle\}$. Then the VS $A = \{\langle x,[0.5,0.7],[0.4,0.6]\rangle\}$ is a VGPOS in X.

Theorem 5.3: For any VTS (X,τ), we have the following:

- Every VOS is a VGPOS
- Every VSOS is a VGPOS
- Every V α OS is a VGPOS
- Every VPOS is a VGPOS. But the converses are not true in general.

Proof: Straight forward.
The converse of the above statements need not be true which can be seen from the following examples.

Example 5.4: Let $X=\{a,b\}$ and $G=\{\langle x,[0.6,0.7],[0.5,0.7]\rangle\}$. Then $\tau=\{0,G,1\}$ is a VT on X. The VS $A=\{\langle x,[0.5,0.6],[0.4,0.7]\rangle\}$ is a VPSOS in (X,τ) but not a VOS in X.

Example 5.5: Let $X=\{a,b\}$ and $G=\{\langle x,[0.5,0.6],[0.6,0.7]\rangle\}$. Then $\tau=\{0,G,1\}$ is a VT on X. The VS $A=\{\langle x,[0.6,0.8],[0.4,0.7]\rangle\}$ is a VPSOS in (X,τ) but not a VSOS in X.

Example 5.6: Let $X=\{a,b\}$ and $G=\{\langle x,[0.4,0.5],[0.4,0.7]\rangle\}$. Then $\tau=\{0,G,1\}$ is a VT on X. The VS $A=\{\langle x,[0.3,0.5],[0.4,0.6]\rangle\}$ is a VPSOS in (X,τ) but not a V α OS in X.

Example 5.7: Let $X=\{a,b\}$ and $G=\{\langle x,[0.6,0.7],[0.5,0.6]\rangle\}$. Then $\tau=\{0,G,1\}$ is a VT on X. The VS $A=\{\langle x,[0.2,0.4],[0.2,0.5]\rangle\}$ is a VPSOS in (X,τ) but not a VPOS in X.

Theorem 5.8: Let (X,τ) be a VTS. If $A\in \mathrm{VGPO}(X)$ then $V\subseteq v\operatorname{int}(vcl(A))$ whenever $V\subseteq A$ and V is VCS in X.

Proof: Let $A\in \mathrm{VGPO}(X)$. Then A^c is a VGPCS in X. Therefore $vpcl(A^c)\subseteq U$ whenever $A^c\subseteq U$ and U is a VOS in X. That is $vcl(v\operatorname{int}(A^c))\subseteq U$. This implies $U^c\subseteq v\operatorname{int}(vcl(A))$ whenever $U^c\subseteq A$ and U^c is VCS in X. Replacing U^c by V, we get $V\subseteq v\operatorname{int}(vcl(A))$ whenever $V\subseteq A$ and V is VCS in X.

Theorem 5.9: Let (X,τ) be a VTS. Then for every $A\in \mathrm{VGPO}(X)$ and for every $B\in \mathrm{VS}(X)$, $vp\operatorname{int}(A)\subseteq B\subseteq A$ implies $B\in \mathrm{VGPO}(X)$.

Proof: By hypothesis $A^c\subseteq B^c\subseteq (vp\operatorname{int}(A))^c$. Let $B^c\subseteq U$ and U be a VOS. Since $A^c\subseteq B^c$, $A^c\subseteq U$. But A^c is a VGPCS, $vpcl(A^c)\subseteq U$. Also $B^c\subseteq (vp\operatorname{int}(A))^c = vpcl(A^c)$. Therefore $vpcl(B^c)\subseteq vpcl(A^c)\subseteq U$. Hence B^c is a VGPCS. Which implies B is a VGPOS of X.

Remark 5.10: The intersection of any two VGPOSs is not a VGPOS in general as seen in the following example.

Example 5.11: Let $X=\{a,b\}$ and let $G=\{\langle x,[0.1,0.2],[0.4,0.5]\rangle\}$. Then $\tau=\{0,G,1\}$ is a VT on X and the VSs $A=\{\langle x,[0.6,0.9],[0.5,0.8]\rangle\}$, $B=\{\langle x,[0.6,0.7],[0.6,0.6]\rangle\}$ are VGPOSs but $A\cap B$ is not a VGPOS in X.

Theorem 5.12: A VS A of a VTS (X,τ) is a VGPOS if and only if $F\subseteq vp\operatorname{int}(A)$ whenever F is a VCS and $F\subseteq A$.

Proof: Necessity: Suppose A is a VGPOS in X. Let F be a VCS and $F\subseteq A$. Then F^c is a VOS in X such that $A^c\subseteq F^c$. Since A^c is a VGPCS, we have $vpcl(A^c)\subseteq F^c$. Hence $(vp\operatorname{int}(A))^c\subseteq F^c$. Therefore $F\subseteq vp\operatorname{int}(A)$.

Sufficiency: Let A be a VS of X and let $F\subseteq vp\operatorname{int}(A)$ whenever F is a VCS and $F\subseteq A$. Then $A^c\subseteq F^c$ and F^c is a VOS. By hypothesis, $(vp\operatorname{int}(A))^c\subseteq F^c$. which implies $vpcl(A^c)\subseteq F^c$. Therefore A^c is a VGPCS of X. Hence A is a VGPOS of X.

Corollary 5.13: A VS A of a VTS (X,τ) is a VGPOS if and only if $F\subseteq v\operatorname{int}(vcl(A))$ whenever F is a VCS and $F\subseteq A$.

Proof: Necessity: Suppose A is a VGPOS in X. Let F be a VCS and $F \subseteq A$. Then F^c is a VOS in X such that $A^c \subseteq F^c$. Since A^c is a VGPCS, we have $vpcl(A^c) \subseteq F^c$. Therefore $vcl(v\,\text{int}(A^c)) \subseteq F^c$. Hence $(v\,\text{int}(vcl(A)))^c \subseteq F^c$. This implies $F \subseteq v\,\text{int}(vcl(A))$.

Sufficiency: Let A be a VS of X and let $F \subseteq v\,\text{int}(vcl(A))$ whenever F is a VCS and $F \subseteq A$. Then $A^c \subseteq F^c$ and F^c is a VOS. By hypothesis, $(v\,\text{int}(vcl(A)))^c \subseteq F^c$. Hence $vcl(v\,\text{int}(A^c)) \subseteq F^c$, which implies $vpcl(A^c) \subseteq F^c$. Hence A is a VGPOS of X.

Theorem 5.14: For a VS A, A is a VOS and a VGPCS in X if and only if A is a VROS in X.

Proof: Necessity: Let A be a VOS and a VGPCS in X. Then $vpcl(A) \subseteq A$. This implies $vcl(v\,\text{int}(A)) \subseteq A$. Since A is a VOS, it is a VPOS. Hence $A \subseteq v\,\text{int}(vcl(A))$. Therefore $A = v\,\text{int}(vcl(A))$. Hence A is a VROS in X.

Sufficiency: Let A be a VROS in X. Therefore $A = v\,\text{int}(vcl(A))$. Let $A \subseteq U$ and U is a VOS in X. This implies $vpcl(A) \subseteq A$. Hence A is a VGPCS in X.

6. Applications of vague generalized pre-closed sets:

In this section we provide some applications of vague generalized pre-closed sets.

Definition 6.1: A VTS (X,τ) is said to be a vague ${}_{p}T_{1/2}$ (V${}_{p}T_{1/2}$ in short) space if every VGPCS in X is a VCS in X.

Definition 6.2: A VTS (X,τ) is said to be a vague ${}_{gp}T_{1/2}$ (V${}_{gp}T_{1/2}$ in short) space if every VGPCS in X is a VPCS in X.

Theorem 6.3: Every V${}_{p}T_{1/2}$ space is a V${}_{gp}T_{1/2}$ space. But the converse is not true in general.

Proof: Let X be a V${}_{p}T_{1/2}$ space and let A be a VGPCS in X. By hypothesis A is a VCS in X. Since every VCS is a VPCS, A is a VPCS in X. Hence X is a V${}_{gp}T_{1/2}$ space.

Example 6.4: Let $X = \{a,b\}$ and $G = \{\langle x,[0.4,0.5],[0.4,0.7]\rangle\}$. Then $\tau = \{0,G,1\}$ is a VT on X. Let $A = \{\langle x,[0.6,0.7],[0.5,0.6]\rangle\}$. Then (X,τ) is a V${}_{gp}T_{1/2}$ space. But it is not a V${}_{p}T_{1/2}$ space since A is VGPCS but not VCS in X.

Theorem 6.5: Let (X,τ) be a VTS and X is a V${}_{p}T_{1/2}$ space then,

i) Any union of VGPCSs is a VGPCS.
ii) Any intersection of VGPOSs is a VGPOS.

Proof: i) Let $\{A_i\}_{i\in J}$ is a collection of VGPCSs in a V${}_{p}T_{1/2}$ space (X,τ). Therefore every VGPCS is a VCS. But the union of VCS is a VCS. Hence the union of VGPCS is a VGPCS in X. (ii) it can be proved by taking complement of (i).

Theorem 6.6: A VTS X is a V${}_{gp}T_{1/2}$ space if and only if $\text{VGPO}(X) = \text{VPO}(X)$.

Proof: Necessity: Let A be a VGPOS in X, then A^c is a VGPCS in X. By hypothesis A^c is a VGPCS in X. Therefore A is a VPOS in X. Hence $\text{VGPO}(X) = \text{VPO}(X)$.

Sufficiency: Let A be a VGPCS in X. Then A^c is a VGPOS in X. By hypothesis A^c is a VGPOS in X. Therefore A is a VPCS in X. Hence X is a V${}_{gp}T_{1/2}$ space.

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