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Characterizations of Soft-πgp-Closed sets in soft topological spaces

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Abstract
The aim of this paper is to characterize the properties of soft-πgp-closed sets in soft topological spaces and investigate its relationship with other soft closed sets. A detail study is carried out on soft-πgp-operators and soft-πgp-$T_{1/2}$ spaces.

Keywords: soft topological spaces, soft-closed, soft-generalized closed, soft-πgp-closed, soft-$T_{1/2}$-space.

1. Introduction
The soft set theory is a rapidly processing field of mathematics. This was introduced by a Russian Researcher Molodtsov [10] in 1999, the concept of soft set theory as a mathematical tool for dealing with uncertainties problems. He proposed soft set theory which contains sufficient parameters such that it is free from the corresponding difficulties and a series of interesting application of the theory instability and regularization, Game theory, Operation Research, Probability and Statistics. The soft theory has a rich potential for application in many directions.

Maji *et al.* [9] proposed several operations on soft set and some basic properties. Shabir and Naz [15] introduced the notion of soft topological spaces. Tridev Jyoti Naog [16] studies a new approach to the theory of soft sets. Kannan [7] introduced soft generalized closed and open sets in soft topological spaces which are defined over an initial universe with a fixed set of parameters. Then Saziye *et al.* [14] studied behavior relative to soft subspaces of soft generalized closed sets and investigate the properties of soft generalized closed and open sets. Pei and Miao [13] investigate the relationship between soft set and information system. Mahanta and Das [11] introduced semi-soft open sets, semi soft-closed sets, semi soft – continuity and related concepts. Palaniappan and Chandrasekhara Rao [12] introduced regular generalized closed sets in Topological spaces. C. Janaki and V. Jeyanthi [5] were introduced and studied soft-πgr-closed sets, soft-πgb-closed in soft topological spaces.

In this paper, we study the properties of soft-πgp-closed sets in soft topological spaces. Also we introduce the soft-πgp-$T_{1/2}$ space and study their basic properties.

2. Preliminaries
Let U be an initial universe set and E be a collection of all possible parameters with respect to U, where parameters are the characteristics or properties of objects in U. Let P(U) denote the power set of U, and let $A \subseteq E$.

Definition: 2.1[10] A pair (F,A) is called a soft set over U, where F is a mapping given by
F: A → P (U).In other words, a soft set over U is a parameterized family of subsets of the universe U. For a particular e ∈ A. F (e) may be considered the set of e-approximate elements of the soft set (F, A).

Definition: 2.2 [3] For two soft sets (F, A) and (G, B) over a common universe U, we say that (F, A) is a soft subset of (G,B) if
(i) $A \subseteq B$, and
(ii) $\forall e \in A$, $F(e) \tilde{\subseteq} G(e)$.

We write (F,A) $\tilde{\subseteq}$ (G,B). (F,A) is said to be a soft super set of (G,B), if (G,B) is a soft subset of (F,A). We denote it by (F,A) $\tilde{\supseteq}$ (G,B).

Definition: 2.3 [9] A soft set (F,A) over U is said to be
(i) Null soft set denoted by ϕ if $\forall e \in A$, $F(e) = \phi$.
(ii) Absolute soft set denoted by A, if $\forall e \in A$, $F(e) = U$.

Definition: 2.4 [9] For two soft sets (F,A) and (G,B) over a common universe U,
(i) Union of two soft sets of (F,A) and (G,B) is the soft set (H,C),where C = A ∪ B, and $\forall e \in C$,

$$H(e) = \begin{cases} F(e) & , if\ e \in A - B \\ G(e) & , if\ e \in B - A \\ F(e) \cup G(e) & , if\ e \in A \cap B \end{cases}$$

We write (F,A) ∪(G,B)=(H,C).

Definition:2.5 [3]The Intersection (H,C) of two soft sets (F,A) and (G,B) over a common universe U denoted (F,A) ∩ (G,B) is defined as C= A∩ B and H(e) = F(e) ∩ G(e) for all e∈C.

Definition: 2.6 [15] Let Y be a non-empty subset of X, then $\tilde{Y}$ denotes the soft set (Y,E) over X for which Y (e) = Y, for all e ∈ E. In particular, (X,E), will be denoted by $\tilde{\tilde{X}}$.

Definition:2.7 [15]For a soft set (F,A) over the universe U, the relative complement of (F,A) is denoted by (F,A)′ and is defined by (F,A)′=(F′,A), where F′: A →P(U) is a mapping defined by F′(e)= U-F(e) for all e∈A.

Definition: 2.8 [15] Let τ be the collection of soft sets over X, then τ is called a soft topology on X if τ satisfies the following axioms:
(i) ϕ, $\tilde{\tilde{X}}$ belong to τ
(ii) The union of any number of soft sets in τ belongs to τ.
(iii) The intersection of any two soft sets in τ belongs to τ.
The triplet (X, τ, E) is called a soft topological space over X. For simplicity, we can take the soft topological space (X, τ, E) as X throughout the work.

Definition: 2.9 [15] Let (X, τ, E) be soft space over X. A soft set (F,E) over X is said to be soft closed in X, if its relative complement (F,E)′ belongs to τ. The relative complement is a mapping F′:E→P(X) defined by F′(e)= X-F(e) for all e ∈A.

Definition: 2.10 [7] Let X be an initial universe set, E be the set of parameters and τ= {ϕ, $\tilde{\tilde{X}}$ }. Then τ is called the soft indiscrete topology on X and (X, τ,E) is said to be a soft indiscrete space over X. If τ is the collection of all soft sets which can be defined over X, then τ is called the soft discrete topology on X and (X, τ, E) is said to be a soft discrete space over X.

Definition: 2.11 [7] Let (X, τ, E) be a soft topological space over X and the soft interior of (F,E) denoted by Int(F,E) is the union of all soft open subsets of (F,E). Clearly, (F,E) is the largest soft open set over X which is contained in (F,E).The soft closure of (F,E) denoted by Cl(F,E) is the intersection of all closed sets containing (F,E). Clearly, (F, E) is smallest soft closed set containing (F,E).
Int (F,E) = ∪ { (O,E): (O,E) is soft open and (O,E) $\tilde{\subseteq}$ (F,E)}.
Cl(F,E) = ∩{ (O,E): (O,E) is soft closed and (F,E) $\tilde{\subseteq}$ (O,E)}.

Definition: 2.12 [7] Let U be the common universe set and E be the set of all parameters. Let (F,A) and (G,B) be soft sets over a common universe set U and A,B $\tilde{\subseteq}$ E. Then (F,A) is a subset of (G,B), denoted by (F,A) $\tilde{\subseteq}$ (G,B). (F,A) equals (G,B), denoted by (F,A)=(G,B) if (F,A) $\tilde{\subseteq}$ (G,B) and (G,B) $\tilde{\subseteq}$ (F,A).

Definition: 2.13 A soft subset (A, E) of X is called
(i) a soft generalized closed (Soft-g-closed)[7], if Cl(A,E) $\tilde{\subseteq}$ U,E) whenever (A,E) $\tilde{\subseteq}$ (U,E) and (U,E) is soft open in X.
(ii) a soft-semi open [2],if (A,E) $\tilde{\subseteq}$ Cl(Int(A,E))
(iii) a soft-regular open[1], if (A,E)= Int(Cl(A,E)).
(iv) a soft-α-open[5], if (A,E) $\tilde{\subseteq}$ Int(Cl(Int(A,E)))
(v) a soft-b-open[6], if (A,E) $\tilde{\subseteq}$ Cl(Int(A,E)) Int(Cl(A,E))
(vi) a soft-pre-open[5], if (A,E) $\tilde{\subseteq}$ Int (Cl(A,E)).
(vii) a soft-clopen[6], if (A,E) is both soft open and soft closed.
(viii) a soft-β-open set[17], if (A,E) $\tilde{\subseteq}$ Cl(Int (Cl(A,E))).
(ix) a soft-πgr-closed[5], if srCl(A,E) $\tilde{\subseteq}$ (U,E) whenever (A,E) $\tilde{\subseteq}$ (U,E) and (U,E) is soft π-open in X.
(x) a soft-πg-closed[1], if Cl(A,E) $\tilde{\subseteq}$ (U,E) whenever (A,E) $\tilde{\subseteq}$ (U,E) and (U,E) is soft π-open in X.
(xi) a soft-πgs-closed[1], if ssCl(A,E) $\tilde{\subseteq}$ (U,E) whenever (A,E) $\tilde{\subseteq}$ (U,E) and (U,E) is soft π-open in X.
(xii) a soft-πgα-closed[5], if sαCl(A,E) $\tilde{\subseteq}$ (U,E) whenever (A,E) $\tilde{\subseteq}$ (U,E) and (U,E) is soft π-open in X.
The complement of the soft semi open, soft regular open, soft α-open, soft b-open, soft pre-open sets are their respective soft semi closed, soft regular closed, soft α-closed, soft pre-closed and soft pre -closed sets.
The finite union of soft regular open sets is called soft π-open set and its complement is soft-π-closed set. The soft regular open set of X is denoted by SRO(X) or SRO(X, τ, E).

Definition: 2.14 [7] A soft topological space X is called a soft $T_{1/2}$-space if every soft-g-closed set is soft closed in X.

Definition: 2.15 [5] The soft regular closure of (A,E) is the intersection of all soft regular closed sets containing (A,E). (i.e)The smallest soft regular closed set containing (A,E) and is denoted by srcl(A,E).
The soft regular interior of (A,E) is the union of all soft regular open set s contained in (A,E) and is denoted by srint(A,E).
Similarly, we define soft α-closure, soft pre-closure, soft semi closure and soft b-closure of the soft set (A,E) of a topological space X and are denoted by sαcl(A,E) or αcl^s(A,E), spcl(A,E) or pcl^s(A,E), sscl(A,E) or $sscl^s$(A,E) and sbcl(A,E) or $sbcl^s$(A,E) respectively.

Proposition: 2.16[4] Let (X, τ, E) be a soft topological space over X and (F,E) and (G,E) be a soft set over X. Then
(1) int (int (F,E)) = int (F,E)
(2) (F,E) $\tilde{\subseteq}$ (G,E) implies int (F,E) $\tilde{\subseteq}$ int(G,E)
(3) cl (cl (F,E)) = cl (F,E)
(4) (F,E) $\tilde{\subseteq}$ (G,E) implies cl (F,E) $\tilde{\subseteq}$ cl(G,E).

Definition: 2.17 [8] A subset A in a topological space is defined to be a Q^s-set iff int(cl(A)=cl(int(A))

Definition 2.18[6] A soft topological space X is said to be soft hyperconnected if the closure of every soft open subset is X.

3. Characterization Soft-πgp -closed sets

Definition 3.1[6]: A soft subset (A,E) of a soft topological space X is called soft-πgp-closed set in X if spcl(A,E) $\tilde{\subseteq}$ (U,E) whenever (A,E) $\tilde{\subseteq}$ (U,E) and (U,E) is soft-π-open in X.By SπGPC(X), we mean the family of all soft-πgp-closed subsets of the space X.

Theorem 3.2
1. Every soft-closed set is soft-πgp-closed
2. Every soft-g-closed is soft-πgp-closed
3. Every soft-α-closed set is soft-πgp-closed
4. Every soft-pre-closed set is soft-πgp-closed
5. Every soft-πgr-closed set is soft-πgp-closed
6. Every soft-πg -closed set is soft-πgp-closed.
7. Every soft-πgα-closed set is soft-πgp-closed
8. Every soft-πgs-closed set is soft-πgp-closed.

Proof: 1.Let (A,E) $\tilde{\subseteq}$ (U,E) and (U,E) be soft-π-open. Then Cl(A,E)=(A,E) $\tilde{\subseteq}$ (U,E). Since every soft-closed set is soft-pre-closed, spcl (A, E) $\tilde{\subseteq}$ Cl (A,E) $\tilde{\subseteq}$ (U,E). Hence (A,E) is soft- πgp-closed.

2. Let (A, E) be soft-g-closed in X and (A, E) $\tilde{\subseteq}$ (U, E) where (U, E) is soft-π- open. Since every soft-π-open set is soft-open and A is soft-g-closed, we have Cl(A,E) $\tilde{\subseteq}$ (U,E). Hence spcl (A,E) $\tilde{\subseteq}$ Cl(A,E) $\tilde{\subseteq}$ (U,E). Then (A,E) is soft-πgp-closed.

3. Let (A,E) $\tilde{\subseteq}$ (U,E) and (U,E) is soft-π-open. Since (A,E) is soft-α-closed, sαcl(A,E) $\tilde{\subseteq}$ (A,E) $\tilde{\subseteq}$ (U,E). we have spcl (A,E) $\tilde{\subseteq}$ sαcl (A,E) $\tilde{\subseteq}$ (U,E). Then (A,) is soft-πgp- closed.

4. Let (A, E) $\tilde{\subseteq}$ (U,E) and (U,E) is soft-π-open. Since (A, E) is soft-pre-closed, spcl (A,E) $\tilde{\subseteq}$ U. Then spcl(A, E) $\tilde{\subseteq}$ (U,E). Then (A,E) is soft-πgp- closed.

5. Let (A,E) $\tilde{\subseteq}$ (U,E) where (A,E) be soft-πgr-closed set in X and (U,E) is soft-π-open. By assumption, srcl (A,E) $\tilde{\subseteq}$ (U,E). we know that spcl(A,E) $\tilde{\subseteq}$ srcl(A,E) $\tilde{\subseteq}$ (U,E). Then (A,E) is soft-πgp-closed.

6. Let (A,E) be soft-πg-closed set in X and (A,E) $\tilde{\subseteq}$ (U,E) where (U,E) is soft-π-open. By assumption Cl(A) $\tilde{\subseteq}$ U. Hence spcl(A,E) $\tilde{\subseteq}$ Cl (A,E) $\tilde{\subseteq}$ (U,E). Then (A,E) is soft-πgp-closed.

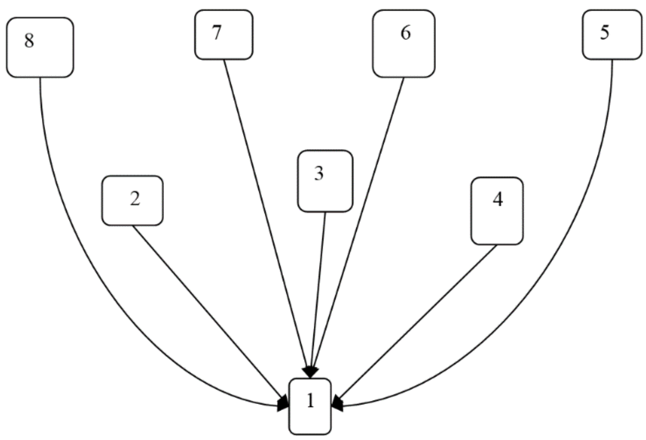
7. Let (A,E) be soft-πgα-closed set in X and (A,E) $\tilde{\subseteq}$ (U,E) where (U,E) is soft-π- open. By assumption sαcl (A,E) $\tilde{\subseteq}$ (U,E). Also spcl (A,E) $\tilde{\subseteq}$ sαcl (A,E) $\tilde{\subseteq}$ (U,E). Then (A,E) is soft- πgp-closed.

8. Let (A,E) be soft-πgs-closed in X and (A,E) $\tilde{\subseteq}$ (U,E) where (U,E) is soft-π- open. By assumption sscl(A) $\tilde{\subseteq}$ (U,E). Hence spcl (A,E) $\tilde{\subseteq}$ sscl (A,E) $\tilde{\subseteq}$ (U,E). Then (A,E) is soft- πgp-closed.

Remark 3.3: Converse of the above need not be true as seen in the following example.

Example 3.4: Let X= {a,b,c,d}, E={e_1,e_2}.Let F_1,F_2,….,F_6 are functions from E to P(X) and are defined as follows:
$F_1(e_1)$={c},$F_1(e_2)$={a}, $F_4(e_1)$={a,d},$F_4(e_2)$={b,d},
$F_2(e_1)$={d},$F_2(e_2)$={b}, $F_5(e_1)$={b,c,d},$F_5(e_2)$={a,b,c},
$F_3(e_1)$={c,d},$F_3(e_2)$={a,b}, $F_6(e_1)$={a,c,d},$F_6(e_2)$={a,b,d},
Then τ = {Φ, X, (F_1, E), (F_6,E)} is a soft topology and elements in τ are sof- open sets.
(i) The soft set (A,E)={{ Φ },{a}} is soft-πgp-closed but not soft-closed.
(ii) The soft set (B,E)={ Φ },{b,d}} is soft-πgp-closed but not soft-g-closed.
(iii) The soft set (C,E)={{ Φ },{a,b}} is soft-πgp-closed but not soft-α-closed.
(iv) The soft set (D,E)={{a},{b}} is soft-πgp-closed but not soft-pre-closed.
(v) The soft set (F,E)={{c},{ Φ }} is soft-πgp-closed but not soft-πgr-closed.
(vi) The soft set (G,E)={{ Φ },{d}} is soft-πgp-closed but not soft-πg-closed.
(vii) The soft set (H,E)={{ Φ },{b}} is soft-πgp-closed but not soft-πgα-closed.
(viii) The soft set (I,E)={{ Φ },{a,d}} is soft-πgp-closed but not soft-πgs-closed.

Remark 3.5: The above discussions are summarized in the following diagram.
1. soft-πgp-closed set, 5.soft-g-closed set,
2. soft-pre -closed set, 6.soft-πgr-closed set,
3. soft-α-closed set, 7.soft- πg-closed set,
4. soft-πgα- closed set, 8.soft- πgs-closed set.
The following diagram shows the relationships of soft-πgp-closed set with other known existing sets. A→B represents A implies B, but not conversely.



Theorem 3.6: If (A, E) is soft-π-open and soft-πgp- closed, then (A, E) is soft-pre-closed.

Proof: Let (A, E) be soft-π-open and soft-πgp-closed. Then spcl (A,E) $\tilde{\subseteq}$ (A,E).But (A,E) $\tilde{\subseteq}$ spcl (A,E).Hence (A,E)=spcl(A,E).This implies (A,E) is soft-pre-closed.

Theorem 3.7: Let (A,E) be soft-πgp-closed in X. Then spcl (A,E) - (A,E) does not contain any non empty soft -π-closed set.

Proof: Let (F,E) be a non empty soft-π-closed set such that (F,E) $\tilde{\subseteq}$ spcl(A,E) $\tilde{\subseteq}$ (A,E). Since (A,E) is soft-πgp-closed, (A,E) $\tilde{\subseteq}$ X-(F,E) where X-(F,E) is soft-π-open implies spcl (A,E) $\tilde{\subseteq}$ X-(F,E). Hence (F,E) $\tilde{\subseteq}$ X-spcl(A,E).Now, (F,E) $\tilde{\subseteq}$ (spcl (A,E))∩ (X-spcl (A,E)) implies (F,E) =ϕ which is a contradiction. Therefore spcl(A,E) -(A,E) does not contain any non empty soft-π- closed set.

Corollary 3.8: Let (A,E) be soft-πgp -closed in X. Then (A,E) is soft-pre-closed set if and only if spcl(A,E) -(A,E) is soft-π-closed.

Proof: Let (A,E) be soft -pre-closed. Then spcl(A,E) = (A,E). This implies spcl(A)-(A,E)= ϕ which is soft -π-closed. Assume that spcl (A,E)-(A,E) is soft-π- closed. Then spcl(A,E) -(A,E) = ϕ. Hence, spcl (A,E) = (A,E).

Theorem 3.9: For a soft subset (A, E) of X, the following statements are equivalent:
(1) (A, E) is soft-π-open and soft-πgp- closed.
(2) (A,E) is soft -regular open.

Proof: (1)⇒(2) Let (A,E) be a soft-π-open and soft-πgp-closed subset of X. Then spcl (A,E)$\tilde{\subseteq}$ (A,E).Hence Int(Cl(A,E)) $\tilde{\subseteq}$ (A,E). Since every soft-π-open is soft open implies (A,E) is soft-pre-open and thus (A,E)$\tilde{\subseteq}$ Int(Cl(A,E)). Therefore, we have Int(Cl(A,E))=(A,E), which shows that (A,E) is soft-regular open.
(2) ⇒(1) Since every soft-regular open set is soft-π-open then spcl(A,E) =(A,E) and spcl(A,E)$\tilde{\subseteq}$ (A,E). Hence (A, E) is soft-πgp-closed.

Theorem 3.10: For a soft subset (A,E) of X, the following statements are equivalent:
(1) (A,E) is soft-π-clopen.
(2) (A,E) is soft-π-open, a Q^s- set and soft-πgp-closed.

Proof: (1) ⇒ (2) Let (A, E) be a soft-π-clopen subset of X. Then (A, E) is soft-π-closed and soft-π-open. Thus (A, E) is soft-closed and soft-open. Therefore, (A, E) is a Q^s-set. Since every soft-π-closed is soft-πgp-closed then (A, E) is soft-πgp-closed.
(2)⇒(1) by above theorem, (A,E) is soft-regular open. Since (A,E) is a Q-set, (A,E) =Int(Cl(A,E))=Cl(Int(A,E)).Therefore (A,E) is soft-regular closed. Then (A, E) is soft-π-closed. Hence (A, E) is soft-π-clopen.

Remark 3.11: Finite union of soft-πgp-closed sets is soft-πgp-closed.

Proof: Let (A,E) and (B,E) be soft-πgp-closed subset of X.Let (U,E) be a soft-π-open in (X,τ,E),such that (AUB,E) $\tilde{\subseteq}$ (U,E).Then pcl(A,E) $\tilde{\subseteq}$ (U,E) and pcl(B,E) $\tilde{\subseteq}$ (U,E).Therefore pcl(AUB,E) $\tilde{\subseteq}$ pcl(A,E)U pcl(B,E) $\tilde{\subseteq}$ (U,E).This implies that pcl(AUB,E) $\tilde{\subseteq}$ (U,E).Hence (A,E)U(B,E) is a soft-πgp-closed set.

Remark 3.12: Finite intersection of soft-πgp -closed sets need not be soft-πgp- closed.

Example 3.13: In example 3.4, let the two soft sets be
$G(e_1)$={a,b,d},$G(e_2)$={b,c,d}
$H(e_1)$={a,b,c},$H(e_2)$={a,c,d}.
Then (G,E) and (H,E) are soft πgp-closed sets over X. But their intersection (A,E)={{a,b},{c,d}} is not soft πgp-closed.

Theorem 3.14: Let X be a soft-hyperconnected space. Then every soft-πgp-closed subset of X is soft-πgs- closed.

Proof: Assume that X is soft-hyperconnected. Let (A,E) be soft-closed and let (U,E) be an soft-π-open set containing (A,E). Then spcl(A,E)=(A,E) $\tilde{\subseteq}$ (A,E)⋃ Int(Cl(A,E)) = sscl(A,E). Since spcl((A,E)) = sscl((A,E)), sscl((A,E)) $\tilde{\subseteq}$ (U,E). Hence, (A,E) is soft-πgs- closed.

Theorem 3.15: If (A,E) is soft-πgp-closed set and (B,E) is any soft subset such that (A,E) $\tilde{\subseteq}$ (B,E) $\tilde{\subseteq}$ spcl (A,E), then (B,E) is soft-πgp-closed set.

Proof: Let (B,E) $\tilde{\subseteq}$ (U,E) and (U,E) be soft-π-open. Given (A,E) $\tilde{\subseteq}$ (B,E). Then (A,E) $\tilde{\subseteq}$ (U,E). Since (A,E) is soft -πgp- closed, (A,E) $\tilde{\subseteq}$ (U,E) implies spcl(A,E) $\tilde{\subseteq}$ (U,E). By assumption spcl(B, E) $\tilde{\subseteq}$ spcl (A, E) $\tilde{\subseteq}$ (U, E). Hence (B, E) is a soft πgp-closed set.

4. Soft-πgp-open sets and Soft-πgp-operators

Definition 4.1: A soft subset (A,E) $\tilde{\subseteq}$ X is called soft-πgp-open if and only if its relative complement is soft-πgp-closed. The soft-πgp-open set of X is denoted by SπGPC(X) which means the family of all soft-πgp-open subsets of the space X.

Remark 4.2: Let (F, A) be a soft subset of a topological space X, then spcl (X-(F, A)) = (X-spint (F,A)).

Proof: Let x∈ X-spint ((F,A)).Then x∉spint(F,A).That is every soft-pre-open set (G,A) containing x is such that (G,A) $\tilde{\subseteq}$ (F,A).Hence every soft-pre-open set (G,A) containing x intersect X-(F,A).This implies x∈spcl(X-(F,A)).Hence X-spint(F,A) $\tilde{\subseteq}$ spcl(X-(F,A)).
Conversely, Let x∈spcl(X-(F,A))Thus every soft-pre-open (H,A) containing x intersect (X-(F,A)).That is every pre-open set (H,A) containing x is such that (H,A) $\tilde{\subseteq}$ (F,A).This implies x∉spint(F,A).Thus spcl(X-(F,A)) $\tilde{\subseteq}$ X-spcl(F,A).Hence spcl(X-(F,A))=(X-spint(F,A)).

Theorem 4.3: The soft subset (A,E) of X is soft-πgp-open iff F$\tilde{\subseteq}$ spint(A,E) whenever (F,E) is soft-π-closed and (F,E) $\tilde{\subseteq}$ (A,E).

Proof: Necessity: Let (A, E) be soft-πgp-open. Let (F, E) be soft-π-closed and (F,E) ⊂(A,E). Then X-(A,E) $\tilde{\subseteq}$ X-(F,E) where X-(F,E) is soft-π-open. By assuming, spcl(X-(A, E)) ⊂ X-(F,E). By above Remark 4.2, X-spint (A,E) $\tilde{\subseteq}$ X-(F,E). Thus (F,E) $\tilde{\subseteq}$ spint (A,E).

Sufficiency: Suppose (F,E) is soft-π- closed and (F,E) $\tilde{\subseteq}$ (A,E) such that (F,E) $\tilde{\subseteq}$ spint(A,E). Let X-(A,E) $\tilde{\subseteq}$ (U,E) where (U,E) is soft-π-open. Then X-(U, E) $\tilde{\subseteq}$ (A, E) where X-(U, E) is soft-π-closed. By hypothesis, X-(U, E) $\tilde{\subseteq}$ spint (A, E).That is X-spint (A, E) $\tilde{\subseteq}$ (U, E). Hence spcl(X-(A, E)) $\tilde{\subseteq}$ (U, E). Thus X-(A, E) is soft-πgp-closed and A is soft-πgp-open.

Theorem 4.4: If spint (A,E) $\tilde{\subseteq}$ (B,E) $\tilde{\subseteq}$ (A,E) and (A,E) is soft-πgp-open then (B,E) is soft-πgp-open.

Proof: Let spint (A,E) $\tilde{\subseteq}$ (B,E) $\tilde{\subseteq}$ (A,E). Thus X-(A,E) $\tilde{\subseteq}$ X-(B,E) $\tilde{\subseteq}$ X-spint(A).That is X-(A,E) $\tilde{\subseteq}$ X-(B,E) $\tilde{\subseteq}$ spcl (X-(A,E)) by the above remark 4.2. Since X-(A,E) is soft-πgp- closed, by theorem 3.15,(X-(A,E)) $\tilde{\subseteq}$ (X-B) $\tilde{\subseteq}$ spcl (X-(A,E)) implies (X-(B,E)) is soft-πgp- closed. Hence (B,E) is soft-πgp-open.

Remark 4.5: For any soft subset (A,E) of X, spint (spcl(A,E) -(A,E))= ϕ.

Theorem 4.6: If (A, E) $\tilde{\subseteq}$ X is soft-πgp- closed, then spcl (A,E) -(A,E) is soft-πgp-open.

Proof: Let (A,E) be soft-πgp-closed let (F,E) be soft-π-closed set such that (F,E) $\tilde{\subseteq}$ spcl (A,E) -(A,E). By theorem 3.7**,** (F,E) = ϕ.Thus (F,E) $\tilde{\subseteq}$ spint (spcl(A,E) -(A,E)).Hence spcl(A,E) -(A,E) is soft-πgp-open.

Theorem 4.7: The intersection of two soft πgp-open sets is again a soft πgp-open.

Proof: Let (A,E) and (B,E) are soft-πgp-open sets. Suppose (G,E) is soft-π-closed set such that (G,E) $\tilde{\subseteq}$ (A∩B,E).Then (G,E) $\tilde{\subseteq}$ (A,E) and (G,E) $\tilde{\subseteq}$ (B,E).Since (A,E) and (B,E)are soft-πgp-open sets,(G,E) $\tilde{\subseteq}$ int(A,E) and (G,E) $\tilde{\subseteq}$ int(B,E).Therefore (G,E) $\tilde{\subseteq}$ int (A,E) ∩ int (B,E). Thus (G,E) $\tilde{\subseteq}$ int (A∩B,E).Hence (A∩B,E) is soft-πgp-open set.

Theorem 4.8: The union of two soft πgp-open sets need not be soft πgp-open sets and is shown.
Example 3.4 let the two soft sets be
$G(e_1)=\{c,d\}, G(e_2)=\{a,b\}$
$H(e_1)=\{a,d\}, H(e_2)=\{b,d\}$.
Then (G,E) and (H,E) are soft-πgp-closed sets over X. But their union (A,E)={{a,c,d},{a,b,d}} is not soft-πgp-closed.

Definition 4.9: Let (X,τ,E) be a soft topological space and (x,E) ∈X. A subset (A,E) of X is called a soft-πgp-neighbourhood (soft-πgp-nbhd) of (x,E),if there exist a soft-πgp-open set (U,E) such that (x,E)∈(U,E) $\tilde{\subseteq}$ (A,E).

Definition 4.10: Let (X,τ,E) be a soft topological space and (A,E) be a subset of X.A point (x,E)∈(A,E) is said to be soft-πgp-interior point of (A,E),if (A,E) is a soft-πgp-nbhd of (x,E).
The set of all soft-πgp-interior points of (A,E) is called the soft-πgp-interior of (A,E) and it is denoted by soft-πgp-int (A,E).

Proposition 4.11: Let (A,E) and (B,E) be a subset of (X,τ,E).Then
(i) soft-πgp-int(ϕ)= ϕ and soft-πgp-int(X)=X.
(ii) soft-πgp-int(A,E) $\tilde{\subseteq}$ (A,E).
(iii) If (B,E) is any soft-πgp-open set contained in (A,E) then (B,E) $\tilde{\subseteq}$ soft-πgp-int(A,E)
(iv) If (A,E) $\tilde{\subseteq}$ (B,E) then soft -πgp-int(A,E) soft-πgp-int(B,E).
(v) soft πgp-int (soft πgp-int (A,E))=soft πgp-int(A,E).

Proof: The proof is Obvious.

Theorem 4.12: If (A,E) and (B,E) are subsets of X,then soft πgp-int(A,E) U soft πgp-int(B,E) $\tilde{\subseteq}$ soft-πgp-int(AUB,E).

Proof: Let (A,E) $\tilde{\subseteq}$ (AUB,E) and (B,E) $\tilde{\subseteq}$ (AUB,E).Then Soft πgp-int(A,E) $\tilde{\subseteq}$ soft πgp-int(AUB,E) and Soft-πgp-int(B,E) $\tilde{\subseteq}$ soft-πgp-int(AUB,E).Therefore soft-πgp-int (A,E)U soft-πgp-int(B,E) $\tilde{\subseteq}$ soft-πgp-int(AUB,E).

Definition 4.13: For a subset (A,E) of (X,τ,E),the soft-πgp-cl(A,E) is the intersection of all soft-πgp-closed sets containing (A,E).

Proposition 4.14: Let (A,E) and (B,E) be subsets of (X,τ,E).Then
(i) soft-πgp-cl(ϕ)= ϕ and soft-πgp-cl(X)=X.
(ii) (A,E) $\tilde{\subseteq}$ soft-πgp-cl(A,E).
(iii) If (B,E) is any soft-πgp-open set contained in (A,E) then soft-πgp-int(A,E) $\tilde{\subseteq}$ (B,E).
(iv) If (A,E) $\tilde{\subseteq}$ (B,E) then soft-πgp-cl(A,E) $\tilde{\subseteq}$ soft-πgp-cl(B,E).
(v) Soft-πgp-cl(A,E)=soft-πgp-cl(soft-πgp-cl(A,E).

Proof: The proof is Obvious.

Proposition 4.15: If (A,E) is a subset of X,Then soft-πgp-cl(A,E) $\tilde{\subseteq}$ cl(A,E).

Proof: Since every soft closed sets is soft-πgp-closed set.cl(A,E)=∩ {(A,E) $\tilde{\subseteq}$ (F,E) ∈ C(X)}.If (A,E) $\tilde{\subseteq}$ (F,E)∈C(X).Then (A,E) $\tilde{\subseteq}$ (F,E)ϵ soft-πgp-C(X).i.e soft-πgp-cl(A,E) $\tilde{\subseteq}$ (F,E).Therefore soft-πgp-cl(A,E) $\tilde{\subseteq}$ ∩{(A,E) $\tilde{\subseteq}$ (F,E)ϵC(X)}=cl(A,E).Hence soft-πgp-cl(A,E) $\tilde{\subseteq}$ (A,E).

Proposition 4.16: Let (A,E) be a soft-πgp-open set and (B,E) be any set in X.If (A,E)∩(B,E) = ϕ.Then (A,E)∩ soft-πgp-cl(B,E)= ϕ.

Proof: Suppose ∩ soft-πgp-cl(B,E) ≠ ϕ and (x,E)∈(A,E)∩ soft-πgp-cl(B,E).Then (x,E)∈(A,E) and (x,E)∈(A,E)∩soft-πgp-cl(B,E).Therefore (A,E)∩(B,E)≠ which is contradiction. Hence (A,E)∩soft-πgp-cl(B,E)=.Hence the proof.

5. Soft-πgp-$T_{1/2}$ spaces

Definition 5.1: A soft topological space X is called soft-πgp-$T_{1/2}$ space if every soft-πgp- closed set is soft pre-closed.

Theorem 5.2: For a soft topological space (X, τ, E) the following are equivalent

(i) The soft topological space X is soft-πgp-$T_{1/2}$ space.

(ii) Every singleton set is either soft-π-closed or soft-pre-open.

Proof: To prove (i) ⇒ (ii): Let X be a soft-πgp-$T_{1/2}$ space. Let (A,E) be a soft singleton set in X and assume that (A,E) is not soft-π-closed. Then X-(A, E) is not soft-π- open. Hence X -(A,E) is trivially a soft-πgp-closed. Since X is soft-πgp- $T_{1/2}$ space, every soft-πgp-closed set is soft-pre-closed. Then X-(A, E) is soft-pre-closed. Therefore (A, E) is soft-pre-open.

(ii) ⇒(i): Assume every singleton of X is either soft-π-closed or soft-pre-open. Let (A, E) be soft-πgp-closed set.

Let (A,E) ∈ spcl (A,E).To Prove: spcl(A,E) (A,E)

Case (i): Let the singleton set (F,E) be soft-π-closed. Suppose (F, E) does not belong to (A,E). Then (F,E) ∈ pcl (A,E) - (A,E). By theorem 3.7, (F,E) ∈ (A,E). Hence spcl (A,E) $\tilde{\subseteq}$ (A,E).

Case (ii): Let the singleton set (F,E) be soft-pre-open. Since (F,E) ∈ spcl (A,E) we have

(F,E)∩ (A,E) ≠ ϕ.Hence (F,E) ∈ (A,E). Therefore spcl (A,E) $\tilde{\subseteq}$ (A,E) or equivalently (A,E) is soft-pre-closed.

Definition 5.3: (i) A soft topological space X is called Soft-πgp- space if every soft-πgp-closed is soft-closed.

(ii)A soft topological space X is called Soft-$T_{\pi gp}$ -space if every soft-πgp-closed set is soft- πg-closed.

Proposition 5.4: (i) Every soft-πgp- space is soft-πgp-$T_{1/2}$ space.

(ii) Every soft-πgp- space is soft-$T_{\pi gp}$ –space.

Proof:

Theorem 5.5: SPO(X, τ, E) $\tilde{\subseteq}$ SπGPC(X, τ, E)

Proof: Let (A,E) be soft-pre-open, then X- (A,E) is soft-pre-closed. So X- (A,E) is soft-πgp-closed. Thus (A, E) is soft-πgp-open. Hence SPO (τ) $\tilde{\subseteq}$ SπGPO(τ).

Theorem 5.6: For a soft topological space (X, τ, E), the following are equivalent

(i) (i)X is soft-πgp-$T_{1/2}$ space.

(ii) For every soft subset (A, E) of X, (A, E) is soft-πgp-open if and only if (A, E) is soft-pre-open.

Proof: (i) ⇒ (ii) Let X be soft-πgp-$T_{1/2}$ space and let (A, E) be a soft-πgp-open subset of X. Then X- (A, E) is soft-πgp-closed set and so X- (A, E) is soft-pre-closed. Hence (A,E) is soft-pre-open.

Conversely, Let (A,E) be soft-pre-open subset of X. Thus X-(A, E) is soft-pre-closed. Since every soft-pre-closed set is soft-πgp-closed then X- (A, E) is soft-πgp-closed. Hence (A, E) is soft-πgp-open.

(ii)⇒ (i) let (A, E) be a soft-πgp-closed subset of X. Then X-(A, E) is soft-πgp-open. By the hypothesis X- (A,E) is soft-pre-open. Thus (A, E) is soft-pre-closed. Since every soft-πgp-closed is soft-pre-closed, X is soft-πgp-$T_{1/2}$ space.

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