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Mobin Ahmad
Professor, Department of Mathematics, Faculty of Science, Jazan University, Saudi Arabia

Correspondence
Mobin Ahmad
Professor, Department of Mathematics, Faculty of Science, Jazan University, Saudi Arabia

A robust study on operations research-optimization theory

Mobin Ahmad

Abstract
In this paper we give introduce to the many faces of Optimization Theory and Operations Research. In the development of Interior-Point Methods has a profound impact on optimization theory as well as practice, influencing the field of Operations Research and related areas. Development of these methods has quickly led to the design of new and efficient optimization codes particularly for Linear Programming. We discuss conceptual as well as technological aspects, and description examples representing impact and excellence. In present time introduce and improvement has a profound impact on optimization theory and Operations Research as well as performs, influencing the field of Operations Research and related areas. Improvement of this technique has speedily led to the plan and resourceful optimization codes particularly for Linear Programming. The two concepts below this theme, “Optimization Theory” and “Operations Research” and their relationship the attempt to explain accurately what is meant by these terms is not equally easy for both concepts. While optimization has developed out of mathematics, a classical science, and thus respite on a hard theoretical foundation and has many academic aspects and roots, Operations Research is a quite youthful, simply practical, and virtually oriented synthesis of different disciplines, including mathematics.

Keywords: Operations Research, Optimization Theory, design, excellence, development, methods, Linear Programming, relationship.

Introduction
Optimization is one of the most fundamental tools in operations research. This is so, not only in the theory and mathematics of operations research but also in applications. Optimization can be seen as a very powerful and elegant generalization of linear optimization. Optimization has had tremendous success since the birth of operations research. Due to the fast advances in science and technology and rapid distribution of newly acquired knowledge as well as very wide accessibility of this knowledge, we see nowadays that those in charge of main decision making functions in many applications are more and more willing to utilize sophisticated operations research techniques.

Optimization is that obedience within applied mathematics that deals with optimization problems, or so-called mathematical programs. In an optimization difficulty or mathematical program we seek to reduce or maximize a (real valued) function over a set of (decision) variables theme to constraints. Since there is no “official” description of Operations Research (OR), the introduction of this concept has to be more expressive. In what follows, we try to summarize the middle center of OR as a “complexity and technique oriented management approach” and demonstrate the “natural world” of this concept, describing several surfaces and characteristic features such as quantification, model based optimization, and interdisciplinary. Or is difficulty oriented. Operations research is a difficulty resolving approach. It is called “operational research” where the term was coined throughout the Second World War when scientists were asked to examine and support the resolution of a range of services problems in connection with the deployment of radar, and so on. The term was invented to differentiate “research on operations” from technological research. Thus, in difference to mathematical programming, the difficulties with which or manages to cope are not conceptual and formal but specific and real.

Motivation and Some Definitions

Consider a typical Linear Programming problem. Let $A \in \mathbb{R}^{m\times n}$ represent the given matrix and assume that the objective function vector $c \in \mathbb{R}^n$ n and the right-hand-side vector $b \in \mathbb{R}^m$ m are also given. Our primal problem, written in a standard equality form is (we will refer to it as (LP)):

$$\begin{array}{lrcl} \min & c^T x & & \\ & Ax & = & b, \\ & x & \geq & 0. \end{array}$$

It's dual (LD) is defined to be

$$\begin{array}{lrcl} \max & b^T y & & \\ & A^T y + s & = & c, \\ & s & \geq & 0. \end{array}$$

We denote by S^n, the space of n-by-n symmetric matrices with real entries. $X \in S^n$ is called positive semi definite if

$$h^T X h \geq 0, \forall h \in \mathbb{R}^n.$$

We denote by $\lambda_j(X)$ the eigenvalues of X. Note that every eigenvalue of every $X \in \mathbb{S}^n$ n is real. We index the eigenvalues so that

$$\lambda_1(X) \geq \lambda_2(X) \geq \cdots \geq \lambda_n(X)$$

Proposition 1

Let $X \in \mathbb{S}^n$. Then, the following are equivalent:

(a) X is positive definite;

(b) $\lambda_j(X) \geq 0, \forall j \in \{1, 2, \ldots, n\};$

(c) There exists $B \in \mathbb{R}^{n\times n}$ nonsingular, such that $X = BB^T;$

(d) For every $J \subseteq \{1, 2, \ldots, n\}, \ \det(X_J) \geq 0, \ where \ X_J := [X_{ij} : i, j \in J]$.

The set of positive semi definite matrices by $\mathbb{S}^n_+$. the sub matrices of X described in part (d) of the above proposition, symmetric minors of X. X ∈ S n is called positive definite if

$$h^T X h > 0, \forall h \in \mathbb{R}^n \setminus \{0\}.$$

Proposition 2

Let $X \in \mathbb{S}^n$. then, the following are equivalent:

(a) X is positive definite;

(b) $\lambda_j(X) > 0, \forall j \in \{1, 2, \ldots, n\};$

(c) There exists $B \in \mathbb{R}^{n\times n}$ nonsingular, such that $X = BB^T;$

(d) For every $J_k := \{1, 2, \ldots, k\}, \ k \in \{1, 2, \ldots, n\}, \ \det(X_J) > 0.$

We denote the set of symmetric positive definite matrices over real's by $\mathbb{S}^n_{++}$. Using Proposition 1 part (c) and Proposition 2 part (c), we deduce

$$\mathbb{S}^n_{++} = \left\{ X \in \mathbb{S}^n_+ : X \text{ is nonsingular} \right\}.$$

For $U, V \in \mathbb{S}^n$, we write $U \succeq V$ to mean $(U - V) \in \mathbb{S}^n_+$ and $U \succ V$ to mean $(U - V) \in \mathbb{S}^n_{++}$.

Another fact about the positive semi definite matrices that is useful in modeling problems is the Complement Lemma:

Lemma 1

Let $X \in \mathbb{S}^n$, $U \in \mathbb{R}^{n\times m}$ and $T \in \mathbb{S}^m_{++}$. Then

$$M := \begin{bmatrix} T & U^T \\ U & X \end{bmatrix} \succeq 0, \iff \left(X - UT^{-1}U^T\right) \succeq 0.$$

The matrix $(X - UT^{-1}U^T)$, the Schurz Complement of *T* in *M* in the above notation

Note that in our standard form (LP) and (LD), the constraints x ≥ 0, s ≥ 0 mean that x and s lie in $\mathbb{R}^n_+$ which is a convex cone (that is, for every positive scalar α, and for every element x of the set, αx is also in the set and, for every pair of elements u, v of the set, (u + v) also lies in the set). In particular, this convex cone is just a direct sum of nonnegative rays:

$$\mathbb{R}^n_+ = \mathbb{R}_+ \oplus \mathbb{R}_+ \oplus \cdots \oplus \mathbb{R}_+.$$

Replacing R_+ by more general convex cones (which contain R+ as a special case) we can generalize (LP). In SDP, we replace $\mathbb{R}_+$ by $\mathbb{S}^{n_i}_+$ for some $n_i \geq 1$. (Note that for $n_i = 1$, we have $\mathbb{S}^{n_i}_+ = \mathbb{R}_+$.) Thus, in this more general optimization problem we write our variable x as a symmetric matrix (possibly with a block diagonal structure) and replace the constraint "x ≥ 0" by

$$X \in \mathbb{S}_+^{n1} \oplus \mathbb{S}_+^{n2} \oplus \cdots \oplus \mathbb{S}_+^{nr}.$$

Clearly, setting $n_1 := n_2 := \cdot \cdot := n_r := 1$ and $r := n$ takes us back to (LP) as a special case

Another very interesting special case is given by the cone:

$$SO^n := \left\{ \begin{pmatrix} x_0 \\ x \end{pmatrix} \in \mathbb{R} \oplus \mathbb{R}^n : x_0 \geq \|x\|_2 \right\}.$$

First notice that

$$SO^n := \mathrm{cl} \left\{ \begin{pmatrix} x_0 \\ x \end{pmatrix} \in \mathbb{R} \oplus \mathbb{R}^n : x_0 > \|x\|_2 \right\},$$

Where cl(·) denotes the closure. Secondly, using the Schurz Complement Lemma, we see that

$$\begin{pmatrix} x_0 \\ x \end{pmatrix} \in SO^n \iff \begin{bmatrix} x_0 & x^T \\ x & x_0 I \end{bmatrix} \succeq 0.$$

Let an: $\mathcal{A}_n : \mathbb{R}^{n+1} \to \mathbb{S}^{n+1}$ denote the linear operator satisfying

$$\mathcal{A}_n \begin{pmatrix} x_0 \\ x \end{pmatrix} = \begin{bmatrix} x_0 & x^T \\ x & x_0 I \end{bmatrix}.$$

Then,

$$\left(x^{(1)}, x^{(2)}, \ldots, x^{(r)}\right) \in SO^{n_1} \oplus SO^{n_2} \oplus \cdots \oplus SO^{n_r}$$

If,

$$\left(\mathcal{A}_{n_1}\left(x^{(1)}\right), \mathcal{A}_{n_2}\left(x^{(2)}\right), \ldots, \mathcal{A}_{n_r}\left(x^{(r)}\right)\right) \in \mathbb{S}_+^{n_1+1} \oplus \mathbb{S}_+^{n_2+1} \oplus \cdots \oplus \mathbb{S}_+^{n_r+1}.$$

For further information on the algebraic and analytic structure

From now on, we will usually write $X \in \mathbb{S}_+^n,\ X \succeq 0,$ etc.; however, depending on the context we might mean

$$X \in \mathbb{S}_+^{n_1} \oplus \mathbb{S}_+^{n_2} \oplus \cdots \oplus \mathbb{S}_+^{n_r},\ \text{where } n_1 + n_2 + \cdots + n_r = n.$$

represent data by $C, A_1, A_2, \ldots, A_m \in \mathbb{S}^n$ and $b \in \mathbb{R}^m$ such that primal is

$$\begin{array}{llrcl} (P) & \inf & \langle C, X \rangle & & \\ & & \langle A_i, X \rangle & = & b_i, \quad \forall i \in \{1, 2, \ldots, m\} \\ & & X & \succeq & 0. \end{array}$$

In the above optimization problem, the inner product is the trace inner product. I.e., for $U, V \in \mathbb{R}^{n \times n}$,

$$\langle U, V \rangle := \mathrm{Tr}\left(U^T V\right).$$

We define the dual of (P) as

$$\begin{array}{llrcl} (D) & \sup & b^T y & & \\ & & \sum_{i=1}^{m} y_i A_i & \preceq & C. \end{array}$$

Or, equivalently

$$\begin{array}{llrcl} (D) & \sup & b^T y & & \\ & & \sum_{i=1}^{m} y_i A_i + S & = & C, \\ & & S & \succeq & 0. \end{array}$$

What is "Optimization"?

Discover integer values for the variables x, y, z and k with $k > 2$ such that $x^k + y^k - z^k$ is minimal.

Thus Optimization is difficulty oriented, yet these problems are merely ideal and formal construct. Carl Friedrich Gauss, the great German mathematician, claimed problems like the Fermat difficulty were of no importance for him since the resolution would yield no generalizable imminent or knowledge. But research on this kind of apparently merely reserved and theoretical difficulties has been the engine for improvement in mathematics important to the improvement of many "appropriate" theories. In linear programming we study optimization problems of the following form:

$$\text{maximize} \quad c_1 x_1 + \cdots + c_n x_n \tag{1}$$

$$\text{subject to} \quad a_{1,1} x_1 + \cdots + a_{1,n} x_n \leq b_1 \tag{2}$$

$$a_{m,1} x_1 + \cdots + a_{m,n} x_n \leq b_m \tag{3}$$

$$x_1 \geq 0, \ldots, x_n \geq 0 \tag{4}$$

Such difficulties are called linear optimization problems or linear programs (LP), since all functions involved are linear. The mathematical behavior of this difficulty has full-grown out of the area of linear algebra, one of the basic mathematical regulation, which, because of its fundamental position, is an important part of each mathematical curriculum. A particular importance of this difficulty group stems from the fact that the linear programming paradigm can provide as the formal model for a number of financial resources allocation tribulations.

Consider for example the difficulty of determining the most beneficial mix of foodstuffs for a manufacturer. For every of the n foodstuffs a resolution variable is introduced to represent its manufacture charge. Let cj be the profit per unit of product j produced. The choice of the product mix is embarrassed by limited capability of the manufacture services which are required for production of these foodstuffs. Let m be the numeral of dissimilar kind of amenities required, for every category the amount accessible during the planning stage, and the amount of capability i used by each unit of product j which is produced. Then resolving the associated linear program gives the optimal product mix.

In combinatorial optimization the mathematical speech is not supported on actual numbers and (in) equalities but on "sets". Let S ⊆ 2E, be a collection on some limited ground set E and c: S → R. Then the task is to find a subset S*∈ S that maximizes (or minimizes) c on S, i.e. c(S*) ≥ c(S) (or c(S*) ≤ c(S)) for all S ∈ S. Set theory, mathematical logic, and diagram theory are the mathematical disciplines, and once more the significance stems from the actuality that fairly dissimilar actual world problems of optimal system plan can be explained using this mathematical formalism and "resolved" by applying the mathematical techniques developed: for instance, the plan of telecommunication networks and distribution systems.

Consider for occurrence the necessity to establish a communiqué network between n sites by subversive cable. The sites can be interpreted as nodes in a diagram, and each connection of two sites constitutes a border among the two nodes with a definite length or cost. Now we want to decide a subset of connections of minimum totality cable length or cost, such that any two sites are attached by a path of involving cables. This difficulty can be resolved by determining the spanning tree of negligible cost in the associated diagram.

In difference to this area, which is based on static and deterministic concepts, the field of stochastic choice development focuses on sequential relationships between random variables and resolution. Here we study dynamic non-deterministic method where at definite decision time points t a controller/decision-maker chooses an action a(t) from a set of accessible measures A based on the observed state S(t) of the system. The optimization problem is to choose a sequence of actions (a(t))t=0,1,2,... , also called a strategy, that will maximize the performance of the system over a given time possibility. Here, the fields of possibility theory, stochastic methods, differential equations, and several others are the mathematical prerequisites, and the special interest stems from the applicability of this concept to describing and controlling difficulties real world processes like queuing, inventorying, and investment.

Consider for example the following production–inventory difficulty. At the beginning of every period t, a firm must choose how many units x(t) should be produced during the present period at a per unit variable production cost of c units. For every period t, the period's random demand d(t) is experiential and met out of present production and inventory, and at the end of each stage the firm holds a definite end of stage inventory i(t) for which a per unit holding cost of h units occurs. Given a certain initial inventory i(0) and a possibility distribution for every period's demand, the difficulty is to determine a production strategy that minimizes the expected net cost incurred during a particular number of periods.

The period "mathematical programming", which is used as a synonym for optimization, refers to the study of these kinds of difficulties, i.e.:

- Their mathematical properties (optimization theory)
- The progress and implementation of algorithms (arithmetical analysis and algorithmic design), and also
- The application of these programs and algorithms to real world problems.

To clarify a common misinterpretation, mathematical programming does not particularly submit to computer programming. Here programming refers to the improvement of a plan or process for dealing with a difficulty.

What is "Operations Research"?

Today the term "Operations Research" means a scientific approach to the solution of problems in the management of complex systems arising in industry, government, and the military and other areas. Under this aspect a frequent substitute for the term "management science" (MS), and often both terms are combined and the discipline is referred as "MS." MS aims to provide a rational basis for decision-making by seeking to understand and structure complex situations and to use this understanding.

System oriented

Operations research adopts an organizational viewpoint and attempts to:

- Search for causes of a problem indicated through system failure, malfunction, or under-performance
- Identify potentials and possibilities for improving system performance
- Evaluate the effect of changes in any part of a system on the performance of the system as a whole, and
- Redesign structures and processes appropriately and implement these changes in the system.

Here usually concerned with systems in which human behavior plays an important role, and it attempts to resolve the conflicts of interest among the components of the organization in a way that is best for the organization as a whole. This distinguishes from systems engineering (SE), which concentrates more on systems in which human behavior is of minor importance.

Decision approach

Operations research is not a science itself, but rather the application of science to the solution of managerial problems. As can be seen from the examples above, the subject matter of consists of decisions that control the operation of systems. Managerial decisions are made but how they should be made. Prescriptive and normative, not a descriptive, approach to decision-making yet Operations Research is not "decision-making", it is only a decision aid or decision support for the (human) decision-maker, the firm's management says. However, the results of project or study must have direct and unambiguous implications for executive actions.

Since the emphasis on making decisions and taking actions is central to all applications, another term related to "decision analysis" (DA). A decision analysis process may take two basic forms: qualitative or quantitative. While qualitative analysis is based primarily on the manager's experience and judgment, in a quantitative approach an analyst will concentrate on the "hard", quantitative facts or the data he or she thinks to be relevant to the problem. The motivation for applying a quantitative approach stems from the complexity, importance, and novelty of many managerial problems, which should prevent the management from relying on purely "soft" information and intuition. If the problem is repetitive, management gains efficiency by creating and applying routine decision recommendations based on quantitative analysis.

From the point of view given so far, one may use the terms Operations Research, management science, and quantitative decision analysis almost interchangeably.

A common definition of Operations Research is the following:

Operations research is concerned with scientifically deciding how to best design and operate man–machine systems, usually under conditions requiring the allocation of scarce resources.

A few examples to help illustrate the scope of Operations Research were given by the Operational Research Society of Great Britain.

- "There are too many lorries on the road." A common cry but something can be done! A bakery used to devise an efficient scheduling system for its delivery vehicles. The new system reduced lorry mileage, road congestion, and pollution as well as saving money for the bakery.
- "I had to wait all morning in hospital." Great pressure on consultants' time, coupled with some patients who do not always keep their appointment times, can cause real problems for hospitals. But by using, appointment systems have been designed that substantially reduce waiting times whilst keeping highly qualified medical staff fully occupied.
- "We've just got to increase our sales." Easier said than done, but proved equal to the task for a mail order firm. The model helped boost catalog sales by designing an ideal mix of discounts, special promotions, and customer incentives.
- We know of some that don't, because they used one manager wanted to ensure the efficient operations of his new automated warehouse by simulating the operation of alternative material handling equipment. This meant that a selection could be made which eliminated any bottlenecks and delivered the required output.

The following two aspects of Operations Research establish the link to optimization.

Modeling

Central for any quantitative decision analysis is the foundation on mathematical models and the application of mathematical theories and methods such as mathematical programming/optimization, simulation, and graph theory. In this respect, model building can be viewed as the essence of the approach to problem solving, and mathematical methods as powerful tool-box. Here the (mathematical) decision model should abstract the crux of the decision-making problem. It should give a suitable and well-structured view of the underlying real situation, that is, a representation of the structure, function, and behavior of the system, the fundamental properties of system operations, and the organizational objective, such that the conclusions obtained from analyzing and manipulating the model are valid for the real problem.

Optimization

Another characteristic is that attempts to find the best possible solution to the problem/model and to identify the course of action which creates optimal performance of the real system. In other words, aims to recommend optimal decisions. This "search for optimality" is an important driver which, although not always realizable in practice, is inherent in the paradigm and distinguishes the goal-oriented approach from decision-making on the basis of "ground rules" representing expertise and (best) practice.

In the following paragraphs we want to focus on two other important features of Operations Research.

Interdisciplinary team approach

It is evident that no individual should be expected to have expertise on all aspects of the work: the problem domain, mathematical modeling, algorithmic implementation, and the design and technical implementation of information systems. Thus applying requires a team of individuals with diverse backgrounds and skills. This interdisciplinary viewpoint contributes to the charm as well as to the significance. It distinguishes from other disciplines, and from optimization, and it has been a central element of the beginning of its development in the middle of the last century. Its versatility – with respect to applications as well as methodologies – has always been the strength and the demand of Operations Research, but more than this, it has been its charm.

Knowledge and information management and processing

Optimization/Operations Research was developed in the middle of the last century, and it has evolved together with disciplines like computer science (CS) and information systems (IS). There has been a link between and CS/IS throughout this time. Solving models representing, for instance, gasoline blending problems in the petroleum industry, was among the first business applications for computers, and problems have constantly been a challenge for computer technology. On the other hand, the implementation of models and methods in a firm depends on the availability of high power computers and requires the integration into a computer (data) based information system. Operations research has been influenced significantly over the last fifty years by developments in the design of user interfaces, as well as in data/knowledge management for information systems, in telecommunications, and in distributed information processing, allowing models to access databases worldwide and in real time. These developments have changed the environment and the requirements as well as the impact, but they have not changed the basic paradigm and intention.

The close linkage between these fields and the change in the focus on information is best represented in the modern definition given by the Institute for Operations Research and the Management Sciences (INFORMS) the society which has been the result of the merger between the American Society of Operations Research (ORSA) and the Institute for Management Science (TIMS):
Operations Research and Management Sciences are the professional disciplines that deal with the application of information technology for informed decision-making.

Conclusion

In this paper we give introduce to the many faces of Optimization Theory and Operations Research. In the development of Interior-Point Methods has a profound impact on optimization theory as well as practice, influencing the field of Operations Research and related areas. Development of these methods has quickly led to the design of new and efficient optimization codes particularly for Linear Programming. We discuss conceptual as well as technical aspects, and report examples demonstrating impact and excellence. In recent years the introduction and development has a profound impact on optimization theory and Operations Research as well as practice, influencing the field of Operations Research and related areas. The concepts under this subject matter, “Optimization Theory” and “Operations Research” and their relationship the attempt to describe precisely is meant by these terms is not equally simple for both concepts.

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