ISSN Print: 2394-7500
ISSN Online: 2394-5869
Impact Factor: 5.2
IJAR 2016; 2(8): 731-734
www.allresearchjournal.com
Received: 17-06-2016
Accepted: 18-07-2016

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Analysis of queuing theory in a bank

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Abstract
In this paper we have used a highly modeling tool for M/M/C queuing model, the stochastic Birth-Death Markov process is also used. The model is multi sever queuing system with Poisson arrival and exponential distribution. Queue discipline is FCFS. This model is applied to Canara Bank Indore. It is analysed that the model is applicable and practical.

Keywords: Markov process, utilization factor, arrival rate, service rate

Introduction
Queuing theory is considered as the branch of operations research because the results are often used when making business decisions about the resource needed to provide service. Queuing theory is the mathematical study of waiting lines, or queues [2]. In queuing theory a model is constructed so that queue lengths and waiting times can be predicted [2]. Queuing theory is not too old. It started with research by A.K. Erlang when he created models to describe the Copenhagen telephone exchange [1]. The ideas have since seen applications including telecommunication [3, 4] and the design of factories, shops, offices and hospitals [5, 6] Queuing theory is concerned with the mathematical modelling and analysis of system that provide service to random demands. Typically the simple queuing models are specified in terms of the arrival process, the service mechanism and the queue discipline [7]. The word queue comes from the Latin word *Cauda*, meaning *tail*. The spelling queuing over queueing is typically encountered in the academic research field.

$$P\left[N(t-m+1)=\frac{n_{m+1}}{N(t_m)}=n_m,\ldots,N(t_1)=n_1\right]$$
$$=P\left[N(t-m+1)=\frac{n_{m+1}}{N(t_m)}=n_m\right] \tag{0.1}$$

And it should be valid for all $t_0 < t_1 < t_2 <\ldots < t_m < t_{m+1}$ at any m Transition Probability.
In equation (0.1) set $t_m = s, n_m = t, t_{m+1} = s + t$ and $n_{m+1} = j$. then the right hand side of the equation expreses the probability that the process makes a transition from state i at time s to state i in time t relative to s. such a probability denoted by $P_{i,j}(s,t)$, is referred to as astate transition probability for Markov process in this paper, we are only concerned with transition probabilities being independent of absolute times, i .e for all $s > 0$

$$P_{i,j}(s,t)=P_{i,j}(t)=P\left[N(t)=\frac{j}{N(0)}=i\right]=P\left[N(s)+\frac{j}{N(s)}=i\right] \tag{0.2}$$

This is called time-Homogeneous or stationary transition probabilities. In other words the intensity of leaving the state is constant in time.

Definition. Let $N(t), t \geq 0$ be a discrete Markov Process. if the conditional probabilities $P\left[N(s)+\frac{j}{N(s)}=i\right]$ For $s,t \geq 0$ do not depend on s, the process is said to be time homogeneous. Then we define the transition probability $P_{i,j}(t) = P\left[N(t)=\frac{j}{N(0)}=i\right]$ and the transition matrix $P(t)$ whose element with index $(i,j) is P_{i,j}(t).P_u(0) = 1$ and $P_{ij}(0) = 0$ for i 6= j so that $P(t) = 1$ For a Markov process with time homogeneous transition probabilities the so called Chapman-Kolmogorov equation implies,

$$P_{ij}(+s) = \sum_{k=0}^{\infty} P_{ik}(t)P_{kj}(s)$$

This equation states that in order to move from state i to J in time $(t+s)$, the queue size process $N(t)$ moves to some intermediate state k in time t and then from k to j in the remaining time$s$. It also says how to compute the long - interval transition probability from a sum of short - interval transition probability components.

An infinitesimal transition probability, denoted by $P_{ij}\Delta t$, specifies the immediate probabilistic behaviour of a Markov process in that$\Delta t \to 0$.

Generalized Markov Birth-Death Process

A birth-death Markov process is characterized by the fact that the discrete state variable changes by at most one, if it changes at all, during an infinitely small time interval. The generalized Markov birth-death process thus satisfies the following criteria: The probability distributions governing the numbers of births and deaths in a specific time interval depends on the length of the interval but not on its starting point. The probability of exactly one birth in a small time interval, t given that the population size at time t is n is $\tau_n\Delta t + 0(\Delta t)$, where τ_n is a constant,

The probability of one death in small time interval t given that the population size at time$t$ is n $\mu_n\Delta t$+$0(\Delta t)$ where μ_n is a constant. The probability of more than one birth and the probability of more than one death in a small time interval t are both $o(\Delta t)$.

Reflecting these facts, the following postulations specify the transition kernel of a general birth-death Markov process.

$$P[N(t+\Delta t) = \frac{n+1}{N(t)} = n] = \tau_n\Delta t + o(\Delta t), n \geq 0$$

$$P[N(t+\Delta t) = \frac{n-1}{N(t)} = n] = \tau_n\Delta t + o(\Delta t), n \geq 0 \quad (0.3)$$

$$P[N(t + \Delta t) = nN(t) = n] = 1 - (\tau_n + \mu_n)\Delta t, n \geq 0$$

$$P[N(t + \Delta t) = kN(t) = n] = 0 = |k - n| \geq 2$$

Here $0(\Delta t)$ is a quantity such that $\lim_{\Delta t} \to \infty = 0$ The first equation handles the case when then state variable increases by one i.e. $N(t) = n + 1$.This is referred to as single birth. Here τ_n is proportionality constant such that the product $\tau_n\Delta t$ should reflect the probability for a single birth to happen during the infinitesimal interval$(t + \Delta t)$.

We can write: It is customary to interpret τ_n as the instantaneous birth rate. Likewise, the second equation is for the case when the state variable is reduced by one i.e.$N(t) = n - 1$. This is referred to as single death. The product $\tau_n(\Delta t)$ signifies the probability that a single death takes place.

i.e.$N(t) = n$, during the infinitely small time interval. Multiple births, multiple deaths and simultaneous births and deaths are taken care of by the $0(\Delta t)$ terms in the equations. This should be interpreted such that the probability for these events to happen is negligible as$\Delta t \to 0$, we say that multiple events are prohibited. τ_n and also the death rate μ_n may depend on the departing state n. A small comment also applies to the second and third equations. Since no death can occur if the state variable is already zero i.e.

if $n = 0$, we always define $\mu_n = 0$

We can write

$$P_n(t+\Delta t) = \sum_{k=0}^{\infty} P\left[N(t+\delta t) = \frac{n}{N(t)} = k\right] P[N(t)k] (Baye's formula) \quad (0.4)$$

$$P_n(t)[1-(\tau_n-\mu_n)\Delta t+0(\Delta t)]+P_{n-1}(t)[\tau_{n-1}(\Delta t)+0(\Delta t)+P_{n+1}(t)[\mu_{n+1}(\Delta t+0(\Delta t)] + 0(\Delta t)$$

By rearranging the terms and dividind by Δt

$$\frac{P_n(t+\Delta) - P_n(t)}{\Delta t} = -(\tau_n + \mu_n)P_n(t) + \mu_{n+1}P_{n+1}(t) + \tau_{n-1}P_{n-1}(t) \quad (0.5)$$

Taking limit as $\Delta t \to 0$, we obtain the differential equation

$$P_n'(t) = \frac{dP_n}{dt} - (\tau_n - \mu_n)P_n(t) + \mu_{n+1}P_{n+1}(t) + \tau_{n-1}P_{n-1}(t)n = 1.2. \ldots, n \quad (0.6)$$

While (0.6) holds for $n = 1,2,...$ we also require an equation for $n = 0$. By following logical, arguement as above we can write $\frac{dP_n(t)}{dt} = -\lambda_0 P_0(t) + \mu_1 P_1(t)$

Equation (0.6) is the general model equation for a birth death Markov process and it essentially captures the probabilistic dynamics of the process

Steady State Solution

We now examine the process when it is in equilibrium (steady state). Under proper conditions, such equilibrium will be reached after the system has been operating for some time. Equilibrium in turn, implies that the state Probabilities $P_n(t)$ eventually become independent of t and approach a set of constant values (if it exists) which is denoted by $P_n, n = 1,2...$ as $t \to \infty$ where $P_n = \lim_{t\to\infty} P_n(t)$. This can be

interpreted as steady state probability that there are n users in the system.
Also under these circumstances in the steady state,

$$\lim_{t\to\infty} P_n'(t) = 0 \quad (0.7)$$

Given the above (.6) and (0.7) are then transformed to:

$$0 = -(\tau n + \mu n)Pn + \mu n{+}1Pn{+}1 + \tau n{-}1Pn{-}1, n = 1,2... \quad (0.8)$$
$$\Rightarrow -(\tau n + \mu n)Pn = +\mu n{+}1Pn{+}1 + \tau n{-}1Pn{-}1$$

and

$$0 = -\tau_0 P_0 + \mu_1 P_1$$
$$\Rightarrow \tau_0 P_0 = \mu_1 P_1 \quad (0.9)$$

Equations (0.8) and (0.9) are called the equilibrium equations or balance equations of birth and death Markov process.

Analysis of M/M/C queuing mode

M/M/C is a multi-server system with customer's arrival follows a Poisson process and exponential service time. When a customer enters an Empty system, he gets the service at once. If the system is non-empty the incoming customer joins the Queue. M/M/C model is a Poisson birth death process [1].
Birth Occurs = Customer arrives Death Occurs = Customer departs both are modelled as Memoryless Markov process.
In M/M/C, M refers to this memoryless/Markov feature of the arrival and service.
M/M/C Model is adopted with Poisson arrival and exponential service rate. The discipline rendered here is FCFS [3]. The main characteristics of this model are given by

1. The expected number of customers waiting in the queue

$$L_q = \left[\frac{1}{(c-1)!}\left(\frac{\tau}{\mu}\right)\frac{\tau\mu}{(c\mu-\tau)^2}\right]P_0$$

Where

$$P_0 = \frac{1}{\sum_{n=0}^{c-1}\frac{1}{n!}\left(\frac{\tau}{\mu}\right)^n \frac{1}{c!}\left(\frac{\tau}{\mu}\right)^c\left(\frac{\mu c}{\mu c-\tau}\right)}$$

2. The expected number of customers in the system is

$$L_s = L_q + \frac{\tau}{\mu}$$

3. Expected number of customers waiting to be served at any t is

$$L_w = \frac{c\mu}{c\mu - \tau}$$

4. The average waiting time of an arrival is

$$W_q = \frac{L_q}{\tau}$$

5. Average time an arrival spends in the system is

$$W_s = \frac{L_s}{\tau}$$

6. Utilization factor is

$$\rho = \frac{\tau}{c\mu}$$

Results

In Canara Bank, Indore, for duration of 15 days, in the working hours between 10.30 a.m. and 4.30 p.m. data regarding arrivals, departures, service patterns are observed. Those data are collected. Moreover service time and waiting time are calculated. A total arrival rate of 3221 customers per a total of 7200 minutes and a total service rate of 4213 customers per a total service times of 6128 is found in final analysis report.

1. $\tau = 3221/7200 = 0.44$
2. $\mu = 4213/6128 = 0.68$
3. $c\mu = 2.75 (c = 4)$
4. Utilization factor=$\rho = 0.1594 < 1$

$P_0 = 0.5284$ L_q
$= 0.0008$ $L_s = 0.6385$
$W_q = 0.0019$ W_s
$= 1.4511$

Utilization factor is greater than probability of Queuing in the system. It clearly indicates that as soon as the customer, enters the system, he is being served. No need for him to wait in the queue. Multi-channel servicing facility makes the Banking service more efficient. As the serving capacity increases the excessive waiting of man hours, congestion in banking halls, suffocation due to congestion are eliminated. Moreover this serving capacity will bring more profit, investments and shareholders to the Banks.
More bank branches to be established with multi-server facility across the country. Effective banking management will lead to a progress.

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