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Arti Tripathi
Department of Physical Science, Mahatma Gandhi Chitrakoot Gramodaya Vishwavidyalaya, Chitrakoot Pin. 485334, Satna, India

V Upadhyay
Department of Physical Science, Mahatma Gandhi Chitrakoot Gramodaya Vishwavidyalaya, Chitrakoot Pin. 485334, Satna, India

AK Agrawal
Department of physical science, Mahatma Gandhi Chitrakoot Gramodaya Vishwavidyalaya, Chitrakoot Pin. 485334, Satna, India

PN Pandey
Department of Mathematics, University of Allahabad, India

Correspondence
Arti Tripathi
Department of Physical Science, Mahatma Gandhi Chitrakoot Gramodaya Vishwavidyalaya, Chitrakoot Pin. 485334, Satna, India

A mathematical modeling on two phase mean blood flow pulmonary arterioles during emphysema due to smoking

Arti Tripathi, V Upadhyay, AK Agrawal and PN Pandey

Abstract
In the present paper, we will discuss the pulmonary blood flow in lungs. We have collected clinical data in case of COPD for Hematocrit v/s Blood pressure. Agrawal and Upadhyay have considered the blood flow of two phase, one of which is red blood cell and other is plasma. They have also applied the Herschel-Bulkley Non-Newtonian model in Bio-fluid mechanical setup. The graphical presentation for particular parametric value in much closed to the clinical observation. The overall presentation is in tensorial form and solution technique adopted is analytical as well as numerical.

Keywords: Pulmonary, Herschel Bulkley, arterioles, parametric value, lungs

Introduction
The lungs are sponge-like structures that lie with in chest, protested by the rib cage. They are made up of progressively branching air passages, the smallest of which end in minute air Sacs, where inhaled oxygen is transferred to the blood stream and carbon-di-oxide is transferred from the blood into the exhaled breath. Inhaled air is directed down the trachea (windpipe), which divides into two passages called bronchi and they distribute the inhaled air one to each lung. The bronchi divide into smaller tubes called bronchiole, and further still into tiny air sacs called alveoli. Each alveolus is enclosed by a fine mesh of capillaries, through which the exchange of oxygen and carbon-di-oxide takes place.

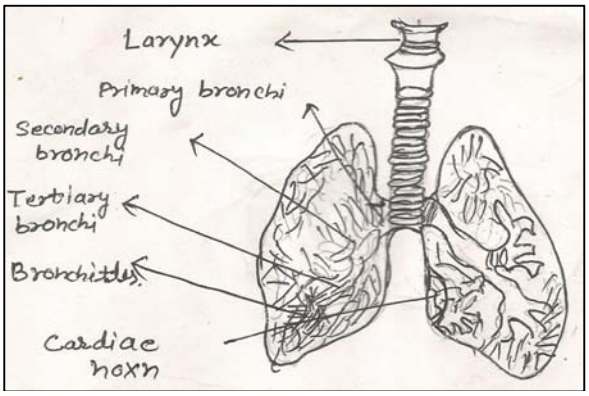


Fig 1

The main function of the lungs is (rapid) gas exchange. This is accomplished by a well-coordinated interaction of the lungs with the central nervous system, the diaphragm and chest wall mascul at use, and the circulatory system. Their principal function is to transport oxygen from the atmosphere into the blood stream, and to release carbon-die-oxide from the blood stream into the atmosphere.

This exchange of gases is a accomplished in the mosaic of specialized cells that from millions of tiny, exceptionally thin-walled air sacs called alveoli.

Arterioles are the blood vessels in the arterial side of the vascular tree that are located proximal to the capillaries and, in conjunction to the regulation of mean arterial pressure and tissue perfusion, their wall consists or cellular and extra cellular components that have been traditionally classified as conforming three layers an intima containing end other cell sited on a basement membrane a media made of an internal elastic lamina exposed by one or two layers of smooth muscle: and adventitia composed mostly of collagen bundle, nerve endings and some fibro blasts. These components of the arteriolar wall are dynamically interconnected, providing a level of plasticity to the arteriolar wall that bluss the traditional boundaries of a rigid layered classification.

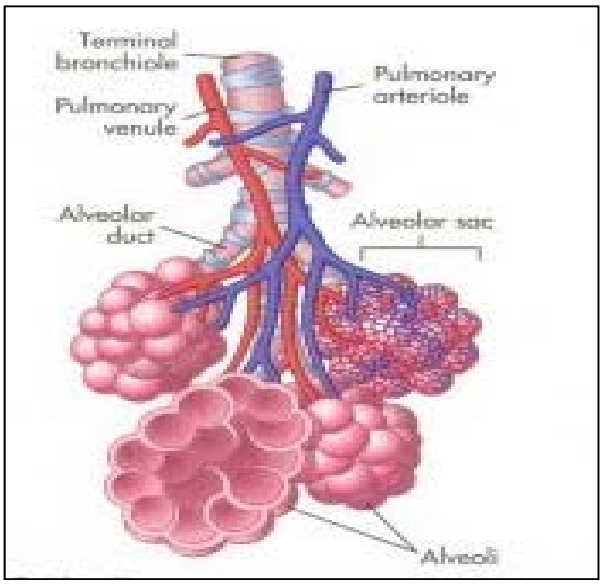


Fig 2: Pulmonary Arterioles

Arterioles are defined as the primary resistance vessels that enter an organ to distribute blood flow in to capillary beds. There blood vessels very significantly in diameter depending on species, vascular bed and state of contraction. Arterioles are considered part of the resistance vasculature that provides in excess of 80% of the resistance to blood flow in the body.

Blood is a vessels fluid composed of cells and plasma more than 99% of the cells are red blood cells. The means that for practical purpose the white cells play almost in determining the physical characteristic of the blood. The percentage of hematocrit, these if a person has a human of 40, 40% of blood volume is covered by the cells and the rest is plasma,. The calibration allows directed reading of the present of cells.

The shape and size of RBC very different classes of vertebrates. Human erythrocytes are 7-8 cm (1 am-10-6m) in diameter and 2 am thick near the rim. Then almost entire cytoplasm is filled with haemoglobin. In the absence of cell organelles, the consumption of oxygen is every lare haemoglobin is a conjugated protein which is made up of a protein called globin and Fe^{2+}porsphyric complex called heme. Thus the total count of RBC is more in man than in woman. The life of a RBC is about 120 days.

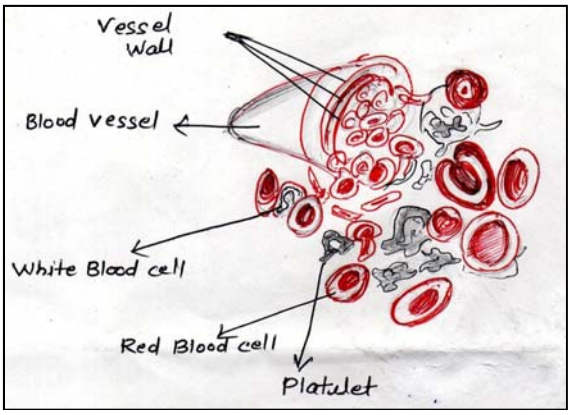


Fig 3

Blood Plasma is basically similar to a dilute solution of sea water and indeed. They share the same salt but in concentrations. Blood plasma has roughly 0.85% salt. The main group of plasma proteins are albumin, globulin and fibrinogen. Their most important role is in the maintenance of proper osmotic pressure between the circulatory fluid and the fluid in the tissue space.

The leucocytes are of two Agranulocytes and Granulocytes. Agranulocytes indeed lymphocytes and monocytes. Granulocytes are ecsinophils basophils and neutrophils.

These are really cell fragment rather than true cells. Blood platelets are much smaller than both the red and white blood corpuscles. Their number harmally varies from 0.15-0.45billions per micro liter of blood when an injury is caused the blood platelets release certain chemical which are termed the plated factor (thromboplastin).

Chronic obstructive pulmonary disease [COPD] is characterized by emphysema, small airways disease, and bronchitis, associates with pulmonary Hypertension. Emphysema is generally caused by cigarette smoking or long –term Exposure to certain industrial pollutants or dusts, a small percentage of cases are caused by familiar or genetic disorder, aipha-1 antitrypsin deficiency, while damaged airways don't regenerate and there is no cure, emphysema is preventable and treatable.

Two phase pulmonary blood flow is a study of measuring the blood pressure if the hemoglobin is known, the percentage of volume covered by blood cells in the whole blood is called hematocrit. This work will focus on two phase pulmonary blood in lungs with special reference to emphysema.

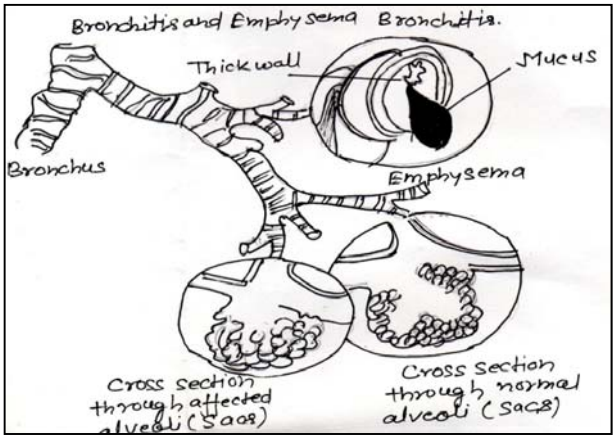


Fig 4

Mathematical Modeling

No of workers as Mishra and Pandey2003, Mishra and Ghosh 2004 and Lifermaggia and other worked on blood transport and have given their viscous. In pulmonary Circulation the blood flow is quite different then the circulation in cylindrical duped and for which several reasons have been observed.

The blood is an important and ideal fluid which is a mixture of plasma and blood cells white flowing through different vessels it changes its shapes and size Veerendra.

In aorta and pulmonary arteries the blood flows due to unusual high Reynolds no of flow as 5000 to 10000 Veerendra.

As we know that principal of conservations and momentum which is applicable to hemodynamics, hence the equation of motion based on the principle has been obtained in tensorial form.

In year 2009 Van Hirtum; Cisonni J; Pelorson, X suggested that the position of flow separation along a construction is important to model fluid structure interactions phenomena. The two dimensional Navier Stokes equations results in an accurate quantitative prediction of flow separation. The assumption applied in Quasi-one dimensional flow description does not accurately predict flow separation.

In year 2010- Proshin, A.P; Solodyannikov, yu. V. Consider a formulation of the problem of parametric identification from measurement of periodic motion.

In year 2011- Mustapha, Norzieha, Mandal, Prashant k. Abdullah llyani; Amin Norsraahaida; Hayat tasawar focus that the numerical in the investigation of the generalized Newtonian blood flow through a couple of irregular atrial stenosis.

In year 2013 Dina Visca, Marina Aiello and Alfredo Chetto consider a formulation of the problem of specifically addressed to cardiovascular system function.

In year 2014 Cheryl R. Laratta and Stephan van Eeden. Consider a formulation acute exacerbation of COPD. The present time, many of these events are unrecognized, despite improve tools for diagnosis and assessment.

Equation of Continuity

The flow of blood is effected by the presence of blood cells. The blood cells effect is directly proportional to the volume. Let the volume be X, this X is replaced by 1/100, where H is the hematocrit the volume % of blood cells. Then the volume portion covered by the plasma will be [1-X].If mass ratio of cells to plasma is r then clearly.

$$r = \frac{X\rho_c}{(1-X)\rho_p} \dots\dots\dots (1)$$

Whereρ_c and ρ_p are densities of blood cells and plasma respectively. Usually this mass ratio is not a constant. Even then this may by supposed to constant in present context.According to the principal of Conservation of mass in pulmonary circulatory system, equation of continuity for two phase are as follows:

$$\frac{\partial X_{\rho c}}{\partial t} + X_{\rho_c} v^i, i = 0 \dots\dots (2)$$

$$\text{And, } \frac{\partial y}{\partial t}(1-X)_{\rho_p} + (1-X)_{\rho_c} v^i, j = 0 \dots\dots (3)$$

Where v is the common velocity of two phase blood cells and plasma. Again $X\rho_{c\,V^{i,}}$ Is covariant derivative of with respect to X^i.In the same way is the covariant derivative of (1-X) with respect to X^i.

If we define the uniform density of bloodρ_m as follows:

$$\frac{1+r}{\rho_m} = \frac{r}{\rho_0} + \frac{1}{\rho_p} \dots\dots\dots\dots (4)$$

The equation 2 and 3 can be combined together as follows-

$$\frac{\partial \rho_m}{\partial t} + (\rho_m v^i), i = 0 \dots\dots\dots (5)$$

Equation of motion of blood flow

The hydrodynamical pressure p between the two phases of blood can be supposed to be uniform because the both phases i.e. always in equilibrium state in blood [12]. Taking viscosity coefficient of blood cells to be $\eta_c$and applying the principle of conservation of momentum in pulmonary circulatory system, we get the equation of motion for the phase of blood cells as follows;

$$\frac{\partial V^i}{\partial t} + (X\rho_c V^i)V_j^i = -X_{p,}\, j\delta^{ij} + X_{\eta c}\left(g^{ij} v_k^i\right) j \dots\dots (6)$$

Similarly the viscosity coefficient of plasma to be η_ρ

$$(1-X)\rho_p \frac{\partial v^i}{\partial t} + \{(1-X)\rho_p v^i\} v_j^i$$
$$-(1-X)\rho, j g^{ij} + (1-X)_{\eta_c}(g^{ij} v_k^i)_j \dots\dots\dots (7)$$

Now adding equation (6) and (7) and using relation (4) the equation of motion for blood flow

With the both phase will be as follows.

$$\rho_m \frac{\partial v^i}{\partial t} + \rho_m v^j v_j^i = -p, j g^{ij} + \eta_m\left(g^{ij} v_k^j\right) j \dots\dots\dots (8)$$

Where $\eta_m = X\eta_c + (1-X)\eta_p$ is the viscosity coefficient of blood as a mixture of two phases.

The pumping effects of heart on these vessels is very low. The yield stress derived from this phenomenon.

Increases the blood viscosity ten times. Hence the pumping of the heart on these vessels relatively low.

(Van Hirtum *et al*……2009)

Secondly these vessels are relatively narrow down more rapidly. In this situation, the blood cells line up on the axis to build up rouleaux.

That's why the Herschel Bulkley law hold good on this two phase blood flow through veins arterioles,

Venules and whose constitutive equation is as follows:

$$T' = \eta_m e^h e^n + T_p \,(T' > T_P)$$

And e = 0 (T" <T_p)

Where T is the yield stress.

When strain rate e=o (T'<T_p) a core region is formed whis flows just like a plug. Let the radias of the

Plug be r_p .

The stress acting on the surface of the plug will be T. equating the forces acting on the plug,

we get

$$p\,\pi\, r_{p^2} = T_p\, 2\pi\, r_p$$

$$r_p = \frac{2T_p}{p} \dots\dots\dots\dots (9)$$

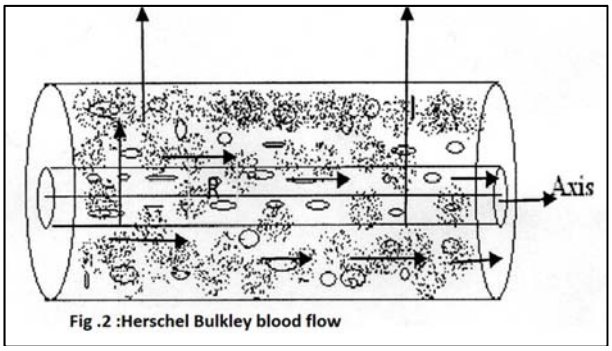


Fig 2: Herschel Bulkley blood flow

The consecutive equation rest part of blood vessels is

$$T' = \eta_m e^h + T_p$$
$$T' - T_p = \eta_m e^h$$

Whose generalized form will be as follows.

$$T^{ij} = -pg^{ij} + T_e^{ij}$$

Where the symbols have their usual meanings.
Now we describe the basic equation for Herschel Bulkley flow as follows.
Equation of continuity

$$\sqrt[1/]{g}\left(\sqrt{g}v^t\right) = 0$$

The equation of motion

$$\rho_m \frac{\partial v^i}{\partial \mathrm{t}} + \rho_m v^i v_j^i = -T^{ij} \text{ j} \ldots\ldots (10)$$

Where all the symbols have their usual meanings

Analysis
Since the blood vessels are cylindrical, the above governing equation are transformed into cylindrical form. As we know earlier
x^1= r, $x^2 = \theta$, $x^3 = z$
Matrix of metric tensor in cylindrical co-ordinate is as follow.

$$[g^{ij}] = \begin{vmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

While matrix of conjugate metric tensor is as follows

$$[g_{ij}] = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1/r^2 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Whereas the Christoffels symbols of 2nd kind as follows;

$$\begin{Bmatrix} 1 \\ 2\,2 \end{Bmatrix} = r \begin{Bmatrix} & 2 \\ 2 & \end{Bmatrix} = \{ \quad {}^{1}_{2}\} = 1/r$$

remaining other are zero.

Relation between contra variant and physical components of velocity of blood flow will be as follows;

$$\sqrt{g_{11}v^1} = v_r = v^1$$
$$\sqrt{g_{22}v^2} = v_\theta = v_\theta = rv^2$$
$$\sqrt{g_{33}v^3} = v_z = v_z = v^3$$

Again the physical components of $-p_j g^{ij}$ are $-\sqrt{g_{ij}p_j g^{ij}}$
Eq. (9) and (10) are transformed into cylindrical form so as solve as power law model to get

$$\frac{dv}{dr} = \left(\frac{p_r}{2\eta_m}\right)^{1/n}$$

Where pressure gradient

$$\frac{dp}{dz} = p$$
$$\frac{dv}{dr} = \frac{p(r - r_p)^{1/n}}{2\eta_m}$$
$$\frac{dv}{dr} = \{\frac{1}{2pr} - \frac{1}{2pr_p}/2\eta_m\}^{\frac{1}{n}}$$

from equation (9)….

$$\frac{dv}{dr} = \left\{\frac{\frac{1}{2}p_r - T_p\}^{\frac{1}{n}}}{\eta_m}\right\} \ldots\ldots (11)$$

Substituting the values of T'_0 from (7) in to (11)

$$\frac{dv}{dr} = [p_{r/2} - pr_{p/2}/\eta_m]^{1/n}$$
$$\frac{dv}{dr} = \frac{p}{2\eta_m}(r - r_p)^{\frac{1}{n}} \ldots\ldots (12)$$

Integrating above equation under the no slip boundary condition $v = 0, r = R$ so as to get

$$v = [p/2\eta_m]^{\frac{1}{n}} \frac{n}{n+1} [(R - r_p)^{\frac{1}{n}+1} - (r - r_p)^{\frac{1}{n}+1}] \ldots\ldots\ldots (13)$$

Which is the formula of velocity of blood flow in arteriole venules and veins putting r = r' to get the velocity of plug flow as follows.

$$v_p = \frac{n}{n+1}\left[\frac{P}{2\eta_m}\right]^{\frac{1}{n}} (R - r_p)^{\frac{1}{n}+1} \ldots\ldots\ldots\ldots\ldots\ldots (14)$$

Where the value of r_p is taken from (7)

Result and Discussion
Observation: Hematocrit and blood pressure from an authorized Lala Lajpat Rai and Associated Hospital Kanpur by Sudhir Chaudhari
Patient Name: Mr. Chandra Pal
Diagnosis – Emphysema

Table

Date	HB (Hemoglobin)	B.P (Blood-pressure in (mm hg)	Mean (Blood-pressure in (mmhg)	Average (in pascal)	Hematocrit
26/4/2012	13.1	106/72	89	11857.73	39.3
27/4/2012	13.2	104/72	88	11724.50	39.6
28/4/2012	14.2	98/66	82	10925.11	42.6
29/4/2012	14.3	118/86	102	13589.76	42.9

The flow flux phased blood flow in arterioles, venules and veins is

$$Q = \int_0^{r_p} 2\pi v_p dr + \int_{r_p}^{R} 2\pi r v dr$$

$$\int_0^{r_p} 2\pi r\, n_{n+1} p/_{2\eta_m}{}^{1/n} R - r_p{}^{\frac{1}{n}+1}{}_{dr} + \int \quad 2\pi r\, n_{n+1} p/2\eta_m{}^{1/n}$$

Using (12) and (14)

$$\frac{2\pi n}{n+1}(p/2\eta_m)^{\frac{1}{n}}(R-r_p)^{\frac{1}{n}+1}[r^2/2]_0^{r_p} + \frac{2\pi n}{n+1}(p/2\eta_m)^{\frac{1}{n}}\left[\frac{r^2}{2}(R-r_p)^{\frac{1}{n}+1} r\left(\frac{r-r_p}{\frac{1}{n}+2}\right) - \frac{(r-r_p)^{\frac{1}{n}+3}}{\frac{1}{n}+2)(\frac{1}{n}+3)}\right]_{r_p}^{R}$$

$$\frac{2\pi n}{n+1}(p/2\eta_m)^{\frac{1}{n}} r_p^2 (R-r_p)^{\frac{1}{n}+1} R^2 (R-r_p)^{\frac{1}{n}+1} - 2R\left(\frac{(r-r_p)^{\frac{1}{n}+2}}{\frac{1}{n}+2}\right) + 2\frac{(R-r_p)^{\frac{1}{n}+3}}{\frac{1}{n}+2)(\frac{1}{n}+3)} - r_p^2 (R-r_p)^{\frac{1}{n}+1}$$

$$\frac{2\pi n}{n+1}(p/2\eta_m)^{\frac{1}{n}} R^{\frac{1}{n}+3}\left[\frac{r_p^2}{R^2}(1-\frac{r_p}{R})^{\frac{1}{n}+1} + \left(1+\frac{r_p}{R}\right)(1-\frac{r_p}{R})^{\frac{1}{n}+2} - 2(1-\frac{r_p}{R})^{\frac{1}{n}+2}/\frac{1}{n} + 2 + 2(1-\frac{\frac{r_p}{R})^{\frac{1}{n}+3}}{\frac{1}{n}+2)(\frac{1}{n}+3)}\right.$$

P = pressure gradient
N= parameter
V=viscosity of mixture
Now we have Q = 425ml/min R=1,$r_p = 1/3$ according to Gustafsonpenile R,(1980)

$$\eta_p = 0.013(\,pascel - \sec)$$

According to Glenn Elert (2010)

$$\eta_m = 0.027(\,pascel - \sec)$$

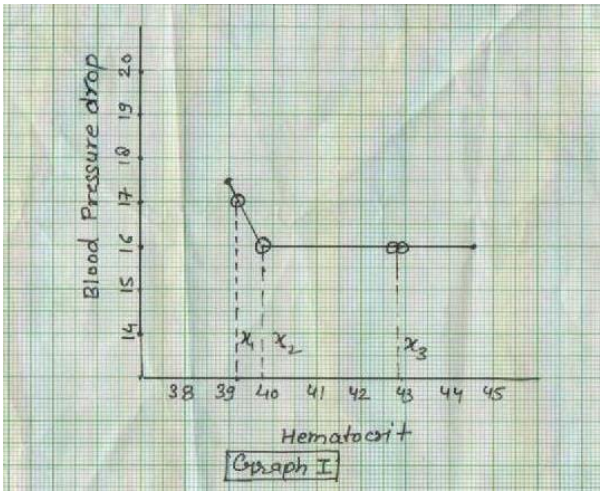
H = 24 $\eta_m = \eta_c X + \eta_p(1-X)$ where

$$X = \frac{H}{100} = \frac{24}{100} = 0.24$$

$$1827.23 = (1728.40)^{\frac{1}{n}}\left[\frac{26n^3+33n^2+9n}{6n^3+11n^2+6n+1}\right]$$

We get n = 0.161
Solved by numerical method.

Hematocrit	39.3	39.6	42.6	42.9
BP drop	17	16	16	16



Conclusion

A simple survey of the graph between blood pressure and hematocrit in emphysema patient show that

1. When Hematocrit is increased then blood pressure drop is decreased from $x_1$to x_2.
2. When Hematocrit is increased then blood pressure drop is constant fron x_2 to x_3.

Acknowledgement

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