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New forms of continuous and closed maps via b-open sets in SETS

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Abstract
In this paper we propose the concept of a new class of functions named strongly g^{*+}b-continuous, perfectly g^{*+}b-continuous, g^{*+}b-closed and open maps in simple extended topological spaces (SETS). Also we intend to define a new class of homeomorphism called g^{*+}b-homeomorphism in SETS. Some of their basic properties and several characterizations of these type of functions are discussed.

Keywords: Strongly g^{*+}b-continuous, perfectly g^{*+}b-continuous, g^{*+}b-closed and open maps, g^{*+}b-homeomorphism

1. Introduction

Levine [9] initiated the concept of generalized closed sets in topological spaces and a class of topological spaces called $T_{1/2}$-spaces. The strong forms of continuous maps named strongly continuous maps, perfectly continuous maps, completely continuous maps and clopen continuous maps were introduced by Levine [10], Noiri [13], Munshi and Bassan [12], Reilly and Vamanamurthy [15] respectively. Bharathi [2] studied the concept of strongly and perfectly g^*b-continuous functions in topological spaces. Malghan [11] delivered the idea of generalized closed mappings in topological spaces. Biswas [1], Sundaram [17], Crossley and Hildebrand [3] have introduced and investigated semi-open maps and several generalized mappings in topological spaces and also discussed a class of homeomorphisms named semi homeomorphism, some what homeomorphism, generalized homeomorphism and gc-homeomorphism respectively. Vidhya and Parimelazhagan [20] devised the concept of g^*b-closed (open) maps and g^*b-homeomorphism in topological spaces.

Levine [8] proposed the concept of extending a topology by a non-open set for τ is defined as $\tau(B) = \{(B \cap O) \cup O' / \ O, O' \in \tau\}$ in 1963. B. Kanchana and F. Nirmala Irudayam [6, 7] formulated the concept of g^{*+}b-closed sets and g^{*+}b-continuity in extended topological spaces. The purpose of the present paper is to study some new forms of g^{*+}b-continuous, g^{*+}b-closed maps and homeomorphism in extended topological spaces and investigate some of their properties.

Throughout this paper X, Y and Z (or (X,τ^+), (Y,σ^+) and (Z,η^+)) are simple extension topological space in which no separation axioms are assumed unless and otherwise stated. For any subset A of X, the interior of A is same as the interior in usual topology and the closure of A is newly defined in simple extension topological spaces.

2. Preliminaries

We recall the following definitions which are useful in the sequel.

Definition 2.1: A subset A of a topological space (X,τ) is called a,

(i) generalized b-closed set (briefly gb-closed) [14], if bcl(A) ⊆ U, whenever A ⊆ U and U is open in X.
(ii) g^*b-closed set [18], if bcl(A) ⊆ U, whenever A ⊆ U and U is g-open in X.

Definition 2.2: A subset A of a topological space (X,τ^+) is called a,

(i) generalized b^+-closed set (briefly gb^+-closed) [6], if $bcl^+(A) \subseteq U$, whenever A ⊆ U and U is open in X.
(ii) g^{*+}b-closed set [6], if $bcl^+(A) \subseteq U$, whenever A ⊆ U and U is g^+-open in X.

Definition 2.3: A function $f:(X,\tau^+)\to(Y,\sigma^+)$ is called,
(i) gb$^+$-continuous [16], if $f^{-1}(V)$ is gb$^+$-closed in X for every closed set V of Y.
(ii) g^{*+}b-continuous [7], if $f^{-1}(V)$ is g^{*+}b-closed in X for every closed set V of Y.

Definition 2.4: A function $f:(X,\tau)\to(Y,\sigma)$ is called [2, 19, 20],
(i) continuous if $f^{-1}(V)$ is closed in X for every closed set V of Y.
(ii) strongly continuous if the inverse image is both open and closed in(X,τ) for each subset V of (Y,σ).
(iii) strongly g-continuous if the inverse image of every g-open set in (Y,σ) is open in(X,τ).
(iv) strongly g*b-continuous if the inverse image of every g*b-open set in (Y,σ) is open in(X,τ).
(v) perfectly g*b-continuous if the inverse image of every g*b-open set in (Y,σ) is open and closed in(X,τ).
(vi) gb-closed map if for each closed set F of X, f(F) is gb-closed set in Y.
(vii) g*b-closed (open) map if for each closed (open) set F of X, f(F) is g*b-closed (open) set inY.

Definition 2.5: A bijection $f:(X,\tau)\to(Y,\sigma)$ is called [20],
(i) gb-homeomorphism if f is both gb-continuous and gb-closed.
(ii) g*b-homeomorphism if f is both g*b-continuous and g*b-closed.

3. Strongly g^{*+}b-Continuous

In this section we device the novel concept of strongly g^{*+}b-continuous functions in extended topological spaces.

Definition 3.1: A function $f:(X,\tau^+)\to(Y,\sigma^+)$ is called strongly$^+$ continuous if the inverse image is both open and closed in(X,τ^+) for each subset V of (Y,σ^+).

Definition 3.2: A function $f:(X,\tau^+)\to(Y,\sigma^+)$ is called strongly g$^+$-continuous if the inverse image of every g$^+$-closed set in (Y,σ^+) is closed in(X,τ^+).

Definition 3.3: A function $f:(X,\tau^+)\to(Y,\sigma^+)$ is called strongly g^{*+}b-continuous if the inverse image of every g^{*+}b-closed set in (Y,σ^+) is closed in(X,τ^+).

Theorem 3.4: If $f:X\to Y$ is strongly g^{*+}b-continuous then it is strongly g$^+$-continuous but not conversely.

Proof: Let $f:X\to Y$ is strongly g^{*+}b-continuous. Let F be any g$^+$-closed set in Y. We know that every g$^+$-closed set is g^{*+}b-closed. Hence F be g^{*+}b-closed in Y. Then the inverse image $f^{-1}(V)$ is closed in X. Therefore *f*is strongly g$^+$-continuous.

Example 3.5: Let X = Y = {a, b, c} with topologies $\tau=\{X,\emptyset,\{a\},\{b\},\{a,b\}\},\tau^+=\{X,\emptyset,\{a\},\{b\},\{a,b\},\{a,c\}\}$ and $\sigma=\{Y,\emptyset,\{a\},\{a,b\}\},\sigma^+=\{Y,\emptyset,\{a\},\{b\},\{a,b\}\}$. Let $f:X\to Y$ be the identity map. Then *f*is strongly g$^+$-continuous but not strongly g^{*+}b-continuous.

Theorem 3.6: If $f:X\to Y$ is strongly$^+$-continuous then it is strongly g^{*+}b-continuous but not conversely.

Proof: Let $f:X\to Y$ is strongly$^+$-continuous. Let F be g^{*+}b-closed in Y. Then the inverse image $f^{-1}(V)$ is closed in X. Therefore *f* is strongly g^{*+}b-continuous.

Example 3.7: Let X = Y = {a, b, c} with topologies $\tau=\{X,\emptyset,\{a\},\{a,b\}\},\tau^+=\{X,\emptyset,\{a\},\{a,b\},\{a,c\}\}$ and $\sigma=\{Y,\emptyset,\{a\}\},\sigma^+=\{Y,\emptyset,\{a\},\{a,c\}\}$. Let $f:X\to Y$ be the identity map. Then *f*is strongly g^{*+}b-continuous but not strongly$^+$-continuous.

Theorem 3.8: If $f:X\to Y$ is strongly g^{*+}b-continuous then it is continuous but not conversely.

Proof: Let $f:X\to Y$ is strongly g^{*+}b-continuous. Let F be any closed set in Y. We know that every closed set is g^{*+}b-closed. Hence F be g^{*+}b-closed in Y. Then the inverse image $f^{-1}(V)$ is closed in X. Therefore *f*is continuous.

Example 3.9: Let X = Y = {a, b, c} with topologies $\tau=\{X,\emptyset,\{a\},\{a,b\}\},\tau^+=\{X,\emptyset,\{a\},\{b\},\{a,b\}\}$ and $\sigma=\{Y,\emptyset,\{b\}\},\sigma^+=\{Y,\emptyset,\{b\},\{a,b\}\}$. Let $f:X\to Y$ be the identity map. Then *f*is continuous but not strongly g^{*+}b-continuous.

Theorem 3.10: A function $f:X\to Y$ is strongly g^{*+}b-continuous if and only if the inverse image of every g^{*+}b-open set in Y is open in X.

Proof: Assume that *f* is strongly g^{*+}b-continuous. Let F be any g^{*+}b-open set in Y. Then Y – F is g^{*+}b-closed in Y. By assumption $f^{-1}(Y-F)$ is closed in X. But $f^{-1}(Y-F)=X-f^{-1}(F)$and so $f^{-1}(F)$ is open in X. Hence the inverse image of every g^{*+}b-open set in Y is open in X. Conversely assume that the condition holds. Let V be any g^{*+}b-closed set in Y. Then Y – V is g^{*+}b-open in Y. Hence $f^{-1}(Y-V)$ is open in X. Therefore $f^{-1}(V)$ is closed in X. Thus *f* is strongly g^{*+}b-continuous.

Theorem 3.11: Let $f:X\to Y$ be a function, then the following are equivalent:
(i) *f*is strongly g^{*+}b-continuous

(ii) *f* is continuous
(iii) *f* is g^{*+}b-continuous.
Proof: The proof is obvious by the definitions.

Theorem 3.12: Let $f: X \to Y$ be onto, g^{*+}b-irresolute and b^+-closed. If X is a T_{b^+}-space then Y is also a T_{b^+}-space.

Proof: Let V be a g^{*+}b-closed subset of Y. Since *f* is g^{*+}b-irresolute then $f^{-1}(V)$ is g^{*+}b-closed in X. Since X is a T_{b^+}-space, then $f^{-1}(V)$ is a b^+-closed in X. Thus V is b^+-closed in Y because *f* is subjective. Hence Y is a T_{b^+}-space.

Theorem 3.13: Let X be a discrete topological space, Y be a g^{*+}b-space and $f: X \to Y$ be a function. Then the following are equivalent:
(i) *f* is strongly$^+$ continuous
(ii) *f* is strongly g^{*+}b-continuous

Proof: (i) ⇒ (ii): Follows from the theorem 3.6.
(ii) ⇒(i): Let U be any g^{*+}b-set in Y. Since Y is a g^{*+}b-space, U is a g^{*+}b-closed subset of Y and by hypothesis, $f^{-1}(U)$ is closed in X. But X is a discrete space and so $f^{-1}(U)$ is also open in X. That is $f^{-1}(U)$ is both open and closed in X and so *f* is strongly$^+$ continuous.

Theorem 3.14: Let $f: X \to Y$ is g^{*+}b-irresolute and closed. If X is almost weakly Hausdroff then Y is almost weakly Hausdroff space.

Proof: Let V be any g^{*+}b-closed set in Y. Since *f* is g^{*+}b-irresolute, $f^{-1}(V)$ is g^{*+}b-closed in X. Since X is almost weakly Hausdroff, $f^{-1}(V)$ is closed in X. Then V is closed because *f* is closed and onto. Hence Y is almost weakly Hausdroff space.

Theorem 3.15: If $f: X \to Y$ and $g: Y \to Z$ are strongly g^{*+}b-continuous, then their composition $g \circ f: X \to Z$ is strongly$^+$ continuous.

Proof: Let U be a g^{*+}b-closed set in Z. Since *g* is strongly g^{*+}b-continuous, $g^{-1}(U)$ is closed in Y. Since $g^{-1}(U)$ is closed, it is g^{*+}b-closed in Y. As *f* is also strongly g^{*+}b-continuous, $f^{-1}(g^{-1}(U)) = (g \circ f)^{-1}(U)$ is closed in X and so $g \circ f$ is strongly$^+$ continuous.

Theorem 3.16: If $f: X \to Y$ and $g: Y \to Z$ be any two functions, then their composition $g \circ f: X \to Z$ is
(i) strongly g^{*+}b-continuous if *g* is strongly g^{*+}b-continuous and *f* is strongly g^{*+}b-continuous.
(ii) g^{*+}b-irresolute if *g* is strongly g^{*+}b-continuous and *f* is g^{*+}b-continuous (or *f* is g^{*+}b-irresolute).
(iii) strongly g^{*+}b-continuous if *g* is strongly g^{*+}b-continuous and *f* is irresolute.
(iv) continuous if *g* is strongly g^{*+}b-continuous and *f* is strongly g^{*+}b-continuous.

Proof: The proof is obvious.

4. Perfectly g^{*+}b-Continuous
In this section we promote the idea of perfectly g^{*+}b-continuous functions in extended topological spaces.

Definition 4.1: A function $f: (X, \tau^+) \to (Y, \sigma^+)$ is called perfectly$^+$-continuous if the inverse image of every closed set in (Y, σ^+) is both open and closed in (X, τ^+).

Definition 4.2: A function $f: (X, \tau^+) \to (Y, \sigma^+)$ is called perfectly g^{*+}b-continuous if the inverse image of every g^{*+}b-closed set in (Y, σ^+) is both open and closed in (X, τ^+).

Theorem 4.3: If $f: X \to Y$ is perfectly g^{*+}b-continuous then it is strongly g^{*+}b-continuous but not conversely.

Proof: Let U be any g^{*+}b-closed set in Y. Since f is perfectly g^{*+}b-continuous, $f^{-1}(U)$ is both open and closed in X, for every g^{*+}b-closed set Y in X. Therefore *f* is strongly g^{*+}b-continuous.

Example 4.4: Let X = Y = {a, b, c} with topologies $\tau = \{X, \emptyset, \{b\}, \{c\}, \{b, c\}\}, \tau^+ = \{X, \emptyset, \{b\}. \{c\}, \{b, c\}, \{a, c\}\}$ and $\sigma = \{Y, \emptyset, \{c\}\}, \sigma^+ = \{Y, \emptyset, \{c\}, \{b, c\}\}$. Let $f: X \to Y$ be the identity map. Then *f* is strongly g^{*+}b-continuous but not strongly$^+$-continuous.

Theorem 4.5: If $f: X \to Y$ is perfectly g^{*+}b-continuous then it is perfectly$^+$-continuous but not conversely.

Proof: Let U be any closed set in Y. Every closed set is g^{*+}b-closed, U is g^{*+}b-closed set in Y. Since f is perfectly g^{*+}b-continuous, $f^{-1}(U)$ is both open and closed in X. Therefore *f* is perfectly$^+$-continuous.

Example 4.6: Let X = Y = {a, b, c} with topologies $\tau = \{X, \emptyset, \{a\}, \{c\}, \{a, c\}\}, \tau^+ = \{X, \emptyset, \{a\}. \{c\}, \{a, b\}, \{a, c\}\}$ and $\sigma = \{Y, \emptyset, \{c\}\}, \sigma^+ = \{Y, \emptyset, \{c\}, \{b, c\}\}$. Let $f: X \to Y$ be the identity map. Then *f* is perfectly$^+$-continuous but not perfectly g^{*+}b-continuous.

Theorem 4.7: If $f: X \to Y$ is strongly$^+$ continuous then it is perfectly g^{*+}b-continuous but not conversely.

Proof: Let U be any subset of Y. Since $f: X \to Y$ is strongly$^+$ continuous, $f^{-1}(U)$ is both open and closed in X, for every g^{*+}b-closed set U in Y. Therefore *f*is perfectly g^{*+}b-continuous.

Example 4.8: Let X = Y = {a, b, c} with topologies $\tau = \{X, \emptyset, \{b\}, \{c,\}\{b, c\}\}, \tau^+ = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}\}$and $\sigma = \{Y, \emptyset, \{a\}\}, \sigma^+ = \{Y, \emptyset, \{a\}, \{a, c\}\}$. Let $f: X \to Y$ be the identity map. Then *f*is perfectly g^{*+}b-continuous but not strongly$^+$-continuous.

Theorem 4.9: A map $f: X \to Y$ is perfectly g^{*+}b-continuous if and only if f^{-1}(A) is both open and closed in X for every g^{*+}b-open set A in Y.

Proof: Assume that *f* is perfectly g^{*+}b-continuous. Let A be any g^{*+}b-open set in Y. Then A^c is g^{*+}b-closed set in Y. Since *f* is perfectlyg^{*+}b-continuous, f^{-1}(A^c) is both open and closed in X.But f^{-1}(A^c) = X − f^{-1}(A) and so f^{-1}(A) is both open and closed in X. Hence f^{-1}(A) is both open and closed in X for every g^{*+}b-open set A in Y.
Conversely assume that the inverse image of every g^{*+}b-open set in Y is both open and closed in X. Let A be any g^{*+}b-closed set in Y. Then A^c is g^{*+}b-open in Y. By assumption, f^{-1}(A^c) is both open and closed in X. But f^{-1}(A^c) = X − f^{-1}(A) and so f^{-1}(A) is both open and closed in X. Therefore *f* is perfectly g^{*+}b-continuous.

Theorem 4.10: Let X be a discrete topological space, Y be any topological space and $f: X \to Y$ be a function. Then the following are equivalent:
(i) *f*is strongly g^{*+}b-continuous
(ii) *f*is perfectly g^{*+}b-continuous

Proof: (i) ⇒ (ii): Let V be any g^{*+}b-closed set in Y. By hypothesis, $f^{-1}(V)$ is closed in X. Since X is a discrete space, $f^{-1}(V)$ is open in X. Hence $f^{-1}(V)$ is both open and closed in X. Therefore, *f* is perfectly g^{*+}b-continuous.
(ii) ⇒(i): Let U be any g^{*+}b-closed set in Y. Then $f^{-1}(U)$ is both open and closed in X. Therefore *f* is strongly g^{*+}b-continuous.

Theorem 4.11: If $f: X \to Y$ and $g: Y \to Z$ are perfectly g^{*+}b-continuous, then their composition $g \circ f: X \to Z$ is perfectly g^{*+}b-continuous.

Proof: Let U be a g^{*+}b-closed set in Z. Since *g* is perfectly g^{*+}b-continuous. We get that $g^{-1}(U)$ is both open and closed in Y. We know that every closed(open) set is g^{*+}b-closed(open), $g^{-1}(U)$ is g^{*+}b-closed in Y. Since *f* is perfectly g^{*+}b-continuous, $f^{-1}(g^{-1}(U)) = (g \circ f)^{-1}(U)$ is both open and closed in X. Hence $g \circ f$ is perfectly g^{*+}b-continuous.

Theorem 4.12: If $f: X \to Y$and $g: Y \to Z$ be any two maps. Then their composition $g \circ f: X \to Z$ is strongly g^{*+}b-continuous if *g* is perfectly g^{*+}b-continuous and *f* is continuous.

Proof: Let U be any g^{*+}b-closed set in Z. Then $g^{-1}(U)$ is both open and closed in Y. Since *f* is continuous, $f^{-1}(g^{-1}(U)) = (g \circ f)^{-1}(U)$ is closed in X. Hence $g \circ f$ is strongly g^{*+}b-continuous.

Theorem 4.13: If $f: X \to Y$ is perfectly g^{*+}b-continuous and $g: Y \to Z$ is strongly g^{*+}b-continuous, then their composition $g \circ f: X \to Z$ is perfectly g^{*+}b-continuous.

Proof: Let U be any g^{*+}b-closed set in Z. Then $g^{-1}(U)$ is closed in Y. Since every closed set is g^{*+}b-closed, $g^{-1}(U)$ is g^{*+}b-closed in Y. By hypothesis, $f^{-1}(g^{-1}(U)) = (g \circ f)^{-1}(U)$ is both open and closed in X. Hence $g \circ f$ is perfectly g^{*+}b-continuous.

5. g^{*+}b-Closed and open Maps

In this section we introduce the concepts of g^{*+}b-closed map, g^{*+}b-open map and gb^+-closed map in extended topological spaces.

Definition 5.1: A map $f: (X, \tau^+) \to (Y, \sigma^+)$ is called g^{*+}b-closed map if for each closed set F of X, f(F) is g^{*+}b-closed set in Y.

Definition 5.2: A map $f: (X, \tau^+) \to (Y, \sigma^+)$ is called g^{*+}b-open map if for each open set F of X, f(F) is g^{*+}b-open set in Y.

Definition 5.3: A map $f: (X, \tau^+) \to (Y, \sigma^+)$ is called gb^+-closed map if for each closed set F of X, f(F) is gb^+-closed set in Y.

Theorem 5.4: Every closed map is a g^{*+}b-closed map but not conversely.

Proof: Let $f: X \to Y$ be an closed map. Let F be any closed set in X. Then $f(F)$ is closed set in Y. Since every closed set is g^{*+}b-closed, $f(F)$ is g^{*+}b-closed set in Y. Therefore *f* is a g^{*+}b-closed map.

Example 5.5: Let X = Y = {a, b, c} with topologies $\tau = \{X, \emptyset, \{b\}\}, \tau^+ = \{X, \emptyset, \{b\}, \{b, c\}\}$ and $\sigma = \{Y, \emptyset, \{b\}\}, \sigma^+ = \{Y, \emptyset, \{b\}, \{a, b\}\}$ Let $f: X \to Y$ be the identity function. Then a map *f* is g^{*+}b-closed map but not closed map.

Theorem 5.6: Every g^{*+}b-closed map is a gb^+-closed map but not conversely.

Proof: Let $f: X \to Y$ be a g^{*+}b-closed map. Let F be a closed set in X. Then $f(F)$ is a g^{*+}b-closed set in Y. Since every g^{*+}b-closed set is gb^+-closed, $f(F)$ is gb^+-closed set in Y. Therefore *f* is a gb^+-closed map.

Example 5.7: Let X = Y = {a, b, c} with topologies $\tau = \{X, \emptyset, \{b\}\}, \tau^+ = \{X, \emptyset, \{b\}, \{a, b\}\}$ and $\sigma = \{Y, \emptyset, \{a\}\}, \sigma^+ = \{Y, \emptyset, \{a\}, \{a, c\}\}$. Let $f: X \to Y$ be the identity function. Then a map *f* is gb^+-closed map but not g^{*+}b-closed map.

Theorem 5.8: If $f : X \to Y$ is closed map and $h : Y \to Z$ is g^{*+}b-closed map, then their composition $h \circ f : X \to Z$ is g^{*+}b-closed map.

Proof: Let F be any closed set in X. Since $f: X \to Y$ is closed, $f(F)$ is closed in Y and since $h: Y \to Z$ is g^{*+}b-closed, $h(f(F))$ is g^{*+}b-closed in Z. Therefore $h \circ f : X \to Z$ is g^{*+}b-closed map.

Remark 5.9: If $f : X \to Y$ is g^{*+}b-closed map and $h : Y \to Z$ is a closed map, then their composition $h \circ f : X \to Z$ need not be g^{*+}b-closed map as seen from the following example.

Example 5.10: Let X = Y = Z = {a, b, c} with topologies $\tau = \{X, \emptyset, \{a, b\}\}, \tau^+ = \{X, \emptyset, \{a\}, \{a, b\}\}$; $\sigma = \{Y, \emptyset, \{a\}\}, \sigma^+ = \{Y, \emptyset, \{a\}, \{a, c\}\}$ and $\eta = \{Z, \emptyset, \{c\}\}, \eta^+ = \{Z, \emptyset, \{c\}, \{a, c\}\}$. Let $f: (X, \tau^+) \to (Y, \sigma^+)$ be the identity map and $h: (Y, \sigma^+) \to (Z, \eta^+)$ is defined by *h*(a) = c, *h*(b) = b, *h*(c) = a. Then *f* is a g^{*+}b-closed map and *g* is a closed map but their composition $h \circ f: (X, \tau^+) \to (Z, \eta^+)$ is not a g^{*+}b-closed map. Since for the closed set {b,c} in X, $h \circ f(\{b, c\})$ = {b,c} is not g^{*+}b-closed in Z and hence $h \circ f$ is not g^{*+}b-closed map.

Remark 5.11: The composition of two g^{*+}b-closed maps need not be g^{*+}b-closed map in general and this is shown by the following example.

Example 5.12: Let X = Y = Z = {a, b, c} with topologies $\tau = \{X, \emptyset, \{b\}\}, \tau^+ = \{X, \emptyset, \{b\}, \{b, c\}\}$; $\sigma = \{Y, \emptyset, \{b\}\}, \sigma^+ = \{Y, \emptyset, \{b\}, \{a, b\}\}$ and $\eta = \{Z, \emptyset, \{c\}\}, \eta^+ = \{Z, \emptyset, \{c\}, \{a, c\}\}$. Let $f: (X, \tau^+) \to (Y, \sigma^+)$ be the identity map and $g: (Y, \sigma^+) \to (Z, \eta^+)$ is defined by *g*(a) = a, *g*(b) = c, *g*(c) = b. Then both *f* and *g* are g^{*+}b-closed map but their composition $g \circ f: (X, \tau^+) \to (Z, \eta^+)$ is not a g^{*+}b-closed map. Since for the closed set {a,c} in X, $g \circ f(\{a, c\})$ = {a,c} is not g^{*+}b-closed in Z and hence $g \circ f$ is not g^{*+}b-closed map.

Theorem 5.13: If $f: X \to Y$ is g^+-closed, $g: Y \to Z$ be g^{*+}b-closed and Y is $T^+_{1/2}$-space then their composition $g \circ f: X \to Z$ is g^{*+}b-closed map.

Proof: Let A be a closed set of X. Since *f* is g^+-closed, *f*(A) is g^+-closed in Y. Since Y is $T^+_{1/2}$-space, *f*(A) is closed in Y. Since *g* is g^{*+}b-closed, *g*(*f*(A)) is g^{*+}b-closed in Z and *g*(*f*(A)) = $g \circ f$(A). Therefore $g \circ f$ is g^{*+}b-closed.

Theorem 5.14: A map $f: X \to Y$ is g^{*+}b-closed if and only if for each subset S of Y and for each open set U containing $f^{-1}(S)$ there is a g^{*+}b-open set V of Y such that S ⊆ V and $f^{-1}(V) \subseteq$ U.

Proof: Suppose f is g^{*+}b-closed. Let S be a subset of Y and U is an open set of X such that $f^{-1}(S) \subseteq$ U. Then V = Y – f(X – U) is g^{*+}b-open set containing S such that $f^{-1}(V) \subseteq$ U. Conversely suppose that F is a closed set in X. Then $f^{-1}(Y - f(F)) = X - F$ and X_F is open. By hypothesis, there is a g^{*+}b-open set V of Y such that Y_{f}(F) ⊆ Y and $f^{-1}(V) \subseteq$ X – F. Therefore F ⊆ X$-f^{-1}(V)$. Hence Y – V $\subseteq f(F) \subseteq f(X - f^{-1}(V)) \subseteq$ Y – V, which implies $f(F)$ = Y – V. Since Y – V is g^{*+}b-closed, $f(F)$ is g^{*+}b-closed and therefore f is g^{*+}b-closed map.

Theorem 5.15: If $f: X \to Y$ is g^{*+}b-closed and A = $f^{-1}(B)$ for some closed set B of Y, then $f_A: A \to Y$ is g^{*+}b-closed.

Proof: Let F be a closed set in A. Then there is a closed set H in X such that F = A ∩ H. Then $f_A(F) = f(A \cap H) = f(H) \cap B$. Since *f* is g^{*+}b-closed, $f(H)$ is g^{*+}b-closed in Y. So $f(H) \cap B$ is g^{*+}b-closed. Since the intersection of a g^{*+}b-closed set and a closed set is a g^{*+}b-closed set. Hence f_A is g^{*+}b-closed.

Remark 5.16: If B is not closed in Y then the above theorem may not hold as seen from the following example.

Example 5.17: Let X = Y = {a, b, c} with topologies $\tau = \{X, \emptyset, \{a\}, \{a,b\}\}, \tau^+ = \{X, \emptyset, \{a\}, \{b\}, \{a,b\}\}$ and $\sigma = \{Y, \emptyset, \{a\}\}, \sigma^+ = \{Y, \emptyset, \{a\}, \{a,b\}\}$. Let $f: X \to Y$ be the identity map. Take B = {a} is not closed in A. Then A = $f^{-1}(B) = f^{-1}(\{a\}) = \{a\}$ and {a} is closed in A. But $f_A(\{a\}) = \{a\}$ is not g*+b-closed in Y. Therefore {a} is also not g*+b-closed in B.

6. g*+b-Homeomorphisms

In this section we promote the ideas of g*+b-homeomorphism and gb+-homeomorphism in extended topological spaces.

Definition 6.1: A bijection $f: (X, \tau^+) \to (Y, \sigma^+)$ is called g*+b-homeomorphism if f is both g*+b-continuous and g*+b-closed.

Definition 6.2: A bijection $f: (X, \tau^+) \to (Y, \sigma^+)$ is called gb+-homeomorphism if f is both gb+-continuous and gb+-closed.

Theorem 6.3: Every homeomorphism is a g*+b-homeomorphism but not conversely.

Proof: Let $f: X \to Y$ be a homeomorphism. Then f is continuous and closed. Since every continuous function is g*+b-continuous and every closed map is g*+b-closed, *f* is g*+b-continuous and g*+b-closed. Hence *f* is a g*+b-homeomorphism.

Example 6.4: Let X = Y = {a, b, c} with topologies $\tau = \{X, \emptyset, \{a,b\}\}, \tau^+ = \{X, \emptyset, \{a\}, \{a,b\}\}$ and $\sigma = \{Y, \emptyset, \{b\}\}, \sigma^+ = \{Y, \emptyset, \{b\}, \{a,b\}\}$. Let $f: X \to Y$ be the identity function. Then *f* is g*+b-homeomorphism but not homeomorphism.

Theorem 6.5: Every g*+b-homeomorphism is a gb+-homeomorphism but not conversely.

Proof: Let $f: X \to Y$ be a g*+b-homeomorphism. Then *f* is a g*+b-continuous and g*+b-closed. Since every g*+b-continuous function is gb+-continuous and every g*+b-closed map is gb+-closed, *f* is gb+-continuous and gb+-closed. Hence *f* is a gb+-homeomorphism.

Example 6.6: Let X = Y = {a, b, c} with topologies $\tau = \{X, \emptyset, \{a\}\}, \tau^+ = \{X, \emptyset, \{a\}, \{a,c\}\}$ and $\sigma = \{Y, \emptyset, \{a\}, \{a,b\}\}, \sigma^+ = \{Y, \emptyset, \{a\}, \{b\}, \{a,b\}\}$. Let $f: X \to Y$ be the identity function. Then *f* is gb+-homeomorphism but not g*+b-homeomorphism.

Theorem 6.7: For any bijection $f: X \to Y$ the following statements are equivalent:
(a) Its inverse map $f^{-1}: Y \to X$ is g*+b-continuous.
(b) *f* is a g*+b-open map.
(c) *f* is a g*+b-closed map.

Proof
(a) ⇒ (b): Let G be any open set in X. Since f^{-1} is g*+b-continuous, the inverse image of G under f^{-1}, namely *f* (G) is g*+b-open in Y and so *f* is a g*+b-open map.
(b) ⇒ (c): Let F be any closed set in X. Then F^c open in X. Since *f* is g*+b-open, *f* (F^c) is g*+b-open in Y. But *f* (F^c) = Y – *f* (F) and so *f* (F) is g*+b-closed in Y. Therefore *f* is a g*+b-closed map.
(c) ⇒ (a): Let F be any closed set in X. Then the inverse image of F under f^{-1}, namely *f* (F) is g*+b-closed in Y. Since *f* is a g*+b-closed map. Therefore f^{-1} is g*+b-continuous.

Theorem 6.8: Let $f: X \to Y$ be a bijective and g*+b-continuous map. Then the following statements are equivalent:
(a) *f* is a g*+b-open map.
(b) *f* is a g*+b-homeomorphism.
(c) *f* is a g*+b-closed map.

Proof
(a) ⇒ (b): Given $f: X \to Y$ be a bijective, g*+b-continuous and g*+b-open. Then by definition, *f* is a g*+b-homeomorphism.
(b) ⇒ (c): Given *f* is g*+b-open and bijective. By theorem 6.7, *f* is a g*+b-closed map.
(c) ⇒ (a): Given *f* is g*+b-closed and bijective. By theorem 6.7, *f* is a g*+b-open map.

Remark 6.9: The following example shows that the composition of two g*+b-homeomorphism is not a g*+b-homeomorphism.

Example 6.10: Let X = Y = Z = {a, b, c} with topologies $\tau = \{X, \emptyset, \{a\}, \{a,b\}\}, \tau^+ = \{X, \emptyset, \{a\}, \{b\}, \{a,b\}\}$; $\sigma = \{Y, \emptyset, \{a,b\}\}, \sigma^+ = \{Y, \emptyset, \{a\}, \{a,b\}\}$ and $\eta = \{Z, \emptyset, \{a\}\}, \eta^+ = \{Z, \emptyset, \{a\}, \{a,c\}\}$. Let $f: (X, \tau^+) \to (Y, \sigma^+)$ and $g: (Y, \sigma^+) \to (Z, \eta^+)$ be the identity map. Both *f* and *g* are g*+b-homeomorphism but their composition $g \circ f: (X, \tau^+) \to (Z, \eta^+)$ is not a g*+b-homeomorphism. Since for the closed set {a,c} in X, $g \circ f(\{a,c\})$ = {a,c} is not g*+b-closed in Z and hence $g \circ f$ is not g*+b-homeomorphism.

7. References

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