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K Ludi Jancy Jenifer
 Research Scholar, Department
 of Mathematics, Nirmala
 College for Women,
 Coimbatore, Tamil Nadu,
 India

K Indirani
 Associate Professor,
 Department of Mathematics,
 Nirmala College for Women,
 Coimbatore, Tamil Nadu,
 India

Correspondence
K Ludi Jancy Jenifer
 Research Scholar, Department
 of Mathematics, Nirmala
 College for Women,
 Coimbatore, Tamil Nadu,
 India

On supra regular generalized star star b- closed sets in supra topological spaces

K Ludi Jancy Jenifer and K Indirani

Abstract

In this paper a new class of $rg^{**}b^{\mu}$ closed sets is introduced and compared with already existing supra closed sets. Also some of the characteristics of $rg^{**}b^{\mu}$ -closed sets are studied, also $rg^{**}b^{\mu}$ neighbourhoods by using $rg^{**}b^{\mu}$ -open sets are discussed.

Keywords: $rg^{**}b^{\mu}$ closed set, $rg^{**}b^{\mu}$ neighbourhood, $rg^{**}b^{\mu}$ -open set

1. Introduction

In 1970, Levine [6], introduced the concept of generalized closed set and discussed the properties of sets, closed and open, maps, compactness, normal and separation axioms. In 1983 Mashhour *et al* [7] introduced supra topological spaces and studied S-continuous maps and S*-continuous maps. In 2008, Devi *et al* [3] introduced and studied a class of sets called supra α -open and a class of maps called $s\alpha$ -continuous between topological spaces, respectively.

In 2010, Sayed and Noiri [11] introduced and studied a class of sets called supra b-open and a class of maps called supra b-continuous. Ravi *et al* [9] introduced and studied a class of sets called supra g-closed and a class of maps called supra g-continuous respectively. In 2013, Indirani and Sindhu [4] introduced a new class of sets called, regular generalized star b-closed set in topological spaces. In 2014, Banupriya and Indirani [1], introduced Regular generalized star star b-closed sets in topological spaces.

In 2015, Chinnapparaj, Sathishmohan. Rajendran. Indirani [2], introduced a new class of sets called, Supra regular generalized star b-closed Set. In this paper a new class of supra closed set called supra regular generalized star star b-closed sets is introduced. We also study $rg^{**}b^{\mu}$ neighbourhoods by using $rg^{**}b^{\mu}$ -open sets.

2. Preliminaries

2.1 Definition: [7] Let X be a non-empty set. The subfamily $\mu \subseteq \mathcal{P}(X)$ where $\mathcal{P}(X)$ is the power set of X is said to be a supra topology on X if $X \in \mu, \emptyset \in \mu$ and μ is closed under arbitrary unions.

The pair (X, μ) is called a supra topological space. The elements of μ are said to be supra open in (X, μ) . Complements of supra open sets are called supra closed sets.

2.2 Definition: [7] Let A be a subset of (X, μ) . Then the supra closure of A is denoted by $cl^{\mu}(A) = \bigcap \{ B / B \text{ is a supra closed set and } A \subseteq B \}$.

2.3 Definition: [7] Let A be a subset of (X, μ) . Then the supra interior of A is denoted by $int^{\mu}(A) = \bigcup \{ B / B \text{ is a supra open set and } A \subseteq B \}$.

2.4 Definition: [7] Let (X, μ) be a topological space and μ be a supra topology on X. μ is supra topology associated with τ if $\tau \subseteq \mu$.

2.5 Definition: A subset A of a supra topological space (X, μ) is called

- (i) Supra semi-open ^[11] if $A \subseteq cl^\mu(int^\mu(A))$.
- (ii) Supra α -open ^[3] if $A \subseteq int^\mu(cl^\mu(int^\mu(A)))$.
- (iii) Supra pre-open ^[10] if $A \subseteq int^\mu(cl^\mu(A))$
- (iv) Supra r-closed if $A = cl^\mu(int^\mu(A))$.
- (v) Supra b-open set ^[11] if $A \subseteq cl^\mu(int^\mu(A)) \cup int^\mu(cl^\mu(A))$.

2.6 Definition: A subset A of a supra topological space (X, μ) is called

- (1) Supra generalized closed set ^[6] (briefly g^μ -closed) if $cl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is supra open in (X, μ) .
- (2) Supra semi-generalized closed set ^[5] (briefly sg^μ -closed) if $scl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is supra semi-open in (X, μ) .
- (3) Supra generalized semi-closed set ^[5] (briefly gs^μ -closed) if $scl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is supra open in (X, μ) .
- (4) Supra generalized α -closed set ^[2] (briefly $g\alpha^\mu$ -closed) if $acl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is supra α -open in (X, μ) .
- (5) Supra α -generalized closed set ^[2] (briefly ag^μ -closed) if $acl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is supra open in (X, μ) .
- (6) Supra generalized pre-closed set ^[2] (briefly gp^μ -closed) if $pcl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is supra open in (X, μ) .
- (7) Supra generalized pre-regular closed set ^[12] (briefly gpr^μ -closed) if $pcl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is supra r-open in (X, μ) .
- (8) Supra regular generalized closed set ^[2] (briefly rg^μ -closed) if $cl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is supra regular open in (X, μ) .
- (9) Supra generalized regular closed set ^[2] (briefly gr^μ -closed) if $rcl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is supra open in (X, μ) .
- (10) Supra generalized star closed set ^[8] (briefly $g^{*\mu}$ -closed) if $cl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is g^μ open in (X, μ) .
- (11) Supra generalized star semi-closed set ^[2] (briefly g^*s^μ -closed) if $scl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is g^μ open in (X, μ) .
- (12) Supra generalized # closed set ^[2] (briefly $g^{\#\mu}$ -closed) if $cl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is αg^μ open in (X, μ) .
- (13) Supra generalized # semi-closed set ^[2] (briefly $g^{\#s^\mu}$ -closed) if $scl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is supra αg^μ open in (X, μ) .
- (14) Supra regular generalized star b-closed set ^[2] (briefly rg^*b^μ -closed) if $bcl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is supra rg-open in (X, μ) .
- (15) Supra generalized star b-closed set ^[2] (briefly g^*b^μ -closed) if $bcl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is supra g-open in (X, μ) .
- (16) Supra generalized αb -closed set ^[1] (briefly gab^μ -closed) if $bcl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is supra α -open in (X, μ) .
- (17) Supra semi generalized b-closed set ^[1] (briefly sgb^μ -closed) if $bcl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is supra semi-open in (X, μ) . The complements of the above mentioned closed sets are called their respective open sets.

3. $rg^{}b^\mu$ -Closed Sets**

3.1 Definition: A subset A of a supra topological space (X, μ) is called supra regular generalized star star b-closed set (briefly $rg^{**}b^\mu$ -closed set) if $rg^*bcl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is supra open in X .

3.2 Theorem: Every supra closed set is $rg^{**}b^\mu$ -closed set but not conversely.

Proof: Let A be a supra closed set in X such that $A \subseteq U$ and U is supra open in X . Since A is supra closed, $cl^\mu(A) = A \subseteq U$. By theorem, "Every supra closed set is rg^*b^μ -closed set", $rg^*bcl^\mu(A) \subseteq cl^\mu(A) = A$. Therefore $rg^*bcl^\mu(A) \subseteq U$. Hence A is $rg^{**}b^\mu$ -closed set.

3.3 Example: Let $X = \{a, b, c\}$, with $\mu = \{X, \emptyset, \{a, b\}, \{b, c\}\}$ and $\mu^c = \{X, \emptyset, \{a\}, \{c\}\}$. Then $\{a, c\}$ is $rg^{**}b^\mu$ -closed set but not supra closed.

3.4 Theorem: Every supra semi-closed set is $rg^{**}b^\mu$ -closed set but not conversely.

Proof: Let A be a supra semi-closed set in X such that $A \subseteq U$ and U is supra open in X . Since A is supra semi-closed, $scl^\mu(A) = A \subseteq U$. But every supra semi-closed set is rg^*b^μ -closed set. Therefore, $rg^*bcl^\mu(A) \subseteq scl^\mu(A) = A \subseteq U$. Therefore $rg^*bcl^\mu(A) \subseteq U$. Hence A is $rg^{**}b^\mu$ -closed set.

3.5 Example: Let $X = \{a, b, c, d\}$, with $\mu = \{X, \emptyset, \{b\}, \{b, c\}, \{c, d\}, \{b, c, d\}\}$. Then $\{a, b, c\}$ is $rg^{**}b^\mu$ -closed set but not supra semi-closed.

3.6 Theorem: Every supra α -closed set is $rg^{**}b^\mu$ -closed set but not conversely.

Proof: Let A be a supra α -closed set in X such that $A \subseteq U$ and U is supra open in X . Since A is supra α -closed, $acl^\mu(A) = A \subseteq U$. By theorem, "Every supra α -closed set is rg^*b^μ -closed set", $rg^*bcl^\mu(A) \subseteq acl^\mu(A) = A \subseteq U$. Therefore $rg^*bcl^\mu(A) \subseteq U$. Hence A is $rg^{**}b^\mu$ -closed set.

3.7 Example: Let $X = \{a, b, c, d\}$, with $\mu = \{X, \emptyset, \{c\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$. Then $\{b, c, d\}$ is $rg^{**}b^\mu$ -closed set but not supra α -closed.

3.8 Theorem: Every supra *pre*-closed set is $rg^{**}b^\mu$ -closed set but not conversely.

Proof: Let A be a supra *pre*-closed set in X such that $A \subseteq U$ and U is supra open in X . Since A is supra *pre*-closed, $pcl^\mu(A) = A \subseteq U$. But every supra *pre*-closed set is $rg^{**}b^\mu$ -closed set. Therefore, $rg^{**}bcl^\mu(A) \subseteq pcl^\mu(A) = A \subseteq U$. Therefore $rg^{**}bcl^\mu(A) \subseteq U$. Hence A is $rg^{**}b^\mu$ -closed set.

3.9 Example: Let $X = \{a, b, c, d\}$, with $\mu = \{X, \emptyset, \{d\}, \{b, c\}, \{c, d\}, \{b, c, d\}\}$. Then $\{a, c, d\}$ is $rg^{**}b^\mu$ -closed set but not supra *pre*-closed.

3.10 Theorem: Every supra *regular*-closed set is $rg^{**}b^\mu$ -closed set but not conversely.

Proof: Let A be a supra *regular*-closed set in X such that $A \subseteq U$ and U is supra open in X . Since A is supra *regular*-closed, $rcl^\mu(A) = A \subseteq U$. But every supra *regular*-closed set is $rg^{**}b^\mu$ -closed set. Therefore, $rg^{**}bcl^\mu(A) \subseteq rcl^\mu(A) = A \subseteq U$. Therefore $rg^{**}bcl^\mu(A) \subseteq U$. Hence A is $rg^{**}b^\mu$ -closed set.

3.11 Example: Let $X = \{a, b, c, d\}$, with $\mu = \{X, \emptyset, \{b\}, \{d\}, \{b, d\}, \{b, c\}, \{c, d\}, \{b, c, d\}\}$. Then $\{a, c\}$ is $rg^{**}b^\mu$ -closed set but not supra *regular*-closed.

3.12 Theorem: Every supra *b*-closed set is $rg^{**}b^\mu$ -closed set but not conversely.

Proof: Let A be a supra *b*-closed set in X such that $A \subseteq U$ and U is supra open in X . Since A is supra *b*-closed, $bcl^\mu(A) = A \subseteq U$. But every supra *b*-closed set is $rg^{**}b^\mu$ -closed set. Therefore, $rg^{**}bcl^\mu(A) \subseteq bcl^\mu(A) = A \subseteq U$. Therefore $rg^{**}bcl^\mu(A) \subseteq U$. Hence A is $rg^{**}b^\mu$ -closed set.

3.13 Example: Let $X = \{a, b, c, d\}$, with $\mu = \{X, \emptyset, \{a\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$. Then $\{a, b, d\}$ is $rg^{**}b^\mu$ -closed set but not supra *b*-closed.

3.14 Theorem: Every supra *g*-closed set is $rg^{**}b^\mu$ -closed set but not conversely.

Proof: Let A be a supra *g*-closed set in X such that $A \subseteq U$ and U is supra open in X . Since A is supra *g*-closed, $cl^\mu(A) \subseteq U$, whenever $A \subseteq U$ and U is supra open in X . But every supra *g*-closed set is $rg^{**}b^\mu$ -closed set. Therefore, $rg^{**}bcl^\mu(A) \subseteq cl^\mu(A) \subseteq U$. Therefore $rg^{**}bcl^\mu(A) \subseteq U$. Hence A is $rg^{**}b^\mu$ -closed set.

3.15 Example: Let $X = \{a, b, c, d\}$, with $\mu = \{X, \emptyset, \{a\}, \{c, d\}, \{d, a\}, \{a, c, d\}\}$. Then $\{a, d\}$ is $rg^{**}b^\mu$ -closed set but not supra *g*-closed.

3.16 Theorem: Every supra *sg*-closed set is $rg^{**}b^\mu$ -closed set but not conversely.

Proof: Let A be a supra *sg*-closed set in X such that $A \subseteq U$ and U is supra open in X . Since A is supra *sg*-closed, $scl^\mu(A) \subseteq U$, whenever $A \subseteq U$ and U is supra semi-open in X . But every supra *sg*-closed set is $rg^{**}b^\mu$ -closed set. Therefore, $rg^{**}bcl^\mu(A) \subseteq scl^\mu(A) \subseteq U$. Therefore $rg^{**}bcl^\mu(A) \subseteq U$. Hence A is $rg^{**}b^\mu$ -closed set.

3.17 Example: Let $X = \{a, b, c, d\}$, with $\mu = \{X, \emptyset, \{a\}, \{c\}, \{a, c\}, \{c, d\}, \{a, d\}, \{a, c, d\}\}$. Then $\{a, b, c\}$ is $rg^{**}b^\mu$ -closed set but not supra *sg*-closed.

3.18 Theorem: Every supra *gs*-closed set is $rg^{**}b^\mu$ -closed set but not conversely.

Proof: Let A be a supra *gs*-closed set in X such that $A \subseteq U$ and U is supra open in X . Since A is supra *gs*-closed, $scl^\mu(A) \subseteq U$, whenever $A \subseteq U$ and U is supra open in X . But every supra *gs*-closed set is $rg^{**}b^\mu$ -closed set. Therefore, $rg^{**}bcl^\mu(A) \subseteq scl^\mu(A) \subseteq U$. Therefore $rg^{**}bcl^\mu(A) \subseteq U$. Hence A is $rg^{**}b^\mu$ -closed set.

3.19 Example: Let $X = \{a, b, c, d\}$, with $\mu = \{X, \emptyset, \{a\}, \{c\}, \{a, c\}, \{a, b, c\}, \{b, c, d\}\}$. Then $\{c\}$ is $rg^{**}b^\mu$ -closed set but not supra *gs*-closed.

3.20 Theorem: Every supra *g α* -closed set is $rg^{**}b^\mu$ -closed set but not conversely.

Proof: Let A be a supra $g\alpha$ - closed set in X such that $A \subseteq U$ and U is supra open in X . Since A is supra $g\alpha$ -closed, $acl^\mu(A) \subseteq U$ whenever, $A \subseteq U$ and U is supra- α open in X . But every supra α -closed set is rg^*b^μ -closed set. Therefore, $rg^*bcl^\mu(A) \subseteq acl^\mu(A) \subseteq U$. Therefore $rg^*bcl^\mu(A) \subseteq U$. Hence A is $rg^{**}b^\mu$ -closed set.

3.21 Example: Let $X = \{a, b, c, d\}$, with $\mu = \{X, \emptyset, \{a\}, \{c\}, \{a, c\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$. Then $\{a\}$ is $rg^{**}b^\mu$ -closed set but not supra $g\alpha$ -closed.

3.22 Theorem: Every supra ag -closed set is $rg^{**}b^\mu$ -closed set but not conversely.

Proof: Let A be a supra ag - closed set in X such that $A \subseteq U$ and U is supra open in X . Since A is supra ag -closed, $acl^\mu(A) \subseteq U$ whenever, $A \subseteq U$ and U is supra open in X . But every supra- α closed set is rg^*b^μ -closed set. Therefore, $rg^*bcl^\mu(A) \subseteq acl^\mu(A) \subseteq U$. Therefore $rg^*bcl^\mu(A) \subseteq U$. Hence A is $rg^{**}b^\mu$ -closed set.

3.23 Example: Let $X = \{a, b, c, d\}$, with $\mu = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}\}$. Then $\{a, c\}$ is $rg^{**}b^\mu$ -closed set but not supra ag -closed.

3.24 Theorem: Every supra gp -closed set is $rg^{**}b^\mu$ -closed set but not conversely.

Proof: Let A be a supra gp - closed set in X such that $A \subseteq U$ and U is supra open in X . Since A is supra gp -closed, $pcl^\mu(A) \subseteq U$ whenever, $A \subseteq U$ and U is supra open in X . But every supra pre -closed set is rg^*b^μ -closed set. Therefore, $rg^*bcl^\mu(A) \subseteq pcl^\mu(A) \subseteq U$. Therefore $rg^*bcl^\mu(A) \subseteq U$. Hence A is $rg^{**}b^\mu$ -closed set.

3.25 Example: Let $X = \{a, b, c, d\}$, with $\mu = \{X, \emptyset, \{c\}, \{d\}, \{c, d\}, \{a, d\}, \{a, c\}, \{a, c, d\}\}$. Then $\{a, d\}$ is $rg^{**}b^\mu$ -closed set but not supra gp -closed.

3.26 Theorem: Every supra gr -closed set is $rg^{**}b^\mu$ -closed set but not conversely.

Proof: Let A be a supra gr - closed set in X such that $A \subseteq U$ and U is supra open in X . Since A is supra gr -closed, $rcl^\mu(A) \subseteq U$ whenever, $A \subseteq U$ and U is supra open in X . But every supra r - closed set is rg^*b^μ -closed set. Therefore, $rg^*bcl^\mu(A) \subseteq rcl^\mu(A) \subseteq U$. Therefore $rg^*bcl^\mu(A) \subseteq U$. Hence A is $rg^{**}b^\mu$ -closed set.

3.27 Example: Let $X = \{a, b, c, d\}$, with $\mu = \{X, \emptyset, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}\}$. Then $\{a\}$ is $rg^{**}b^\mu$ -closed set but not supra gr -closed.

3.28 Theorem: Every supra g^* -closed set is $rg^{**}b^\mu$ -closed set but not conversely.

Proof: Let A be a supra g^* - closed set in X such that $A \subseteq U$ and U is supra open in X . Since A is supra g^* -closed, $cl^\mu(A) \subseteq U$, whenever $A \subseteq U$ and U is supra g -open. But every supra g -closed set is rg^*b^μ -closed set and every supra closed set is $rg^{**}b^\mu$ set. Therefore, $rg^*bcl^\mu(A) \subseteq cl^\mu(A) \subseteq U$. Therefore $rg^*bcl^\mu(A) \subseteq U$. Hence A is $rg^{**}b^\mu$ -closed set.

3.29 Example: Let $X = \{a, b, c, d\}$, with $\mu = \{X, \emptyset, \{a\}, \{d, c\}, \{a, d\}, \{a, c, d\}\}$. Then $\{a\}$ is $rg^{**}b^\mu$ -closed set but not supra g^* closed.

3.30 Theorem: Every supra g^*s -closed set is $rg^{**}b^\mu$ -closed set but not conversely.

Proof: Let A be a supra g^*s - closed set in X such that $A \subseteq U$ and U is supra open in X . Since A is supra g^*s -closed, $scl^\mu(A) \subseteq U$ whenever, $A \subseteq U$ and U is supra g -open in X . But every supra $semi$ -closed set is rg^*b^μ -closed set. Therefore, $rg^*bcl^\mu(A) \subseteq scl^\mu(A) \subseteq U$. Therefore $rg^*bcl^\mu(A) \subseteq U$. Hence A is $rg^{**}b^\mu$ -closed set.

3.31 Example: Let $X = \{a, b, c, d\}$, with $\mu = \{X, \emptyset, \{c, a\}, \{a, b\}\}$. Then $\{a\}$ is $rg^{**}b^\mu$ -closed set but not supra g^*s closed.

3.32 Theorem: Every supra $g^\#$ -closed set is $rg^{**}b^\mu$ -closed set but not conversely.

Proof: Let A be a supra $g^\#$ - closed set in X such that $A \subseteq U$ and U is supra open in X . Since A is supra $g^\#$ -closed, $cl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is supra ag - open in X . But every supra closed set is rg^*b^μ -closed set. Therefore, $rg^*bcl^\mu(A) \subseteq cl^\mu(A) \subseteq U$. Therefore $rg^*bcl^\mu(A) \subseteq U$. Hence A is $rg^{**}b^\mu$ -closed set.

3.33 Example: Let $X = \{a, b, c, d\}$, with $\mu = \{X, \emptyset, \{b\}, \{c\}, \{b, c\}, \{b, d\}, \{c, d\}, \{b, c, d\}\}$. Then $\{b, d\}$ is $rg^{**}b^\mu$ -closed set but not supra $g^\#$ closed.

3.34 Theorem: Every supra $g^\#s$ -closed set is $rg^{**}b^\mu$ -closed set but not conversely.

Proof: Let A be a supra $g^\#s$ -closed set in X such that $A \subseteq U$ and U is supra open in X . Since A is supra $g^\#s$ -closed, $scl^\mu(A) \subseteq U$ whenever, $A \subseteq U$ and U is supra αg -open in X . But every supra semi-closed set is rg^*b^μ -closed set. Therefore, $rg^*bcl^\mu(A) \subseteq scl^\mu(A) \subseteq U$. Therefore $rg^*bcl^\mu(A) \subseteq U$. Hence A is $rg^{**}b^\mu$ -closed set.

3.35 Example: Let $X = \{a, b, c, d\}$, with $\mu = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{b, c, d\}\}$. Then $\{b, d\}$ is $rg^{**}b^\mu$ -closed set but not supra $g^\#s$ closed.

3.36 Theorem: Every supra rg^*b -closed set is $rg^{**}b^\mu$ -closed set but not conversely.

Proof: Let A be a supra rg^*b -closed set in X such that $A \subseteq U$ and U is supra open in X . Since A is supra rg^*b -closed, $rg^*bcl^\mu(A) \subseteq U$. Therefore $rg^*bcl^\mu(A) \subseteq U$. Hence A is $rg^{**}b^\mu$ -closed set.

3.37 Example: Let $X = \{a, b, c, d\}$, with $\mu = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, d, c\}, \{b, c, d\}\}$. Then $\{a, b, c\}$ is $rg^{**}b^\mu$ -closed set but not supra rg^*b closed.

3.38 Theorem: Every supra g^*b -closed set is $rg^{**}b^\mu$ -closed set but not conversely.

Proof: Let A be a supra g^*b -closed set in X such that $A \subseteq U$ and U is supra open in X . Since A is supra g^*b -closed, $bcl^\mu(A) \subseteq U$, whenever, $A \subseteq U$ and U is supra g -open in X . But every supra b -closed set is rg^*b^μ -closed set. Therefore, $rg^*bcl^\mu(A) \subseteq bcl^\mu(A) \subseteq U$. Therefore $rg^*bcl^\mu(A) \subseteq U$. Hence A is $rg^{**}b^\mu$ -closed set.

3.39 Example: Let $X = \{a, b, c, d\}$, with $\mu = \{X, \emptyset, \{a\}, \{d\}, \{a, d\}, \{a, b, c\}, \{b, c, d\}\}$. Then $\{a, c, d\}$ is $rg^{**}b^\mu$ -closed set but not supra g^*b closed.

3.40 Theorem: Every supra gab -closed set is $rg^{**}b^\mu$ -closed set but not conversely.

Proof: Let A be a supra gab -closed set in X such that $A \subseteq U$ and U is supra open in X . Since A is supra gab -closed, $bcl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is supra α -open in X . But every supra b -closed set is rg^*b^μ -closed set. Therefore, $rg^*bcl^\mu(A) \subseteq bcl^\mu(A) \subseteq U$. Therefore $rg^*bcl^\mu(A) \subseteq U$. Hence A is $rg^{**}b^\mu$ -closed set.

3.41 Example: Let $X = \{a, b, c, d\}$, with $\mu = \{X, \emptyset, \{d\}, \{a\}, \{a, c\}, \{c, d\}, \{a, d\}, \{c, d, a\}\}$. Then $\{a, b, d\}$ is $rg^{**}b^\mu$ -closed set but not supra gab closed.

3.42 Theorem: Every supra sgb -closed set is $rg^{**}b^\mu$ -closed set but not conversely.

Proof: Let A be a supra sgb -closed set in X such that $A \subseteq U$ and U is supra open in X . Since A is supra sgb -closed, $bcl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is supra semi-open in X . But every supra b -closed set is rg^*b^μ -closed set. Therefore, $rg^*bcl^\mu(A) \subseteq bcl^\mu(A) \subseteq U$. Therefore $rg^*bcl^\mu(A) \subseteq U$. Hence A is $rg^{**}b^\mu$ -closed set.

3.43 Example: Let $X = \{a, b, c, d\}$, with $\mu = \{X, \emptyset, \{d\}, \{c\}, \{b, d\}, \{b, c\}, \{c, d\}, \{b, c, d\}\}$. Then $\{a, c, d\}$ is $rg^{**}b^\mu$ -closed set but not supra sgb closed.

3.44 Remark: $rg^{**}b^\mu$ -closed set and rg^μ -closed set are independent to each other as seen from the following examples.

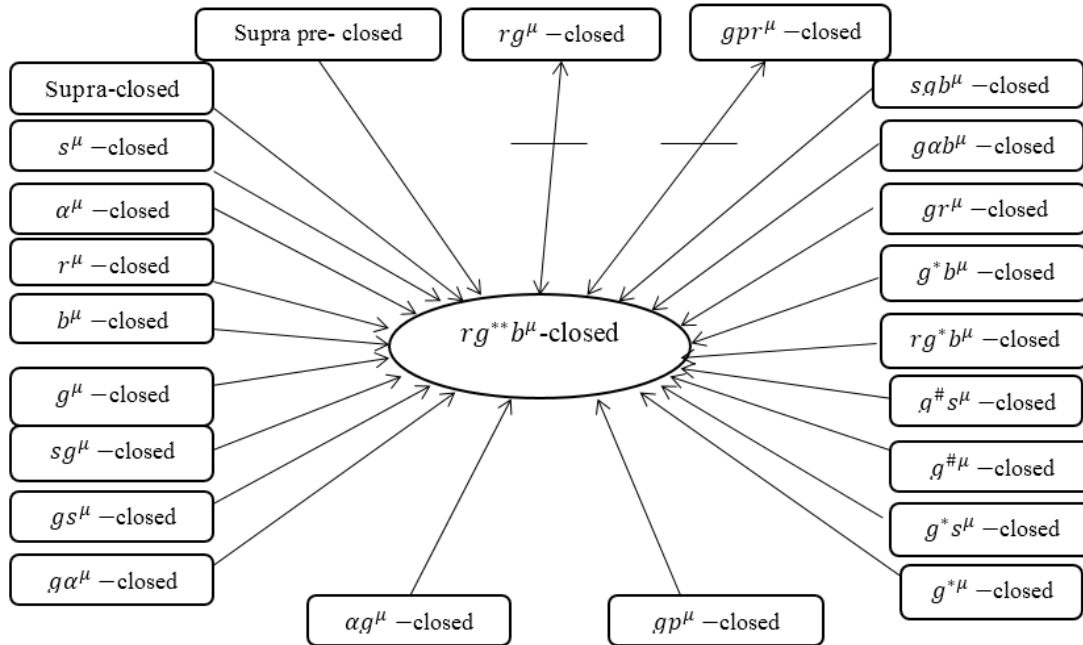
3.45 Example: Let $X = \{a, b, c, d\}$, with $\mu = \{X, \emptyset, \{b\}, \{d\}, \{b, d\}, \{b, c, d\}, \{a, b, c\}\}$. Then $\{b, c\}$ is rg^μ -closed set but not $rg^{**}b^\mu$ -closed set.

3.46 Example: Let $X = \{a, b, c, d\}$, with $\mu = \{X, \emptyset, \{e\}, \{d\}, \{e, d\}, \{c, d\}, \{b, d\}, \{c, d, e\}, \{b, d, e\}, \{c, b, d\}, \{a, c, d, e\}, \{a, b, d, e\}, \{b, c, d, e\}\}$. Then $\{b, c, d\}$ is $rg^{**}b^\mu$ -closed set but not rg^μ closed set.

3.47 Remark: $rg^{**}b^\mu$ -closed set and gpr^μ -closed set are independent to each other as seen from the following examples.

3.48 Example: Let $X = \{a, b, c, d\}$, with $\mu = \{X, \emptyset, \{a\}, \{c\}, \{a, c\}, \{b, c, d\}, \{a, c, d\}\}$. Then $\{a, c\}$ is $-gpr^\mu$ -closed set but not $rg^{**}b^\mu$ closed set.

3.49 Example: Let $X = \{a, b, c, d\}$, with $\mu = \{X, \emptyset, \{c\}, \{d\}, \{c, d\}, \{a, d\}, \{a, c\}, \{a, c, d\}\}$. Then $\{a, c\}$ is $-rg^{**}b^\mu$ -closed set but not gpr^μ closed set. From the



4. Some Characteristics of $rg^{}b^\mu$ -Closed Sets**

4.1 Theorem: A set A is $rg^{**}b^\mu$ -closed set then $rg^*bcl^\mu(A) \setminus A$ contains no non-empty supra closed set.

Proof: Let F be a supra closed set in X such that $F \subseteq rg^*bcl^\mu(A) - A$. Then $A \subseteq X \setminus F$. Since A is $rg^{**}b^\mu$ -closed set and $X \setminus F$ is supra open then $rg^*bcl^\mu(A) \subseteq X \setminus F$. That is $F \subseteq X \setminus rg^*bcl^\mu(A)$. So $F \subseteq (X \setminus rg^*bcl^\mu(A)) \cap (rg^*bcl^\mu(A) \setminus A)$. Therefore $F = \emptyset$.

4.2 Remark: The intersection of any two $rg^{**}b^\mu$ -closed sets in X need not be $rg^{**}b^\mu$ -closed set in X.

4.3 Example: Let $X = \{a, b, c, d\}$, with $\mu = \{X, \emptyset, \{b\}, \{d\}, \{b, d\}, \{a, b, c\}, \{a, c, d\}\}$. The sets $\{X, \emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{a, b, d\}, \{b, c, d\}\}$ are $rg^{**}b^\mu$ -closed in X. But $\{a, b, d\} \cap \{b, c, d\} = \{b, d\}$ which is not $rg^{**}b^\mu$ -closed set in X.

4.4 Remark: The union of any two subsets of $rg^{**}b^\mu$ -closed sets in X need not be $rg^{**}b^\mu$ -closed set in X.

4.5 Example: Let $X = \{a, b, c\}$ with $\mu = \{X, \emptyset, \{b, c\}, \{a, c\}\}$. The sets $\{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}\}$ are $rg^{**}b^\mu$ -closed set in X. But $\{a\} \cup \{c\} = \{a, c\}$ which is not $rg^{**}b^\mu$ -closed set in X.

4.6 Theorem: If $A \subseteq Y \subseteq X$ and suppose that A is $rg^{**}b^\mu$ -closed set in X, then A is $rg^{**}b^\mu$ -closed set relative to Y.

Proof: Given that $A \subseteq Y \subseteq X$ and A is $rg^{**}b^\mu$ -closed set in X. To prove A is $rg^{**}b^\mu$ -closed set relative to Y. Let us assume that $A \subseteq Y \cap U$, where U is supra open in X. Since A is $rg^{**}b^\mu$ -closed set, $A \subseteq U$ implies $rg^*bcl^\mu(A) \subseteq U$. It follows that $Y \cap rg^*bcl^\mu(A) \subseteq Y \cap U$, that is A is $rg^{**}b^\mu$ -closed set relative to Y.

5. Supra Regular Generalized Star Star b-Open Sets

5.1 Definition: A subset A of a supra topological space (X, μ) , is called regular generalized star star b-open set (briefly $rg^{**}b^\mu$ -open set) if A^c is $rg^{**}b^\mu$ -closed set in X. We denote the family of all $rg^{**}b^\mu$ -open sets in X by $RG^{**}B^\mu-O(X)$.

5.2 Remark: The union of two $rg^{**}b^\mu$ -open sets in X is generally not an $rg^{**}b^\mu$ -open set in X.

5.3 Example: $X = \{a, b, c, d\}$, with $\mu = \{X, \emptyset, \{a, b, c\}, \{b, c, d\}, \{a\}, \{d\}, \{a, d\}\}$. The sets $\{X, \emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{c, d\}, \{b, d\}, \{a, d\}, \{a, c\}, \{a, b\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}, \{a, b, d\}\}$ are $rg^{**}b^\mu$ -open in X. But $\{b\} \cup \{c\} = \{b, c\}$ which is not $rg^{**}b^\mu$ -open in X.

5.4 Remark: The intersection of two $rg^{**}b^\mu$ -open sets in X is generally not an $rg^{**}b^\mu$ -open set in X.

5.5 Example: Let $X = \{a, b, c\}$ with $\mu = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$. The sets $\{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ are $rg^{**}b^\mu$ -open in X. But $\{a, c\} \cap \{b, c\} = \{c\}$ is not an $rg^{**}b^\mu$ -open set in X.

6. Supra Regular Generalized Star Star b-Neighbour hood S

6.1 Definition: Let x be a point in a supra topological space X. A subset N of X is said to be $rg^{**}b^\mu$ -neighbourhood of x iff there exists $rg^{**}b^\mu$ -open set G such that $x \in G \subseteq N$.

6.2 Definition: A subset N of a space X is called $rg^{**}b^\mu$ -neighbourhood of $A \subset X$ iff there exists an $rg^{**}b^\mu$ -open set G such that $A \subset G \subset N$.

6.3 Theorem: Every supra neighbourhood N of $x \in X$ is $rg^{**}b^\mu$ -neighbourhood of x .

Proof: Let N be a supra neighbourhood of a point $x \in X$. To prove that N is an $rg^{**}b^\mu$ -neighbourhood of x . By the definition of supra neighbourhood, there exists an supra open set G such that $x \in G \subset N$. But every supra open set is $rg^{**}b^\mu$ -open set, G is $rg^{**}b^\mu$ -open set such that $x \in G \subset N$. Hence N is an $rg^{**}b^\mu$ -neighbourhood of x .

6.4 Remark: In general, an $rg^{**}b^\mu$ -neighbourhood of $x \in X$ need not be a supra neighbourhood of x in X as seen from the example below.

6.5 Example: Let $X = \{a, b, c\}$ with $\mu = \{X, \emptyset, \{c\}, \{a, b\}, \{a, c\}\}$. The sets $\{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$. The set $\{b, c\}$ is $rg^{**}b^\mu$ -neighbourhood of the point $\{b\}$, there exists an $rg^{**}b^\mu$ -open set $\{b, c\}$ such that $b \in \{b, c\} \subseteq \{b, c\}$. But $\{b, c\}$ is not an supra neighbourhood of the point $\{b\}$, since no supra open set exists such that $b \in G \subseteq \{b, c\}$.

6.6 Remark: The $rg^{**}b^\mu$ -neighbourhood N of $x \in X$ need not be $rg^{**}b^\mu$ -open in X .

6.7 Example: $X = \{a, b, c, d\}$ with $\mu = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, c, d\}\}$. The sets $\{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$ are $rg^{**}b^\mu$ -open in X . The set $\{b, d\}$ is $rg^{**}b^\mu$ -neighbourhood of $\{b\}$, since $b \in \{b\} \subseteq \{b, d\}$. But $\{b, d\}$ is not $rg^{**}b^\mu$ -open in X .

6.8 Theorem: If a subset N of a space X is $rg^{**}b^\mu$ -open, then N is $rg^{**}b^\mu$ -neighbourhood of each of its points.

Proof: Suppose N is $rg^{**}b^\mu$ -open. Let $x \in N$. We claim that N is $rg^{**}b^\mu$ -neighbourhood of x . For, N is $rg^{**}b^\mu$ -open set such that $x \in N \subset N$. Then N is $rg^{**}b^\mu$ -neighbourhood of x . Since x is an arbitrary point of N , it follows that N is an $rg^{**}b^\mu$ -neighbourhood of each of its points.

6.9 Theorem: Let X be a supra topological space. If F is $rg^{**}b^\mu$ -closed subset of X and $x \in F^c$. Then there exists an $rg^{**}b^\mu$ -neighbourhood N of x such that $N \cap F = \emptyset$.

Proof: Let F be $rg^{**}b^\mu$ -closed subset of X and $x \in F^c$. Then F^c is of $rg^{**}b^\mu$ -open set of X . So by theorem 6.8, F^c contains an $rg^{**}b^\mu$ -neighbourhood N of each of its points. Hence there exists an $rg^{**}b^\mu$ -neighbourhood N of x such that $N \subset F^c$, (i.e.) $N \cap F = \emptyset$.

6.10 Definiton: Let x be a point in supra topological space. The set of all $rg^{**}b^\mu$ -neighbourhood of x is called the $rg^{**}b^\mu$ -neighbourhood system at x and is denoted by $rg^{**}b^\mu$ - $N(x)$.

6.11 Theorem: Let N be $rg^{**}b^\mu$ -neighbourhood of $x \in X$ and X be a supra topological space. Let $rg^{**}b^\mu$ - $N(X, \mu)$ be the collection of all $rg^{**}b^\mu$ -neighbourhood of x . Then we have the following results.

- (i) For all $x \in X, rg^{**}b^\mu$ - $N(x) \neq \emptyset$.
- (ii) $(ii) N \in rg^{**}b^\mu$ - $N(x) \implies x \in N$.
- (iii) $N \in rg^{**}b^\mu$ - $N(x), M \supset N \implies M \in rg^{**}b^\mu$ - $N(x)$.
- (iv) $N \in rg^{**}b^\mu$ - $N(x) \implies$ there exists $M \in rg^{**}b^\mu$ - $N(x)$ such that $M \subset N$ then $M \in rg^{**}b^\mu$ - $N(y)$ for every $y \in M$.

Proof

- (i) Obvious.
- (ii) If $N \in rg^{**}b^\mu$ - $N(x)$, then N is $rg^{**}b^\mu$ -neighbourhood of x . Therefore $x \in N$.
- (iii) Let $N \in rg^{**}b^\mu$ - $N(x)$, and $M \supset N$. Then there is an $rg^{**}b^\mu$ -open set G such that $x \in G \subset N$. Since $N \subset M, x \in G \subset N$ and so M is $rg^{**}b^\mu$ -neighbourhood of x . Hence $M \in rg^{**}b^\mu$ - $N(x)$.
- (iv) Let $N \in rg^{**}b^\mu$ - $N(x)$, then there is an $rg^{**}b^\mu$ -open set M such that $x \in M \subset N$. Since M is $rg^{**}b^\mu$ -neighbourhood of each of its points. Therefore $M \in rg^{**}b^\mu$ - $N(y)$ for every $y \in M$.

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