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# Some forms of nano generalized b-closed maps in nano topological spaces

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**Abstract**
In this paper we introduce the concept of nano*generalized b-closed maps and we obtain the basic properties and their relationships with other forms of nano*generalized b- closed maps in nano topological spaces

**Keywords:** Nano*generalized b-closed maps, almost nano*generalized b-closed maps, strongly nano*generalized b-closed maps.

## 1. Introduction

Levine [1] derived the concept of generalized closed sets in topological space. Al Omari and Mohd. Salmi Md. Noorani [2] studied the class of generalized b-closed sets. The notation of nano topology was introduced by Lellis Thivagar [10] which was defined in terms of approximations and boundry regions of a subset of an universe using an equivalence relation on it and also defined nano closed sets, nano interior and nano-closure. Nano gb-closed set was initiated by Dhanis Arul Mary and I. Arockiarani [6]. The purpose of the paper is to introduce and investigate some of the fundamental properties of nano*generalized b-closed maps and almost nano*generalized b-closed maps, strongly nano*generalized b-closed maps and study some of its properties.

## 2. Preliminaries

**2.1 Definition [15]: Let U be a non-empty finite set of objects called the universe and R be** an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be the approximation space. Let $X \subseteq U$

1. The lower approximation of X with respect to R is the set of all objects, which can be for certainly classified as X with respect to R and it is denoted by $L_R(X)$. That is $L_R(X) = \underset{x\in U}{U}\{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by $X \in U$ 2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is $U_R(X) = \underset{x\in U}{U}\{R(x) : R(X) \cap X \neq \varphi\}$

3. The boundary of X with respect to R is the set of all objects, which can be classified neither as X nor as not-X with respect to R and it is denoted by $B_R(X)$. That is $B_R(X) = U_R(X) - L_R(X)$.

**Definition 2.2 [10]**: If $(U, R)$ is an approximation space and $X, Y \subseteq U$, then

(i) $L_R(X) \subseteq X \subseteq U_R(X)$

(ii) $L_R(\varphi) = U_R(\varphi) = \varphi$ and $L_R(U) = U_R(U) = U$
(iii) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
(iv) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$
(v) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$
(vi) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
(vii) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$
(viii) $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$
(ix) $U_R U_R(X) = L_R U_R(X) = U_R(X)$
(x) $L_R L_R(X) = U_R L_R(X) = L_R(X)$

**2.3 Definition [9]:** Let U be non-empty, finite universe of objects and R be an equivalence relation on U. Let $X \subseteq U$. Let $\tau_R(X) = \{U, \varphi, L_R(X), U_R(X), B_R(X)\}$. Then $\tau_R(X)$ is a topology on U, called as the nano topology with respect to X. Elements of the nano topology are known as the nano-open sets in U and $(U, \tau_R(X))$ is called the nano topological space. $[\tau_R(X)]^c$ is called as the dual nano topology of $\tau_R(X)$. Elements of $[\tau_R(X)]^c$ are called as nano closed sets.

**2.4 Definition [10]:** If $\tau_R(X)$ is the nano topolopy on U with respect to X, then the set $B = \{U, L_R(X), U_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$

**2.5 Definition [10]:** If $(U, \tau_R(X))$ is a nano topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then the nano interior of A is defined as the union of all nano-open subsets of A and it is denoted by $N\operatorname{int}(A)$. That is $N\operatorname{int}(A)$, is the largest nano open subset of A. The nano closure of A is defined as the intersection of all nano closed sets containing A and is denoted by $Ncl(A)$. That is $Ncl(A)$, is the smallest nano closed set containing A.

**2.6 Definition [6]:** A subset A of a nano topological space $(U, \tau_R(X))$ is called nano generalized b-closed( briefly, nano gb-closed),if Nbcl(A)⊆V whenever A⊆V and V is nano open in U.

**2.7 Definition [7]:** A subset A of a nano topological space $(U, \tau_R(X))$ is called nano*generalized b-closed if $Nbcl(A) \subseteq V$ whenever $A \subseteq V$ and V is nano gb-open in U

**2.8 Definition [10]:** Let $(U, \tau_R(X))$ be a nano topological space and $A \subseteq U$. Then A is said to be Nano semi open If $A \subseteq Ncl(N\operatorname{int}(A))$

Nano pre-open if $A \subseteq N\operatorname{int}(Ncl(A))$

Nano $\alpha$ -open if $A \subseteq N\operatorname{int}(Ncl(N\operatorname{int}(A)))$

Nano b-open if $A \subseteq Ncl(N\operatorname{int}(A)) \cup N\operatorname{int}(Ncl(A))$

Nano regular-open if $A = N\operatorname{int}(Ncl(A))$

**2.9 Definition:** Let (U,$_R$(X)) and (V,$\tau_{R'}$(Y)) be a nano topological spaces, then a map f:(U,$\tau_R$(X))→(V,$\tau_{R'}$(Y)) is said to be
(i) Nano continuous if $f^{-1}(V)$is nano closed in (U,$\tau_R$(X)) for each nano closed set V in (V,$\tau_{R'}$(Y))
(ii) Nano*generalized b-continuous if $f^{-1}(V)$is nano b-closed in (U,$_R$(X)) for each nano closed set V in (V,$\tau_{R'}$(Y).

**2.10 Definition:** A bijection f:(U,$\tau_R$(X)) → (V,$\tau_{R'}$(Y)) is called nano-homeomorphism if f is both nano-continuous and nano-open.

## 3. NANO*GENERALIZED b-CLOSED MAPS

**3.1 Definition:** A map f:(U,$\tau_R$(X)) →(V,$\tau_{R'}$(Y)) is said to be nano*generalized b-closed if the image of every nano closed set in (U, $\tau_R$(X)) is nano*generalized b-closed in (V, $\tau_{R'}$(Y)).

**3.2 Definition:** A map f:(U,$\tau_R$(X)) → (V,$\tau_{R'}$(Y)) is said to be nano*generalized b-open if f(A) is nano*generalized b-open for each nano open set A in (U,$\tau_R$(X)).

**3.3 Theorem**

(i) Every nano closed map is nano*generalized b-closed map.
(ii) Every nano c-closed map is nano*generalized b-closed
(iii) Every nano r-closed map is nano*generalized b-closed map.
(iv) Every nano pre-closed map is nano*generalized b-closed map.
(v) Every nano semi closed map is nano*generalized b-closed map.
(vi) Every nano α-closed map is nano*generalized b-closed map.

**Proof:** Let f: (U,$_R$(X))→ (V,$\tau_{R'}$(Y)) be a nano closed map. Let B be nano closed set in U, Since f is nano closed map then f(B) is nano closed set in V. We know that every nano closed set is nano* generalized b-closed, then f(B) is nano* generalized b-closed in V. Therefore f is nano* generalized b-closed map. Proof is obvious for others.

**3.4 Remark:** The Converse of the above theorem need not be true. It is shown by the following examples.

**3.5 Example:** Let U={a,b,c,d} with U/R={{b},{c},{a,d}}. Let X={a,c}⊆U. Then $\tau_R$(X)={U,Φ,{c}, {a,d},{a,c,d}}. Let V={a,b,c,d} with V/R'={V,Φ,{a},{c},{b,d}}. Let Y={a,b}⊆V. Then $\tau_{R'}$(Y)={{a}{b, d},{a, b, d}}. Define f:(U,$\tau_R$(X)) → (V,$\tau_{R'}$(Y)) be a function defined by f(a)=a, f(b)=c, f(c)=d, f(d)=b. Here f is nano*generalized b-closed map but not nano closed map. Since A={b,c} is closed in (U,$\tau_R$(X)) but f({b,c})={c,d} is nano*generalized b-closed set but not nano closed set in (V,$\tau_{R'}$(Y)).

**3.6 Example:** Let U={a,b,c} with U/R={{b},{a,c}}. Let X= {a,b}⊆U. Then $\tau_R$(X)={U,Φ,{b},{a,c}} Let V={a,b,c} with V/R'={{a},{b,c}}.Let Y={a,c}⊆V. Then $\tau_{R'}$(Y)={V,Φ,{a},{b,c}}. Define f:(U,$\tau_R$(X))→(V,$\tau_{R'}$(Y)) be a function defined by f(a)=b, f(b)=c, f(c)=a. Here f is nano*generalized b-closed map but not nano c-closed map. Since A={a,c} is closed in (U, $\tau_R$(X)) but f({a,c})={a,b} is nano*generalized b-closed set but not nano c-closed set in (V, $\tau_{R'}$(Y)).

**3.7 Example:** Let U={a,b,c} with U/R={{c},{a,b}}. Let X={a,c}⊆U. Then $\tau_R$(X)={U,Φ,{c},{a,b}} Let V={a,b,c} with V/R'={{a},{b,c}}.Let Y={a,c}⊆V. Then $\tau_{R'}$(Y)={V,Φ,{a},{b,c}}. Define f:(U,$\tau_R$(X))→(V,$\tau_{R'}$(Y)) be a function defined by f(a)=a, f(b)=b, f(c)=c. Here f is nano*generalized b-closed map but not nano r-closed map. Since A={c} is closed in (U,$_R$(X)) but f({c})={c} is nano*generalized b-closed set but not nano r-closed set in (V,$\tau_{R'}$(Y)).

**3.8 Example:** Let U={a,b,c,d} with U/R={{c},{d},{a,b}}. Let X={a,d}⊆U. Then $\tau_R$(X)={U,Φ,{d},{a,b},{a,b,d}}. Let V={a,b,c,d} with V/R'={{b},{c}{a,d}}. Let Y={b,d}⊆V. Then $\tau_{R'}$(Y)={V,Φ,{b},{a,d},{a,b,d}}. Define f:(U,$\tau_R$(X)) → (V, $\tau_{R'}$(Y)) be a function defined by f(a)=b, f(b)=c, f(c)=d, f(d)=a. Here f is nano*generalized b-closed map but not nano pre-closed map. Since A={c,d} is closed in (U,$\tau_R$(X)) but f({c,d})={a,d} is nano*generalized b-closed set but not nano pre-closed set in (V,$\tau_{R'}$(Y)).

**3.9 Example:** Let U={a,b,c,d} with U/R={{a},{c},{b,d}}. Let X={a,b}⊆U. Then $\tau_R$(X)={U,Φ,{a},{b,d},{a,b,d}}. Let V={a,b,c,d} with V/R'={{a},{b}{c,d}}. Let Y={a,c}⊆V. Then $\tau_{R'}$(Y)={V,Φ,{a},{c,d},{a,c,d}}. Define f:(U,$\tau_R$(X)) → (V,$\tau_{R'}$(Y)) be a function defined by f(a)=c, f(b)=a, f(c)=b,f(d)=d. Here f is nano*generalized b-closed map but not nano s-closed map. Since A={a,c} is closed in (U,$\tau_R$(X)) but f({a,c})={b,c} is nano*generalized b-closed set but not nano s-closed set in (V,$\tau_{R'}$(Y)).

**3.10 Example:** Let U={a,b,c} with U/R={{a},{b,c}}. Let X={a,b}⊆U. Then $\tau_R$(X)={U,Φ,{a},{b,c}} Let V={a,b,c} with V/R'={{b},{a,c}}.Let Y={b,c}⊆V. Then $\tau_{R'}$(Y)={V,Φ,{b},{a,c}}. Define f:(U,$\tau_R$(X))→(V,$\tau_{R'}$(Y)) be a function defined by f(a)=a, f(b)=b, f(c)=c. Here f is nano*generalized b-closed map but not nano α-closed map. Since A={a} is closed in (U,$\tau_R$(X)) but f({a})={a} is nano*generalized b-closed set but not nano α-closed set in (U,$\tau_R$(X)).

**3.11 Remark:** If f:(U,$\tau_R(X)$) → (V,$\tau_{R'}(Y)$) is nano*generalized b-closed map and g:(V,$\tau_{R'}(Y)$) →(W, $\tau_{R''}(Z)$) is nano*generalized b-closed map then gof:(U,$\tau_R(X)$)→ (W,$\tau_{R''}(Z)$) need not be nano*generalized b-closed map in general and this is shown by the following example.

**3.12 Example:** Let U={a,b,c,d} with U/R={{a},{b},{c,d}} and X={b,d}, then $\tau_R(X)$={U,Φ,{b},{c,d},{b,c,d}}. Let V={a,b,c,d} with V/R'={{a},{c},{b,d}} and Y={a,b},then $\tau_{R'}(Y)$={V,Φ,{a},{b,d},{a,b,d}} and W={a,b,c,d} with W/R''={{b},{c},{a,d}} and Z={a,c}, then $\tau_{R''}(Z)$= {W,Φ,{c},{a,d},{a,c,d}}. Define f:(U,$\tau_R(X)$)→(V, $\tau_{R'}(Y)$) be the function defined by f(a)=c, f(b)=b, f(c)=a, f(d)=d. and g:(V,$\tau_{R'}(Y)$)→ (W,$\tau_{R''}(Z)$) be the function defined by g(a)=a, g(b)=b, g(c)=d, g(d)=c. Here f and g is nano*generalized b-closed map, but its composition is not nano*generalized b-closed map, since gof({a,c,d})={a,c,d}is not nano*generalized b-closed map (W, $\tau_{R''}(Z)$).

**3.13 Theorem:** If f:(U,$\tau_R(X)$→(V,$\tau_{R'}(Y)$)is nano closed map and g:(V,$\tau_{R'}(Y)$)→(W,$\tau_{R''}(Z)$) is nano*generalized b-closed map then the composition gof: (U,$\tau_R(X)$→ (W, $\tau_{R''}(Z)$) is nano*generalized map.

**Proof:** Let B be nano closed set in (U, $\tau_R(X)$). Since f is a nano closed map, f(B) is nano closed set in (V,$\tau_{R'}(Y)$). Since g is nano*generalized b-closed map, g(f(B)) is nano*generalized b-closed in (W, $\tau_{R''}(Z)$). This implies gof is nano*generalized b-closed map.

**Almost Nano*Generalized b-Closed Map and Strongly Nano*Generalized b-Closed Map**

**3.14 Definition:** A map f:(U,$\tau_R(X)$) → (V, $\tau_{R'}(Y)$) is said to be almost nano*generalized b-closed map if for every nano regular closed set F of (U,$\tau_R(X)$), f(F) is nano*generalized b-closed in (V,$\tau_{R'}(Y)$).

**3.15 Definition:** A map f:(U,$_R$(X))→(V, $\tau_{R'}(Y)$) is said to be strongly nano*generalized b-closed map if for every nano*generalized b-closed set F of (U,$\tau_R(X)$), f(F) is nano*generalized b-closed set F of (V,$\tau_{R'}(Y)$).

**3.16 Definition:** A map f: (U,$_R$(X))→ (V, $\tau_{R'}(Y)$) is said to be strongly nano*generalized b-open map if for every nano*generalized b-open set F of (U,$\tau_R(X)$), f(F) is nano*generalized b-open set F of (V,$\tau_{R'}(Y)$).

**3.17 Theorem:** Every strongly nano*generalized b-closed map is nano*generalized b-closed map.

**Proof:** Let B be nano closed set in (U,$_R$(X)). Since every nano closed set is nano*generalized b-closed set, then B is nano*generalized b-closed in (U,$_R$(X)). Since f is strongly nano*generalized b-closed map, f(B) is nano*generalized b-closed set in(V, $\tau_{R'}(Y)$). Therefore f is nano*generalized b-closed map.

**3.18 Remark:** The converse of the above theorem need not be true. It is shown by the following example.

**3.19 Example:** Let U={a,b,c,d} with U/R={{b},{c},{a,d}} and X={a,c}, then $\tau_R(X)$={U,Φ,{c},{a,d},{a,c,d}}. Let V={a,b,c,d} with V/R'={{a},{c},{b,d}} and Y={a,b}, then $\tau_{R'}(Y)$={V,Φ,{a},{b,d},{a,b,d}}. Define f:(U,$\tau_R(X)$)→(V, $\tau_{R'}(Y)$) be the function defined by f(a)=a, f(b)=b,f(c)=c,f(d)=d. Here f is nano*generalized b-closed map, but not strongly nano*generalized b-closed map, since A={a,d}is not nano*generalized b-closed set in (U,$\tau_R(X)$), but f({a,d})={a,d}is not nano*generalized b-closed set in (V,$\tau_{R'}(Y)$).

**3.20 Theorem:** Every nano*generalized b-closed map is almost nano*generalized b-closed map.

**Proof:** It is obvious

**3.21 Remark:** The converse of the above theorem need not be true. It is shown by the following example.

**3.22 Example:** Let U={a,b,c,d} with U/R={{a,b},{c,d}} and X={a,b}, then $\tau_R(X)$={U,Φ,{a,b}. Let V={a,b,c,d} with V/R'={{a},{b},{c,d}} and Y={a,c}, then $\tau_{R'}(Y)$={V,Φ,{a},{c,d},{a,c,d}}. Define f: (U,$\tau_R(X)$)→(V, $\tau_{R'}(Y)$) be the function defined by f(a)=a, f(b)=c,f(c)=b,f(d)=d. Here f is almost nano*generalized b-closed map, but it is not nano*generalized b-closed map, since A={c,d}is nano closed set in (U,$\tau_R(X)$), but f({c,d})={b,d} is not nano*generalized b-closed set in (V,$\tau_{R'}(Y)$).

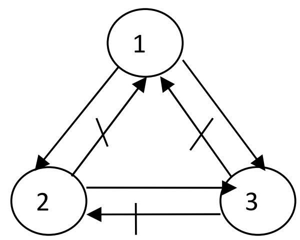
**3.23 Theorem:** Every strongly nano*generalized b-closed map is almost nano*generalized b-closed map.

**Proof:** Let B be nano regular closed set in $(U,_{R}(X))$. We know that every nano regular closed set is nano closed set and every nano closed set is nano*generalized b-closed set0 Therefore B is nano*generalized b-closed set in $(U,_{R}(X))$. Since f is strongly nano*generalized b-closed map, f(B) is nano*generalized b-closed set in $(V,\tau_{R'}(Y))$. Therefore f is almost nano*generalized b-closed map.

**3.24 Remark:** The converse of the above theorem need not be true. It is shown by the following example.

**3.25 Example:** Let U={a,b,c,d} with U/R={{b},{c},{a,d}} and X={a,c}, then $\tau_R(X)$={U,Φ,{c},{a,d},{a,c,d}}}. Let V={a,b,c,d} with V/R'={{a},{c},{b,d}}and Y={a,c},then $\tau_{R'}(Y)$={V,Φ,{a},{b,d},{a,b,d}}. Define f: $(U,_{R}(X)) \rightarrow (V,\tau_{R'}(Y))$ be the function defined by f(a)=a, f(b)=b, f(c)=d, f(d)=c. Here f is almost nano*generalized b-closed map, but it is not strongly nano*generalized b-closed map, since A={a,b}is nano closed set in $(U,\tau_R(X))$, but f({a,b})={a,b} is not nano*generalized b-closed set in $(V,\tau_{R'}(Y))$.

**3.26 Remark:** From the above theorem and examples, we have the following diagrammatic representation:




In the above diagram, the numbers 1 – 3 represent the following:
1. strongly nano*generalized b-closed map 2. nano*generalized b-closed map 3. almost nano*generalized b-closed map

**3.27 Theorem:** If $f:(U,\tau_R(X))\rightarrow(V, \tau_{R'}(Y))$ is strongly nano*generalized b-closed map and $g:(V,\tau_{R'}(Y))\rightarrow(W, \tau_{R''}(Z))$ is strongly nano*generalized b-closed map then its composition gof is strongly nano*generalized b-closed map.

**Proof:** Let B be nano*generalized b-closed set in $(U,_{R}(X))$. Since f is strongly nano*generalized b-closed, then f(B) is nano*generalized b-closed in $(V,\tau_{R'}(Y))$. Since g is strongly nano*generalized b-closed, then g(f(B)) is f is nano*generalized b-closed in $(W,\tau_{R''}(Z))$. Therefore gof: $(U,_{R}(X))\rightarrow (W,\tau_{R''}(Z))$ is strongly nano*generalized b-closed map.

**3.28 Theorem:** If $f:(U,\tau_R(X))\rightarrow(V, \tau_{R'}(Y))$ is almost nano*generalized b-closed map and $g:(V,\tau_{R'}(Y))\rightarrow(W, \tau_{R''}(Z))$ is strongly nano*generalized b-closed map then its composite gof: $(U,\tau_R(X))\rightarrow (W,\tau_{R''}(Z))$ is almost nano*generalized b-closed map.

**Proof:** It is obvious

**3.29 Theorem:** If $f:(U,\tau_R(X))\rightarrow(V,\tau_{R'}(Y))$ and $g:(V,\tau_{R'}(Y))\rightarrow(W,\tau_{R''}(Z))$ be two mappings such that their composition $gof:(U,\tau_R(X))\rightarrow(W,\tau_{R''}(Z))$ be a nano*generalized b-closed mapping then the following statements are true:

(i) If f is nano continuous and surjective then g is nano*generalized b-closed map.
(ii) If g is nano*generalized b-irresolutee and injective then f is nano*generalized b-closed map.

**Proof**

(i) Let B be a nano closed set in $(V,_{R'}(Y))$.Since f is nano continuous $f^{-1}(B)$ is nano*generalized b-closed set in $(U,\tau_R(X))$. Since gof is nano*generalized b-closed map, we have $(gof)(f^{-1}(B))$ is nano*generalized b-closed in $(W, \tau_{R''}(Z))$. Therefore g(B) is nano*generalized b-closed $(W,\tau_{R''}(Z))$,since f is surjective. Hence g is nano*generalized b-closed map.
(ii) Let B be nano closed set of $(U,\tau_R(X))$.Since gof is nano*generalized b-closed, we have gof(B) is nano*generalized b-closed in $(W,\tau_{R''}(Z))$.Since g is injective and nano*generalized b-irresolute $g^{-1}(B)(gof(B))$ is nano*generalized b-closed in $(V,\tau_{R'}(Y))$.Therefore f(B) is nano*generalized b-closed in $(V,\tau_{R'}(Y))$.Hence f is nano*generalized b-closed map.

**3.30 Proposition:** For any bijection $f:(U,\tau_R(X))\rightarrow(V,\tau_{R'}(Y))$ the following statements are equivalent.

(i) f is a nano*generalized b-open map
(ii) f is a nano*generalized b-closed map.
(iii) $f^{-1}:(V,\tau_{R'}(Y))\rightarrow(U,\tau_R(X))$ is nano*generalized b-continuous.

**Proof**
(i)⟹(ii) Let f be nano*generalized b -open map. Let B be nano closed in $(U,_R(X))$. Then $X - B$ is nano open in $(U,_R(X))$. By assumption, $f(X - B)$ is a nano*generalized b -open map and it implies $Y - f(B)$ is a nano*generalized b-open map and hence f(B) is a nano*generalized b- closed map.
(ii)=>(iii) Let B be nano closed in $(U,\tau_R(X))$. By (ii) $f(B)=(f^{-1})^{-1}(B)$ is nano*generalized b-closed in $(V,\tau_{R'}(Y)$.
(iii)=>(i) Let B be nano open in $(U,\tau_R(X))$. By (iii) $(f^{-1})^{-1}(B)=f(B)$ is nano*generalized b-open in $(V,\tau_{R'}(Y))$.

**3.31 Proposition:** For any bijection $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ the following statements are equivalent.
(i) $f^{-1}:(V,\tau_{R'}(Y))\to(U,\tau_R(X))$ is nano*generalized b -irresolute.
(ii) f is a strongly nano*generalized b-open map
(iii) f is a strongly nano*generalized b-closed map.

**Proof:** It is obvious

## 4. Nano*Generalized b-Homeomorphisms
**4.1 Definition:** A bijection $f:(U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is called nano*generalized b-homeomorphism if f is both nano*generalized b-continuous and nano*generalized b- open.

**4.2 Theorem**
(i) Every nano homeomorphism is nano*generalized b- homeomorphism
(ii) Every nano r-homeomorphism is nano*generalized b- homeomorphism
(iii) Every nano c-homeomorphism is nano*generalized b- homeomorphism
(iv) Every nano s-homeomorphism is nano*generalized b- homeomorphism
(v) Every nano pre-homeomorphism is nano*generalized b- homeomorphism
(vi) Every nano α-homeomorphism is nano*generalized b- homeomorphism

**Proof**: Let $f: (U,_R(X)) \to (V,\tau_{R'}(Y))$ be a nano homeomorphism. Then f is nano continuous and nano open. Since every nano continuous function is nano*generalized b-continuous and every nano open map is nano*generalized b-open, f is a nano*generalized b-continuous and nano*generalized b-open. Hence f is a nano*generalized b-homeomorphism. Proof is obvious for others

**4.3 Remark:** The converse of the above theorem need not be true. It is shown by the following examples.

**4.4 Example:** Let U={a,b,c,d} with U/R={{a},{c},{b,d}} and X={a,b}, then $\tau_R(X)$={U,Φ,{a},{b,d},{a,b,d}}}. Let V={a,b,c,d} with V/R'={{a,b},{c,d}} and Y={a,b}, then $\tau_{R'}(Y)$={V,Φ,{a,b}}. Define f: $(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ be the function defined by f(a)=b, f(b)=d, f(c)=a, f(d)=c. Here f is nano*generalized b-homeomorphism, but it is not nano homeomorphism Since $f^{-1}${a,b}={a,c} is nano*generalized b-open but nano open.

**4.5 Example:** Let U={a,b,c} with U/R={{a},{b,c}}and X={a,c}, then $\tau_R(X)$={U,Φ,{a},{b,c}} and
Let V={a,b,c} with V/R'={{b},{a,c}} and Y={a,b}, then $\tau_{R'}(Y)$={V,Φ,{b},{a,c}} Define $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ be the function defined by f(a)=c, f(b)=b, f(c)=a. Here f is nano*generalized b-homeomorphism, but it is not nano r-homeomorphism, since $f^{-1}${b}={b} is nano*generalized b-open but nano r- open.

**4.6 Example:** Let U={a,b,c} with U/R={{a},{b,c}}and X={a,c}, then $\tau_R(X)$={U,Φ,{a},{b,c}} and
Let V={a,b,c} with V/R'={{c},{a,b}} and Y={b,c}, then $\tau_{R'}(Y)$={V,Φ,{c},{a,b}} Define $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ be the function defined by f(a)=a, f(b)=b, f(c)=c. Here f is nano*generalized b-homeomorphism, but it is not nano c-homeomorphism, since $f^{-1}${a,b}={a,b} is nano*generalized b-open but nano c-open.

**4.7 Example:** Let U={a,b,c,d} with U/R={{a},{b},{c,d}}and X={a,c}, then $\tau_R(X)$={U,Φ,{a},{c,d},{a,c,d}} and Let V={a,b,c,d} with V/R'={{b},{c},{a,d}} and Y={a,b}, then $\tau_{R'}(Y)$={V,Φ,{b},{a,d},{a,b,d}}. Define f: $(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ be the function defined by f(a)=a, f(b)=c, f(c)=b,f(d)=d. Here f is nano*generalized b-homeomorphism, but it is not nano s- homeomorphism, since $f^{-1}${b}={c} is nano*generalized b-open but nano s- open.

**4.8 Example:** Let U={a,b,c,d} with U/R={{b},{c},{a,d}}and X={b,d}, then $\tau_R(X)$={U,Φ,{b},{a,d},{a,b,d}} and Let V={a,b,c,d} with V/R'={{a},{c},{b,d}} and Y={a,d}, then $\tau_{R'}(Y)$={V,Φ,{a},{b,d},{a,b,d}}. Define $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ be the function defined by f(a)=a, f(b)=b, f(c)=d,f(d)=c. Here f is nano*generalized b-homeomorphism, but it is not nano pre- homeomorphism, since $f^{-1}${b,d}={b,c} is nano*generalized b-open but nano pre- open.

**4.9 Example:** Let U={a,b,c,} with U/R={{b},{a,c}}and X={b,c}, then $\tau_R$(X)={U,Φ,{b},{a,c}} and Let V={a,b,c}with V/R'={{a},{b,c}} and Y={a}, then $\tau_{R'}$(Y)={V,Φ,{a}}.
Define f:(U,$\tau_R$(X))→(V,$\tau_{R'}$(Y)) be the function defined by f(a)=b, f(b)=c, f(c)=a. Here f is nano*generalized b-homeomorphism, but it is not nano α-homeomorphism, since $f^{-1}${a}={c} is nano*generalized b-open but nano α- open.

**4.10 Proposition:** For any bijection f:(U,$\tau_R$(X))→(V,$\tau_{R'}$(Y)) the following statements are equivalent.
(i) f is a nano*generalized b-open map
(ii) f is a nano*generalized b-homeomorphism
(iii) .f is a nano*generalized b-closed map.

**Proof**
(i)⟹(ii) Given f:(U,$\tau_R$(X))→(V,$\tau_{R'}$(Y)) be a bijective, nano*generalized b-continuous and nano*generalized b-open. Then by definition, f is a nano*generalized b-homeomorphism.
(ii)⟹(iii) Given f is nano*generalized b-open and bijective by proposition 3.32, f is a nano*generalized b-closed map.
(iii)⟹(i) Given f is nano*generalized b-closed and bijective by propoition 3.32, f is a nano*generalized b-closed map.

**4.11 Remark:** Composition of two nano*generalized b-homeomorphism need not be nano*generalized b- homeomorphism.

**4.12 Example:** Let U={a,b,c,d}with U/R={{a},{b},{c,d}} and X={b,d}, then $\tau_R$(X)={U,Φ,{b},{c,d},{b,c,d}}. Let V={a,b,c,d} with V/R'={{a},{c},{b,d}} and Y={a,b}, then $\tau_{R'}$(Y)={V,Φ,{a},{b,d},{a,b,d}} and W={a,b,c,d with W/R''={{b},{c},{a,d}}and Z={a,c},then$\tau_{R''}$(Z)={W,Φ,{c},{a,d},{a,c,d}}.
Define f:(U,$\tau_R$(X))→(V,$\tau_{R'}$(Y)) be the function defined by f(a)=d, f(b)=b, f(c)=c, f(d)=a. and g:(V,$\tau_{R'}$(Y))→(W, $\tau_{R''}$(Z)) be the function defined by g(a)=a, g(b)=b,g(c)=d,g(d)=c. Here f and g is nano*generalized b-homeomorphism, but its composition is not nano*generalized b-homeomorphism, since gof({c})={b}is not nano*generalized b-open map (W,$\tau_{R''}$(Z)).

**4.13 Theorem:** Let f:(U,$\tau_R$(X))→(V,$\tau_{R'}$(Y)) be one to one onto mapping. Then f is nano*generalized b-homeomorphism if and only if f is nano*generalized b-closed and nano*generalized b-continuous.

**Proof:** Let f be an nano*generalized b-homeomorphism. Then f is nano*generalized b-continuous. Let B be an arbitrary nano closed set in (U, $\tau_R$(X)). Then $U - B$ is nano open. Since f is nano*generalized b-open, $f(U - B)$ is nano*generalized b-open in (V,$_{R'}$(Y)). That is $V - f(B)$ is nano*generalized b-open in (V,$_{R'}$(Y)). Thus the image of every nano closed set in (U, $\tau_R$(X)) is nano*generalized b-closed in (V,$_{R'}$(Y)).
Conversely, let f be nano*generalized b-closed and nano*generalized b-continuous. Let B be a nano open set in (U, $\tau_R$(X)). Then $U - B$ is nano closed in (U,$_R$(X)). Since f is nano*generalized b-closed, $f(U - B) = V - f(B)$ is nano*generalized b-closed in (V,$_{R'}$(Y)). Therefore f(B) is nano*generalized b-open in (V,$\tau_{R'}$(Y)).Thus f is nano*generalized b-open and nano*generalized b-homeomorphism.

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