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On contra Čech Mp-continuous functions in Čech closure spaces

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Abstract
The purpose of this paper is to introduce the concept of contra Čech MP-continuous mappings in closure spaces. The properties and relationship with other types of mappings in closure spaces are obtained with examples.

Keywords: Contra Čech MP-continuous, contra Čech MP-irresolute functions, almost contra Čech MP-continuous, Čech MP-closed maps

1. Introduction
Every topological space is a closure space, we define the closure operator of the space as a function that takes any subset to its closure. Norman Levine introduced the concept of generalized closed sets as a generalization of closed sets, to investigate some topological properties. The concept of closure space is the generalization of a topological space. E. Čech [4] introduced the concept of Čech-closure spaces. Closure functions that are more general than the topological ones have been studied already by many authors [5, 6].

Let (X, k) or simply X denote a Čech-closure space. For any subset A⊆X,int(A) and k(A) denote the Čech interior and Čech closure of a set A with respect to the function k. The ideas about the concept of a continuous mapping and of a set endowed with continuous operation (Composition) play a fundamental role in general mathematical analysis. Dontchev [7] introduced the notions of contra continuity in topological spaces. In this paper, the present work has as its purpose to investigate some fundamental properties of contra Čech MP-continuous, contra Čech MP-irresolute maps. Furthermore we extend and study their characterizations.

2. Preliminaries
A map k: P(X)⟶P(X) defined on the power set P(X) of a set X is called a closure operator on X and the pair (X, k) is called a closure space if the following axioms are satisfied.
$k(\phi)=\phi$
A⊆k(A) for every A⊆X
k(A∪B)=k(A)∪k(B) for all A,B⊆X
A closure operator k on a set X is called idempotent if k(A)=k[k(A)] for all A⊆X.

2.1 Definition: A subset A of a Čech-closure (X, k) will be called Čech closed if k(A)=A and Čech-open if its complement is closed. i.e., k(X-A)=X-A.

2.2 Definition: A subset A of a Čech closure space (X, k) is said to be
1. Čech regular open if A=int(k(A)) and Čech regular closed if A=k(int(A)).
2. Čech pre open if A⊆int(k(A)) and Čech pre closed if k(int(A))⊆A. Čech semi open if A⊆k(int(A))
3. Čech α-open if A⊆int(k(int(A))) and Čech α-closed if k(int(k(A)))⊆A.
4. Čech β-open if A⊆k(int(k(A))) and Čech β-closed if int(k(int(A)))⊆A.

2.3 Definition: A Čech closure space (Y, I) is said to be a subspace of (X, k) if Y⊆X and k(A)=k(A)⋂Y for each subset A⊆Y. if Y is closed in (X, k) then the subspace (Y,I) of (X, k) is said to be closed too.

2.4 Definition: Let (X, k) and (Y, l) be Čech closure space. A map f: (X, k) ⟶(Y, l) is said to be continuous, if f(kA)⊆k f(A) for every subset A⊆F.

2.5 Definition: Let (X, k) and (Y, l) be Čech closure spaces. A map f: (X, k) ⟶(Y, l) is said to be closed (res. open) if f(F) is a closed (res. open) subset of (Y,I) whenever F is a closed (res. open) subset of (X, k).

2.6 Definition: Let (X, u) and (Y, v) be closure spaces. A map f: (X, u) ⟶(Y, v) is called Čech M-continuous (resp. Čech N-continuous, Čech T-continuous, Čech D-continuous) [6] if the inverse image of every open set in (Y, v) is M-open in (X, u)(resp. N-open, T-open, D-open).

2.7 Definition: Let (X, u) and (Y, v) be closure spaces. A map f: (X, u) ⟶(Y, v) is called Čech MP-continuous if the inverse image of every Čech open set in (Y, v) is Čech MP-open in (X, u).

2.8 Definition: Let (X, u) and (Y, v) be closure space and a map f: (X, u) ⟶(Y, v) is called Čech MP-Irresolute, if f^{-1}(G) is Čech MP-open (closed) in (X,u) for every Čech MP-open set (closed) G in (Y,v).

2.9 Definition: Let (X, u) and (Y, v) be closure spaces and a map f: (X, u) ⟶(Y, v) is called open map (closed map) if f (B) is open in (Y, v) for every open set (closed set) B in (X, u).

2.10 Definition: A closure space (X, u) is said to be T_m-space if every Čech MP-open set in (X, u) is open.

3. Contra Čech MP-continuous and contra Čech MP-irresolute functions

3.1 Definition: Let (X, u) and (Y, v) be closure spaces. A map f: (X, u) ⟶(Y, v) is called contra Čech MP-continuous if the inverse image of every Čech open set in (Y, v) is Čech MP-closed in (X, u).

3.2 Proposition: Let f: (X, u) ⟶(Y, v) be a map where (X, u) and (Y, v) are closure spaces. Then f is contra Čech MP-continuous if and only if the inverse image of every closed subset of (Y, v) is Čech MP-open in (X, u).

Proof: Consider F to be a closed subset in (Y, v). Then Y-F is open in (Y, v). Since f is contra Čech MP-continuous, f^{-1}(Y-F) is Čech MP-closed. But f^{-1}(Y-F) = X-f^{-1}(F). Thus f^{-1}(F) is Čech MP-open in (X, u). Conversely let G be an open subset in (Y, v). Then Y-G is closed in (Y, v). Since the inverse image of each closed subset in (Y, v) is Čech MP-open in (X, u), f^{-1}(Y-G) is Čech MP-open in (X, u). But f^{-1}(Y-G) = X-f^{-1}(G). Thus f^{-1}(G) is Čech MP-closed. Therefore f is contra Čech MP-continuous.

3.3 Proposition: Let (X, u), (Y, v) and (Z, w) be closure spaces. If f: (X, u) ⟶(Y, v) and g: (Y, v)⟶(Z, w) be maps. If gof is contra Čech MP-continuous and g is a closed injection. Then f is contra Čech MP-continuous.

Proof: Let H be a closed subset of (Y, v). since g is closed, g(H) is closed in (Z, w) as gof is contra Čech MP-continuous, $(gof)^{-1}$(g(H))= f^{-1}(g^{-1}((g(H))) is Čech MP-open in (X, u). But g is injective, hence f^{-1}(g^{-1}((g(H))) =f^{-1}(H), f^{-1}(H) is Čech MP-open. Therefore, f is contra Čech MP-continuous.

3.4 Proposition: Let (X, u) and (Z, w) be closure space and (Y, v) be a T_m -space. If f: (X, u)⟶(y, v) and g: (Y, v)⟶(Z, w) are contra Čech MP-continuous maps. Then gof is contra Čech MP-continuous.

Proof: Let H be closed in (Z, w). Since g is contra Čech MP-continuous, g^{-1}(H) is Čech MP-open in (Y, v). But (Y, v) is a T_m-space, Hence g^{-1}(H) is open in (Y,v). As f is contra Čech MP-continuous, f^{-1}(g^{-1}(H))=$(gof)^{-1}$(H) is Čech MP-closed in (X, u). Therefore, gof is Čech MP-continuous.

3.5 Proposition: Assume (X, u), (Y, v) and (Z, w) be closure spaces, where f: (X, u) ⟶(Y, v), g: (Y, v)⟶(Z, w) be two maps. If f is contra Čech MP-continuous and g is continuous. Then gof is contra Čech MP-continuous.

Proof: Consider gof: (X, u) ⟶ (Z, w). Let H be closed in Z. Since g is continuous, g^{-1}(H) is closed in Y. As f is contra Čech MP-continuous, f^{-1}(g^{-1}(H)) is Čech MP-open in X. Hence $(gof)^{-1}$(H) is Čech MP-open in X. Therefore, (gof) is contra Čech MP-continuous.

3.6 Definition: A function f: (X, u) ⟶(Y, v) is said to be contra Čech MP-irresolute if f^{-1}(V) is Čech MP-open in (X, u) for every Čech MP-closed set V in (Y, v).

3.7 Definition: A function f: (X, u) ⟶(Y, v) is said to be totally continuous if f^{-1}(V) is clopen in (X, u) for every open set V in (Y, v).

3.8 Theorem

1. Every contra Čech continuous function is contra Čech MP-continuous.
2. Every contra Čech w-continuous function is contra Čech MP-continuous.
3. Every contra Čech g-continuous function is contra Čech MP-continuous.
4. Every contra Čech αψ-continuous function is contra Čech MP-continuous.
5. Every contra J-Čech continuous function is contra Čech MP-continuous.
6. Every contra Čech πgβ-continuous function is contra Čech MP-continuous.
7. Every contra Čech M-continuous function is contra Čech MP-continuous.
8. Every contra Čech N-continuous function is contra Čech MP-continuous.
9. Every contra Čech T-continuous function is contra Čech MP-continuous.
10. Every contra Čech D-continuous function is contra Čech MP-continuous.

Proof: (1) Let f: (X, u) ⟶(Y, v) be a contra Čech continuous map. Let V be a Čech open set in (Y, v). Since f is contra Čech continuous map, f^{-1}(V) is Čech closed set of (X, u). Every Čech closed set is Čech MP-closed set. That implies f^{-1}(V) is Čech MP-closed set of (X, u), for every Čech open set V in (Y, v). (i.e.,) f is contra Čech MP-continuous. Therefore every contra Čech continuous map is contra Čech MP-continuous.
Proof of the following statements are similar.

3.9 Remark: The converse of the above need not be true may be seen by the following example.

3.10 Example: Let X= {a, b, c}, Y={1, 2, 3}. Define a closure operator u on X by u{ϕ}=ϕ, u{a}={a}, u{b}={b, c}, u{c}=u{a, c}={a, c}, u{a, b}=u{b, c}=uX=X. Define a closure operator v on Y by v{ϕ}=ϕ, v{1}=v{2}=v{1, 2}={1, 2}, v{3}=v{1, 3}={1, 3}, v{2, 3}=vY=Y. Let f: (X, u) ⟶(Y, v) be defined by f(a)=2, f(b)=3, f(c)=1. Here f is contra Čech MP-continuous but not contra Čech continuous. Since for the closed set {1,2} in Y the inverse image f^{-1}{1,2}={a,c} is not Čech open in X.

3.11 Example: Let X= {a, b, c}, Y= {1, 2, 3}. Define a closure operator u on X by u{ϕ}=ϕ, u{a}={a}, u{b}={b, c}, u{c}=u{a, c}={a, c}, u{a, b}=u{b, c}=uX=X. Define a closure operator v on Y by v{ϕ}=ϕ, v{1}=v{2}=v{1, 2}={1, 2}, v{3}=v{1, 3}={1, 3}, v{2, 3}=vY=Y. Let f: (X, u) ⟶(Y, v) be defined by f(a)=2, f(b)=3, f(c)=1. Here f is contra Čech MP-continuous but not contra Čech w-continuous. Since for the closed set {1, 2} in Y the inverse image f^{-1}{1,2}={a,c} is not Čech w-open in X.

3.12 Example: Let X= {a, b, c}, Y= {1, 2, 3}. Define a closure operator u on X by u{ϕ}=ϕ, u{a}=u{a, b}={a, b}, u{b}={b}, u{c}={c}, u{a, c}=u{b, c}=uX=X. Define a closure operator v on Y by v{ϕ}=ϕ, v{1}=v{1, 2}={1, 2}, v{2}={2}, v{3}={3}, v{2, 3}=v{1, 3}=vY=Y. Let f: (X, u) ⟶(Y, v) be defined by f(a)=3, f(b)=2, f(c)=1. Here f is contra Čech MP-continuous but not contra Čech g-continuous. Since for the closed set {2} in Y the inverse image f^{-1}{2}={b} is not Čech g-open in X.

3.13 Example: Let X= {a, b, c}, Y={1, 2, 3}. Define a closure operator u on X by u{ϕ}=ϕ, u{a}=u{a, b}={a, b}, u{b}={b}, u{c}={c}, u{a, c}=u{b, c}=u X=X. Define a closure operator v on Y by v{ϕ}=ϕ, v{1}={1, 2}, v{2}={2, 3}, v{3}={3}, v{1, 3}={1, 3}, v{1, 2}={1, 2}, v{2, 3}=v Y=Y. Let f: (X, u) ⟶(Y, v) be defined by f(a)=2, f(b)=1, f(c)=3. Here f is contra Čech MP-continuous but not contra Čech αψ-continuous. Since for the closed set {1, 3} in Y the inverse image of f^{-1}{1,3}={b,c} is not Čech αψ-open in X.

3.14 Example: Let X= {a, b, c}, Y={1, 2, 3}. Define a closure operator u on X by u{ϕ}=ϕ, u{a}={a}, u{b}={b, c}, u{c}=u{a, c}={a, c}, u{a, b}=u{b, c}=u X=X. Define a closure operator v on Y by v{ϕ}=ϕ, v{1}={1}, v{2}={2}, v{3}=v{1, 3}={1, 3}, v{1, 2}=v{2, 3}=v Y=Y. Let f: (X, u) ⟶(Y, v) be defined by f(a)=1, f(b)=3, f(c)=2. Here f is contra Čech MP-continuous but not contra J-Čech continuous. Since for the closed set {1} in Y, the inverse image f^{-1}{1} ={a} is not in J-Čech open in X.

3.15 Example: Let X ={a, b, c}, Y={1, 2, 3}. Define a closure operator u on X by u{ϕ}=ϕ, u{a}={a, b}, u{b}=u{b, c}={b, c}, u{c}={c}, u{a, b}={a, b}, u{a, c}=u X=X. Define a closure operator v on Y by v{ϕ}=ϕ, v{1}={1}, v{2}={2}, v{3}={3}, v{1, 2}={1, 2}, v{2, 3}= v{1, 3}=v Y=Y. Let f: (X, u) ⟶(Y, v) be defined by f(a)=1, f(b)=3, f(c)=2. Here f is contra Čech MP-continuous but not contra Čech πgβ-continuous. Since for the closed set {1, 2} in Y the inverse image f^{-1}{1, 2} ={a, c} is not in Čech πgβ-open in X.

3.15 Example: Let X ={a, b, c}, Y={1, 2, 3}. Define a closure operator u on X by u{ϕ}=ϕ, u{a}={a}, u{b}={b}, u{c}={c}, u{a, c}={a, c}, u{a, b}=u{b, c}=u X=X. Define a closure operator v on Y by v{ϕ}=ϕ, v{1}=v{1, 3}={1, 3}, v{2}=v{2, 3}={2, 3}, v{3}={3},v{1, 2}= vY=Y. Let f: (X, u) ⟶(Y, v) be defined by f(a)=3, f(b)=2, f(c)=1. Here f is contra Čech MP-continuous but not contra Čech M-continuous. Since for the closed set {3} in Y the inverse image f^{-1}{3} ={a} is not in Čech M-open in X.

3.16 Example: Let X = {a, b, c}, Y={1, 2, 3}. Define a closure operator u on X by u{ϕ}={ϕ}, u{a}={a, b}, u{b}=u{b, c}={b, c}, u{c}={c}, u{a, b}={a, b}, u{a, c}=u X=X. Define a closure operator v on Y by v{ϕ}={ϕ}, v{2}={2}, v{3}=v{1, 3}={1, 3}, v{1}=v{1, 2}=v{2, 3}=v Y=Y. Let f: (X, u) ⟶(Y, v) be defined by f(a)=2, f(b)=1, f(c)=3. Here f is contra Čech MP-continuous but not contra Čech N-continuous. Since for the closed set {1, 3} in Y the inverse image f^{-1}{1, 3} ={b, c} is not in Čech N-open in X.

3.17 Example: Let X ={a, b, c}, Y={1, 2, 3}. Define a closure operator u on X by u{ϕ}=ϕ, u{a}={a}, u{b}={b, c}, u{c}=u{a, c}={a, c}, u{a, b}=u{b, c}=u X=X. Define a closure operator v on Y by v{ϕ}=ϕ, v{1, 2}={1, 2}, v{2, 3}={2, 3}, v{1, 3}={1, 3}, v{1}=v{2}=v{3}=v Y=Y. Let f: (X, u) ⟶(Y, v) be defined by f(a)=1, f(b)=3, f(c)=2. Here f is contra Čech MP-continuous but not contra Čech T-continuous. Since for the closed set {1, 2} in Y the inverse image f^{-1}{1, 2} = {a, c} is not in Čech T-open in X.

3.18 Example: Let X = {a, b, c}, Y= {1, 2, 3}. Define a closure operator u on x by u{ϕ}=ϕ, u{a}={a}, u{b}={b, c}, u{c}=u{a, c}={a, c}, u{a, b}=u{b, c}=u X=X. Define a closure operator v on Y by v{ϕ}=ϕ, v{1, 2}={1, 2}, v{2, 3}={2, 3}, v{1, 3}={1, 3}, v{1}={1}, v{2}=v{3}= vY=Y. Let f: (X, u) ⟶(Y, v) be defined by f(a)=1, f(b)=2, f(c)=3. Here f is contra Čech MP-continuous but not contra Čech D-continuous. Since for the closed set {1} in Y the inverse image f^{-1}{1} = {a} is not in Čech D-open in X.

3.19 Proposition: Let f: (X, u) ⟶(Y, v) be a surjection function. Suppose Čech MP(X) is closed under arbitrary union. Then the following statements are equivalent.

1. f is contra Čech MP-continuous.
2. For every Čech closed subset F of Y, f^{-1}(F) is Čech MP-open in X
3. For each x∈X and each Čech closed set F in Y containing f(X), there exists Čech MP-open set U in X containing x such that f(U)⊆F

Proof: (1) ⇒ (2) Let F be Čech closed in Y. Then Y-F is Čech open in Y. by (1), f^{-1}(Y-F)=X-f^{-1}(F) is Čech MP-closed in X which implies f^{-1}(F) is Čech MP-open.
(2) ⇒ (1) Suppose V is Čech open in Y. Then Y-V is Čech closed in Y by (2), f^{-1}(Y-V)=X-f^{-1}(V) is Čech MP-open in X which implies f^{-1}(V) is Čech MP-closed in X thus f is contra Čech MP-continuous.
(2) ⇒ (3) Let F be Čech closed set in Y containing f(X). by(2), f^{-1}(F) is Čech MP-open for x∈f^{-1}(F) take U=f^{-1}(F) then f(U)⊆F
(3) ⇒ (2) Let F be any Čech closed set in Y and x∈f^{-1}(F) by(3), there exists Čech MP-open set U_x⊆f^{-1}(F), hence f^{-1}(F)=$\bigcup_{x\in X} U_x$. thus f^{-1}(F) is Čech MP-open.

3.20 Proposition

1. If f: (X, u)⟶(Y, v) be a map, then the following are equivalent
2. F is contra Čech MP-irresolute
3. For each x∈X and any Čech MP-open set V of (Y, v) containing f(x) there exists a Čech MP-closed set U such that x∈U and f(U)⊆V
4. The inverse image of every Čech MP-closed set in (Y, v) is Čech MP-open in (X, u).

The proof is obvious.

4. Almost Contra ČEch Mp-Continuous Maps

4.1 Definition: A function f: (X, u) ⟶ (Y, v) is said to be almost contra Čech MP-continuous if f^{-1}(V) is Čech MP-closed set in (X, u) for every Čech regular open set V of (Y, v).

4.2 Definition: A function f: (X, u) ⟶ (Y, v) is said to be perfectly Čech continuous if f^{-1}(V) is Čech clopen set in (X, u) for each Čech open set V of (Y, v).

4.3 Theorem: Let (X, u) and (Y, v) be closure spaces. Then the following statements are equivalent for a function f: (X, u) ⟶ (Y, v).

1. f is almost contra Čech MP-continuous.
2. f^{-1}(F) is a Čech MP-open set of (X, u), for every F in Čech regular closed set of (Y, v).
3. For each x∈ X and each Čech regular closed set F of (Y, f(x)), there exist Čech MP-open set of (X, x) such that f(U)⊂F.
4. For each x∈ X and each Čech regular open set V in Y not containing f(x) there exist a Čech MP-closed set A in X not containing x such that f^{-1}(U)⊂A.
5. f^{-1}(int(cl(G))) is a Čech MP-closed set of (X, u) for every open subset G of Y.
6. f^{-1}(cl(int(F))) is a Čech MP-open set of X, for every Čech closed subset F of Y.

Proof: (1) ⇒ (2) Let F be Čech regular closed set of (Y, v). that implies Y-F is Čech regular open in (Y, v). Since f is almost contra Čech MP-continuous, f^{-1}(Y-F) = X-f^{-1}(F) is Čech MP-closed in (X, u). that implies f^{-1}(F) is Čech MP-open set in (X, u).
(2) ⇒ (1) Let V be Čech regular open set in (Y, v), then Y-V is Čech regular closed set in (Y, v). By (2) f^{-1}(Y-V) = X-f^{-1}(V) is Čech MP-open set in (X, u). Thus f^{-1}(V) is Čech MP-closed set in (X, u).
(2) ⇒ (3) Let F be any Čech regular closed set in Y containing f(x). By (2) f^{-1}(F) is Čech MP-open in (X, u) and x∈f^{-1}(F). Take U=f^{-1}(F) then U is Čech MP-open set in X containing x such that f(U)⊆F.
(3) ⇒ (2) Let F be Čech regular closed set $U_x \subset f^{-1}(F)$, we have $f^{-1}(F)=\cup\{ U_x / x\in f^{-1}(F)\}$ is Čech MP-open set in X. Therefore f^{-1}(F) is Čech MP-open.
(3) ⇒ (4) Let V be any Čech regular open in Y not containing f(x). Then Y-V is a Čech regular closed set containing f(x). By (3) there exist a Čech MP-open set U in X containing x such that f(U)⊂Y-V. Hence U⊂f^{-1}(Y-V)=X-f^{-1}(V) and f^{-1}(V)⊂X-U. Take A=X-U. Therefore we get Čech MP-closed set A in X not containing x.
(4) ⇒ (3) Let F be a Čech regular closed set Y containing f(x). Then Y-F is a Čech regular set in Y not containing f(x). By (4) there exist a Čech MP-closed set A in X not containing x such that f^{-1}(Y-F)⊂A. that implies X-f^{-1}(F)⊂A. Therefore f(X-A)⊂F. Take U=X-A. Then U is Čech MP-open set in X containing x such that f(U)⊂F.
(1) ⇒ (5) Let G be an Čech open subset of Y. Since int(cl(G)) is Čech regular open, then by (1) f^{-1}(int(cl(G))) is Čech MP-closed set of X.
(5) ⇒ (1) Let V be a Čech regular open set in Y, then V is Čech open in Y. Therefore by (5) f^{-1}(int(cl(V))) is Čech MP-closed set in (X, u). that implies f^{-1}(V) is Čech MP-closed set in X. Therefore f is almost contra Čech MP-continuous.
(2) ⇒ (6) Let F be Čech closed subset of Y. Since cl(int(F)) is Čech regular closed, then by (2) f^{-1}(cl(int(F))) is Čech MP-open set in (X, u).
(6) ⇒ (2) Let F be Čech regular closed set of (Y, v). Then F is Čech closed set in Y. By (6) f^{-1}(cl(int(F))) is Čech MP-open set in X.

4.4 Theorem: If f: (X, u)⟶(Y, v) is contra Čech MP-continuous, then it is almost contra Čech MP-continuous.

Proof: Let V be Čech regular open in Y. Then V is Čech open in X. By assumption f^{-1}(V) is Čech MP-closed in (X, u). Thus f is almost contra Čech MP-continuous.

4.5 Proposition: If f: (x, u)⟶(Y, v) be Čech regular set connected then it is almost contra Čech MP-continuous.

Proof: If V is a Čech regular open set in (Y, v). Then f^{-1}(V) is Čech clopen in (X, u), since f is Čech regular set connected. That implies f^{-1}(V) is Čech closed set in (X, u). But every Čech closed set is Čech MP-closed set. That implies f^{-1}(V) is Čech MP-closed in (X, u). That implies f is almost contra Čech MP-continuous.

4.6 Proposition: If f: (X, u)⟶(Y, v) is contra Čech continuous then it is almost contra Čech MP-continuous.

Proof: Let V be Čech regular open in (Y, v). Then V is Čech open in (Y, v). Since f is contra Čech continuous, f^{-1}(V) is Čech closed set in (X, u). Every Čech closed set is Čech MP-closed in (X, u) that implies f is almost contra Čech MP-continuous.

4.7 Definition: A function f: (X, u)⟶(Y, v) is called perfectly Čech continuous if f^{-1}(V) is Čech clopen in X, for each Čech open set V in Y.

4.8 Definition: A function f: (X, u)⟶(Y, v) is called Perfectly Čech MP-continuous if f^{-1}(V) is Čech MP-clopen in X, for each Čech open set V in Y.

4.9 Proposition: Every Perfectly Čech continuous function is contra Čech MP-continuous.

Proof: Suppose f is a Perfectly Čech continuous function and V is a Čech open set in Y. Then f^{-1}(V) is Čech clopen in X. (i.e.,) f^{-1}(V) is both Čech closed and Čech open. But every Čech closed set is Čech MP-closed set. That implies f^{-1}(V) is Čech MP-closed set. Hence f is contra Čech MP-continuous.

4.10 Proposition: Every Perfectly Čech MP-continuous function is contra Čech MP-continuous.

Proof: If V is a Čech open set in Y. Then f^{-1}(V) is Čech MP-clopen in X, Since f is a Perfectly Čech MP-continuous. (i.e.,) f^{-1}(V) is both Čech MP-closed and Čech MP-open. That implies f^{-1}(V) is Čech MP-closed set. Hence f is contra Čech MP-continuous.

5. Čech MP-closed maps

5.1 Definition: A map f: (X, u) ⟶(Y, v) is called Čech MP-closed map if for each Čech closed set of A of (X, u), f(A) is a Čech MP-closed set of (Y, v).

5.2 Definition: A map f: (X, u) ⟶(Y, v) is called Čech T-closed(res. Čech M-closed, Čech N-closed, Čech D-closed) map if for each Čech closed set of A of (X,u),f(A) is a Čech T-closed set(res. Čech M-closed, Čech N-closed, Čech D-closed) of (Y, v).

5.3 Proposition: Let (X, u) be a closure space and A⊆X. Then the following properties hold

1. $k_\beta(A)$ is the smallest Čech MP-closed set containing A and
2. A is Čech MP-closed if and only if $k_\beta(A)=A$

Proof

1. It follows from the definition of k_β.
2. If A is Čech MP-closed, then A itself is the smallest Čech MP-closed set containing $k_\beta(A)$. (i.e.,) $k_\beta(A)\subseteq A$.

We know that $A\subseteq k_\beta(A)$ and hence $k_\beta(A)=A$. Conversely, Let $k_\beta(A)=A$, then by (i) $k_\beta(A)$ is Čech MP-closed set. Therefore A is Čech MP-closed set.

5.4 Proposition: For any two subsets A and B of (X, u)

1. If A⊆B then $k_\beta(A)\subseteq k_\beta(B)$ and
2. $k_\beta(A\cap B) \subseteq k_\beta(A)\cap k_\beta(B)$.

Proof

1. We know that $B \subseteq k_\beta(B)$, Since A⊆B, we have $A\subseteq k_\beta(B)$. Thus $k_\beta(B)$ is a Čech MP-closed set containing A. Since $k_\beta(A)$ is the smallest closed set containing A, we have $k_\beta(A) \subseteq k_\beta(B)$.
2. $A\cap B\subseteq A\Rightarrow k_\beta(A\cap B)\subseteq k_\beta(A)$ and $A\cap B\subseteq B\Rightarrow k_\beta(A\cap B)\subseteq k_\beta(B)$ Hence $k_\beta(A\cap B)\subseteq k_\beta(A)\cap k_\beta(B)$.

5.5 Proposition: A mapping f: (X, u) ⟶(Y, v) is Čech MP-closed if and only if $k_\beta(f(A))\subseteq f(k(A))$ for every subset A of (X, u).

Proof: Suppose that f is Čech MP-closed and A⊆X. Then f(k(A)) is Čech MP-closed in (Y,v). we have f(A)⊆f(k(A)) and by proposition 5.3 and 5.4, $k_\beta(f(A))\subseteq k_\beta(f(k(A)))=f(k(A))$. Conversely, Let A be any closed set in (X,u). Then by hypothesis, A=k(A), so $f(A)=f(k(A))\supseteq k_\beta(f(A))$. We have $f(A)\subseteq k_\beta(f(A))$ by proposition 5.3. Therefore, $f(A)= k_\beta(f(A))$. (i.e.,) f(A) is Čech MP-closed by proposition 5.3. Hence f is Čech MP-closed.

5.6 Theorem: A map f: (X, u) ⟶(Y, v) is Čech MP-closed if and only if for each subset S of Y and for each Čech open set U containing f(S), there exists an Čech MP-open set V of Y containing S and $f^{-1}(V)\subset U$.

Proof

Necessity: Suppose that f is a Čech MP-closed map. Let S be a subset of Y and U be an Čech open set of X such that $f^{-1}(S) \subset U$. Then V=Y-f(X-U), is an Čech MP-open set containing S such that $f^{-1}(V)\subset U$.

Sufficiency: Let F be a Čech closed set of X. Then $f^{-1}(Y\text{-}f(F))\subset X\text{-}F$ and X-F is Čech open. By taking S=Y-f(F) and U=X-F in hypothesis there exists an Čech MP-open set V of Y containing Y-f(F) and $f^{-1}(V)\subset X\text{-}F$. Then we have $F\subset X\text{-}f^{-1}(V)$. Hence $Y\text{-}V\subseteq f(F)\subseteq f(X\text{-}f^{-1}(V))\subseteq Y\text{-}V$. that implies Y-V=f(F). Since Y-V is Čech MP-closed, f(F) is Čech MP-closed and thus f is an Čech MP-closed map.

5.7 Remark: The following example shows that the composition of two Čech MP-closed maps need not be Čech MP-closed.

5.8 Example: Let X = {p, q, r, s},Y={a, b, c, d},Z={1,2,3,4}. Define a function f: (X, u)⟶(Y, v) such that f(p)=a, f(q)=c, f(r)=d, f(s)=b and g: (Y, v)⟶(Z, w) such that g(a)=2, g(b)=4, g(c)=1, g(d)=3 and gof: (X, u)⟶(Z, w) such that gof(p)=2,gof(q)=1,gof(r)=3,gof(s)=4.

Let u, v and w be closure operators of X,Y and Z defined as u{ϕ}=ϕ, u{p}=u{p, q}={p, q}, u{p, r}=u{p, q, r}={p, q, r}, u{p, s}=u{r, s}=u{p, r, s}={p, r, s}, u{r}={r}, u{q}={q}, u{q, r}={q, r}, u{s}=u{q, s}=u{p, q, s}=u{q, r, s}=u X=X and v{ϕ}=ϕ, v{a}=v{a, b}={a, b}, v{b}=v{b, c}=v{a, b, c}={a, b, c}, v{c}=v{a, c}={a, c}, v{d}=v{a, d}=v{a, b, d}={a, b, d}, v{b, d}=v{c, d}=v{b, c, d}=v{a, c, d}=v Y=Y and w{ϕ}=ϕ, w{1}=w{4}=w{1, 2}= w{1, 4}=w{1, 2, 4}={1, 2, 4}, w{2}={2}, w{3}=w{2, 4}=w{3, 4}=w{2, 3, 4}={2, 3, 4}, w{1, 3}=w{1, 3, 4}={1, 3, 4}, w{2, 3}=w{1, 2, 3}=w Z=Z. Here f and g are Čech MP-closed maps, but g of {q} = {1} is not Čech MP-closed in (Z,w). Therefore gof is not a Čech MP-closed map.

5.9 Proposition: If f: (X, u) ⟶(Y, v) is Čech closed map and g: (Y, v)⟶(Z, w) is Čech MP-closed map, then their composition gof: (X, u)⟶(Z, w) is Čech MP-closed.

Proof: It is obvious.

5.10 Theorem: Let f: (X, u) ⟶(Y, v) and g: (Y, v)⟶(Z, w) be two mappings such that their composition gof: (X, u)⟶(Z, w) be a Čech MP-closed map. Then the following statements are true

1. If f is Čech continuous and surjective then g is Čech MP-closed.
2. If g is Čech MP-irresolute and injective then f is Čech MP-closed.

Proof: (1) If A is a Čech closed set of (Y, v) then f^{-1}(A) is Čech closed in (X,u) since f is Čech continuous. Since gof is Čech MP-closed, (g of) (f^{-1}(A)) is Čech MP-closed in (Z,w). (i.e.,) g(A) is a Čech MP-closed in (Z,w)[since f is surjective]. Therefore g is a Čech MP-closed map.
(2) Let B be a Čech closed set of (X, u). Since gof is Čech MP-closed, (gof) (B) is Čech MP-closed in (Z,w). Here g is Čech MP-irresolute and hence g^{-1}((gof)(B)) is Čech MP-closed in Y. (i.e.,) f(B) is Čech MP-closed in (Y, v), since g is injective. Thus f is a Čech MP-closed map.

5.11 Proposition: For any bijective f: (X, u) ⟶(Y, v), the following statements are equivalent:
1. f^{-1}: (Y, v) ⟶(X, u) is Čech MP-continuous
2. f is a Čech MP-open map and
3. f is a Čech MP-closed map.

Proof: (1) ⇒ (2) Let U be a Čech open set of (X, u). by assumption $(f^{-1})^{-1}$(U)=f(U) is Čech MP-open in (Y,v) and so f is Čech MP-open.
(2) ⇒ (3) Let F be a Čech closed set of (X,u). Then X-F is Čech open in (X,u). By assumption, f(X-F) is Čech MP-open in (Y,v). (i.e.,) f(X-F)=Y-f(F) is Čech MP-open in (Y,v) and therefore f(F) is Čech MP-closed in (Y,v). Hence f is Čech MP-closed.
(3) ⇒ (1) Let F be a Čech closed set of (X,u). By assumption, f(F) is Čech MP-closed in (Y,v). But f(F)=$(f^{-1})^{-1}$(F). Therefore f^{-1} is Čech MP-continuous on Y.

5.12 Proposition: Every Čech closed map is Čech MP-closed map.

Proof: Suppose f be a Čech closed map and A is a Čech closed set in X. Since f is a Čech closed map,f(A) is a Čech closed set in (Y,v). But every Čech closed set is Čech MP-closed. Therefore f(A) is Čech MP-closed set. Hence f is a Čech MP-closed map.

5.13 Proposition
1. Every Čech w-closed map is Čech MP-closed map.
2. Every Čech g-closed map is Čech MP-closed map.
3. Every Čech $\alpha\psi$-closed map is Čech MP-closed map.
4. Every J-Čech closed map is Čech MP-closed map.
5. Every Čech πgβ-closed map is Čech MP-closed map.
6. Every Čech M-closed map is Čech MP-closed map.
7. Every Čech N-closed map is Čech MP-closed map.
8. Every Čech T-closed map is Čech MP-closed map.
9. Every Čech D-closed map is Čech MP-closed map.

Proof: (1) Assume that f is a Čech w-closed map and A is a Čech closed set in X. Then f(A) is a Čech w-closed set in (Y,v). Since f is a Čech w-closed map, But every Čech w-closed set is Čech MP-closed. Therefore f(A) is Čech MP-closed set. Thus f is a Čech MP-closed map.
The proof of others is obvious.

5.14 Remark: The converse of the above theorem need not be true which can be seen from the following example.
5.15 Example: Let X = {1, 2, 3, 4},Y={a, b, c, d}. Define a closure operator u on X by u{ϕ}=ϕ, u{1}=u{1, 2}={1, 2}, u{2}={2}, u{3}={3}, u{2, 3}=u{1, 3}=uX=X. Define a closure operator v on Y by v{ϕ}=ϕ, v{a}={a}, v{b}={b, c}, v{c}=v{a, c}={a, c}, v{a, b}=v{b, c}=v Y=Y. Let f: (X, u)⟶(Y, v) is defined by f(1)=a, f(2)=b, and f(3)=c. Here {2} is a Čech closed set of X. But f{2}={b} is Čech MP-closed set of Y but not Čech w-closed set of Y. Therefore Čech MP-closed map need not be a Čech w-closed map.

5.16 Example: Let X={1, 2, 3}, Y={a, b, c}. Define a closure operator u on X by u{ϕ}=ϕ, u{1}=u{2}=u{1, 2}={1, 2}, u{3}=u{1, 3}={1, 3}, u{2,3}=uX=X. Define a closure operator v on Y by v{ϕ}=ϕ, v{a}=v{a, b}={a, b}, v{b}={b}, v{c}={c}, v{a, c}=v{b, c}=v Y=Y. Let f: (X, u)⟶(Y, v) is defined by f(1)=c, f(2)=b, and f(3)=a. Here {1,3} is a Čech closed set of X. But f{1,3}={a,c} is Čech MP-closed set of Y but not Čech g-closed set of Y. Therefore Čech MP-closed map need not be a Čech g-closed map.

5.17 Example: Let X={1,2,3},Y={a, b, c}. Define a closure operator u on X by u{ϕ}= ϕ, u{1}={1}, u{2}={2}, u{3}=u{1,3}={1,3}, u{1,2}=u{2,3}=uX=X. Define a closure operator v on Y by v{ϕ}=ϕ, v{a}=v{a, b}={a, b}, v{b}={b}, v{c}={c}, v{a, c}=v {b, c}=vY=Y. Let f: (X, u)⟶(Y, v) is defined by f(1)=a, f(2)=c and f(3)=b. Here {1} is a Čech closed set of X. But f{1}={a} is Čech MP-closed set of Y but not Čech αψ-closed set of Y. Therefore Čech MP-closed map need not be a Čech αψ-closed map.

5.18 Example: Let X={1, 2, 3},Y={a, b, c}. Define a closure operator u on X by u{ϕ}= ϕ, u{1}={1,2}, u{2}={2,3}, u{3}={3}, u{1,3}={1,3}, u{1,2}={1,2}, u{2,3}=uX=X. Define a closure operator v on Y by v{ϕ}=ϕ, v{a}={a}, v{b}={b, c}, v{c}=v{a, c}={a, c}, v{a, b}= v {b, c}=vY=Y. Let f: (X, u)⟶(Y, v) is defined by f(1)=b, f(2)=c and f(3)=a. Here {1,2} is a Čech closed set of X. But f{1,2}={b,c} is Čech MP-closed set of Y but not J- Čech closed set of Y. Therefore Čech MP-closed map need not be a J- Čech closed map.

5.19 Example: Let X={1, 2, 3},Y={a, b, c}. Define a closure operator u on X by u{ϕ}= ϕ, u{1}={1}, u{2}={2}, u{3}={3}, u{1, 2}={1, 2}, u{2, 3}=u{1, 3}=u X=X. Define a closure operator v on Y by v{ϕ}=ϕ, v{a}={a, b}, v{b}=v{b, c}={b, c}, v{c}={c}, v{a, b}={a, b}, v{a, c}=v Y=Y. Let f: (X, u)⟶(Y, v) is defined by f(1)=a, f(2)=c and f(3)=b. Here {3} is a Čech closed set of X. But f{3}={b} is Čech MP-closed set of Y but not Čech πgβ-closed set of Y. Therefore Čech MP-closed map need not be a Čech πgβ-closed map.

5.20 Example: Let X={1, 2, 3},Y={a, b, c}. Define a closure operator u on X by u{ϕ}= ϕ, u{1}=u{1, 3}={1, 3}, u{2}=u{2, 3}={2, 3}, u{3}={3}, u{1, 2}=u X=X. Define a closure operator v on Y by v{ϕ}=ϕ, v{a}={a}, v{b}={b}, v{c}={c},v{a, c}={a, c}, v{a, b}=v{b, c}= v Y=Y. Let f: (X, u)⟶(Y, v) is defined by f(1)=c, f(2)=b and f(3)=a. Here {2,3} is a Čech closed set of X. But f{2,3}={a,b} is Čech MP-closed set of Y but not Čech M-closed set of Y. Therefore Čech MP-closed map need not be a Čech M-closed map.

5.21 Example: Let X = {1, 2, 3}, Y={a, b, c}. Define a closure operator u on X by u{ϕ}= ϕ, u{2}={2}, u{3}=u{1, 3}={1, 3}, u{1}=u{1, 2}=u{2, 3}, u{2}=u X=X. Define a closure operator v on Y by v{ϕ}=ϕ, v{a}={a, b}, v{b}=v{b, c}={b, c}, v{c}={c}, v{a, b}={a, b}, v{a, c}=v Y=Y. Let f: (X, u) ⟶ (Y, v) is defined by f (1) =c, f(2)=a and f(3)=b. Here {2} is a Čech closed set of X. But f {2}={a} is Čech MP-closed set of Y but not Čech N-closed set of Y. Therefore Čech MP-closed map need not be a Čech N-closed map.

5.22 Example: Let X = {1, 2, 3}, Y={a, b, c}. Define a closure operator u on X by u{ϕ}=ϕ, u{1, 2}={1, 2}, u{2, 3}={2, 3}, u{1, 3}={1, 3}, u{1}=u{2}=u{3}=u X=X. Define a closure operator v on Y by v{ϕ}=ϕ, v{a}={a}, v{b}={b, c}, v{c}=v{a, c}={a, c}, v{a, b}= v{b, c}=vY=Y. Let f: (X, u) ⟶(Y, v) is defined by f(1)=a, f(2)=c and f(3)=b. Here {3} is Čech closed set of X. But f {3}={b} is Čech MP-closed set of Y but not Čech T-closed set of Y. Therefore Čech MP-closed map need not be a Čech T-closed map.

5.23 Example: Let X = {1, 2, 3}, Y={a, b, c}. Define a closure operator u on X by u{ϕ}=ϕ, u{1, 2}={1, 2}, u{2, 3}={2, 3}, u{1, 3}={1, 3}, u{1}=u{2}=u{3}=u X=X. Define a closure operator v on Y by v{ϕ}=ϕ, v{a}={a}, v{b}={b, c}, v{c}=v{a, c}={a, c}, v{a, b}=v{b, c}= v Y=Y. Let f: (X, u) ⟶ (Y, v) is defined by f(1)=c, f(2)=a and f(3)=b. Here {2, 3} is Čech closed set of X. But f {2,3}={a,b} is Čech MP-closed set of Y but not Čech D-closed set of Y. Therefore Čech MP-closed map need not be a Čech D-closed map.

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