ISSN Print: 2394-7500
ISSN Online: 2394-5869
Impact Factor: 5.2
IJAR 2017; 3(1): 476-478
www.allresearchjournal.com
Received: 03-11-2016
Accepted: 04-12-2016

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Construction of group acceptance sampling plan based on Pareto distribution (Second Kind) using Poisson distribution

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Abstract
Acceptance sampling plans developed relating to groups of items on testers is called the group acceptance sampling plan (GASP). In this scheme, a sample of items is distributed into different groups and a lot of product is rejected if more than a specified number of failures are recorded in any group. A group acceptance sampling schedule can be used to save the time and cost in inspection as compared with classical acceptance sampling. In this paper, a procedure for constructing a group acceptance sampling plan (GASP) using Poisson distribution in the construction of sampling plan is proposed. Suitable tables and examples are also provided for easy selection of the plans.

Keywords: Acceptance sampling plan, group sampling, Pareto distribution (second kind), consumer’s risk, producer's risk, poisson distribution

1. Introduction

Sample size and the length of experiment time are two major factors regarding cost in a life test. How to design an acceptance sampling scheme to conduct a life test in short time with small sample size is mainly concerned for experimenters. Typically, ordinary acceptance sampling plans regarding lifetimes are developed to satisfy two points on the operating characteristic (OC) curve to protect producers and consumers simultaneously or based on the economic viewpoint. Some ordinary acceptance sampling plans have been developed in the literature by several researchers, see e.g., Refs 1–15. In some situations, life tests may be conducted with multiple items in testers. That is, more than one item can be tested in the experiment simultaneously.

In this situation experimenters/practitioner cannot use the ordinary acceptance sampling plan to test the items. Experimenters/practitioners can treat items in a tester as a group. Acceptance sampling plans developed regarding groups of items on testers are called the group acceptance sampling plan (GASP). In this scheme, a sample size of items is distributed into different groups and a lot of product is rejected if more than specified failures are recorded in any group. Therefore a group acceptance sampling schedule can be used to save the test time and cost as compared with the life test scheme in which only a single item is put on test in a tester, or the ordinary acceptance sampling scheme. Moreover, the group acceptance sampling scheme can be used for the strict inspection of items than the ordinary acceptance sampling before it will be realized for consumers’ use.

Sudden death testing is an example using this type of testers. Some applications of sudden death testing can be found in Refs 16–18. Reference 19 introduced the idea of GASP to truncated life tests. They developed GASPs under the inverse Rayleigh and log logistic lifetime distributions, respectively.

The group sampling can be used to save time and cost and for more strict inspection of submitted product than the ordinary acceptance sampling. No attention has been paid to develop the group acceptance sampling plan based on truncated life tests. In this paper, we develop GASPs based on truncated life test assuming that the lifetimes of a product follows the Pareto distribution of the second kind with known shape parameter. Moreover, the determinations of the group number, the OC value and the mean ratio of the True mean life to the specified mean life regarding the GASP is discussed. The GASPs are constructed for satisfying the specified consumer’s risk at the low quality level.

The rest of this paper is organized as follows: The Pareto distribution of the second kind is brief discussed in "The Design of Group Acceptance Sampling Plans." Moreover, the design of the GASP for the Pareto distribution of the second kind is developed.
An example is given in "Example" for illustration. A comparative study is given in "Comparative Study." Some conclusion remarks are given in "Conclusions.

2. The Pareto Distribution of the Second Kind
The probability density function and the cumulative distribution function (c.d.f.) of the Pareto distribution of the second kind are given respectively by

$$f(t,\sigma,\lambda)=\frac{\lambda}{\sigma}\left(t+\frac{t}{\sigma}\right)^{-(\lambda+1)},t>0,\lambda,\sigma,>0 \quad (1)$$

$$F(t,\sigma,\lambda)=1-\left(t+\frac{t}{\sigma}\right)^{-\lambda},t>0,\lambda,\sigma,>0 \quad (2)$$

where λ and σ are shape and scale parameters respectively. The mean life of the model (eqn. 2) is given by

$$\mu = \sigma / (\lambda-1),\ \lambda>1 \quad (3)$$

It is important to note that the mean function is only valid when shape values is larger than 1.

3. The Group Acceptance Sampling Plans
The producer's risk and the consumer's risk are represented by α and β respectively in the development of acceptance sampling plans. The producer's risk is defined as the probability of rejecting a good lot. The consumer's risk is the probability of accepting a bad lot. We are interested in constructing a GASP to ensure that the mean life μ is greater than a specified mean life μ_0.

3.1 Operating Procedure
1. Determine the group size g. Sample n=gr items from a lot randomly and allocate r items to each group for the life test.
2. Determine the acceptance number c for every group and specify the termination time of the life test t_0 and
3. Implement the life test based on the g groups of items simultaneously. Accept the lots if at most c failed items are found in every group by the termination time. Truncate the life test and reject the lot if more than c failures are found in any group.

The parameters in the GASP to be determined are the group size g, the sample size in each group r, the acceptance number c and the termination ratio t/μ_0. If r = l, the GASP reduces to the ordinary acceptance sampling plan. As the decision is either to accept or to reject the lot on the basis of a sample selected from a big lot, the Poisson distribution can also be used to develop the GASPs. The lot acceptance probability for the proposed plan is given by

$$L(P)=\left[\frac{\sum_{x=0}^{c} e^{-rp} rp^{x}}{x!}\right]^{g} \leq \beta \quad (4)$$

where p = function of cumulative distribution function given in eqn 2.
It would be convenient to take the termination time as a multiple of the specified number *a*, i.e., $t_0 = a\mu_0$. Therefore the cdf in Eq 2 can be presented as

$$L(p)=\left[1+\frac{a}{(\lambda-1)(\mu/\mu_0)}\right]^{-\lambda} \quad (5)$$

The parameters g, r, c and μ/μ_0 are to be determined. The minimal group size (g) and minimal ratio (μ/μ_0) of the true mean life to the specified mean life can be determined from the following equations (6) and (7)

$$L(p)=\left[1+\frac{a}{(\lambda-1)(\mu/\mu_0)}\right]^{-\lambda} \leq \beta \quad (6)$$

When consumer risk (β) and producers risk (α) are specified.

4. Example 1
For a specified β = 0.05, a = 0.8, r = 4, c = 2 and λ = 2, the value of g can be obtained from table 1 as 6. The relevant GASP is n = 24, g = 6, r = 4, c = 2 for a specified β = 0.05 and a = 0.8.

4.1 Explanation
If the consumer fixes β = 0.05 (5 non-confirmative at of 100) the manufacturer has to select 24 items from the lot and allocate them into 6 groups of 4 items each and allow for life test till the time t_0 and the life test is stopped when more than 2 failed items are recorded during the time and the lot is rejected, otherwise the lot is accepted. If the lot is rejected inform the management for corrective action.

5. Construction of Tables
The minimum group size (g) is obtained using equations (4), (5), (6) & (7) for various values of β, α, termination ratio (a) with r = 2, a = 0.7 and c = 1 for a specified value of r = 2, c =1, α = 0.05, γ = 2 and with the help of Excel package the results are calculated and presented in table 1, table 2 and table 3 and table 4.

Table 1: Minimal Group Sizes for GASP for r =2, c = 1, where γ=2

β \ a	0.7	0.8	1.0	1.2	1.5	2.0
0.25	1	1	1	1	1	1
0.10	2	2	2	2	2	2
0.05	2	2	2	2	2	2
0.01	2	2	2	2	2	2

Table 2: OC values for GASP for r = 2, c = 1, where $\gamma = 2$

β \ μ/μ_0	2	4	6	8	10	12
0.25	0.3272	0.4177	0.4764	0.5205	0.5537	0.5803
0.10	0.2371	0.3247	0.3852	0.4311	0.4669	0.4961
0.05	0.1937	0.2773	0.3376	0.3834	0.4196	0.4496
0.01	0.1319	0.2045	0.2613	0.3055	0.3441	0.3715

Table 3: OC values for GASP for r = 3, c = 1, where $\gamma = 2$

β \ μ/μ_0	2	4	6	8	10	12
0.25	0.3982	0.5035	0.5702	0.6175	0.6526	0.6798
0.10	0.2881	0.3946	0.4661	0.5186	0.5594	0.5906
0.05	0.2340	0.3373	0.4093	0.4636	0.5062	0.5391
0.01	0.1560	0.2499	0.3159	0.3720	0.4146	0.4510

6. Example 2

For a specified $\beta = 0.05$, a = 0.8, r = 2, c = 1 and $\gamma = 2$, the value of g can be obtained from table1 as 5. The relevant GASP is n = 10, g = 5, r = 2, c = 1 for a specified $\beta = 0.05$ and a = 0.8.

6.1 Explanation

If the consumer fixes $\beta = 0.05$ (5 non-confirmative at of 100) the manufacturer has to select 10 items from the lot and allocate them into 5 groups of 2 items each and allow for life test till the time t_0 and the life test is stopped when more than 1 failed items are recorded during the time and the lot is rejected, otherwise the lot is accepted. If the lot is rejected inform the management for corrective action.

7. Conclusion

In this paper a new procedure for the construction of GASP using Poisson distribution as the base line distribution with gamma distribution in life testing is presented. Tables are also presented for the easy selection of groups by the engineers and additional tables can also be developed depending on the choice of a, α, β, γ and c.

8. References

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