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## Voronoi cell to find optimal paths for the transportation problem

**S Lakshmi and P Jeevitha**

### Abstract

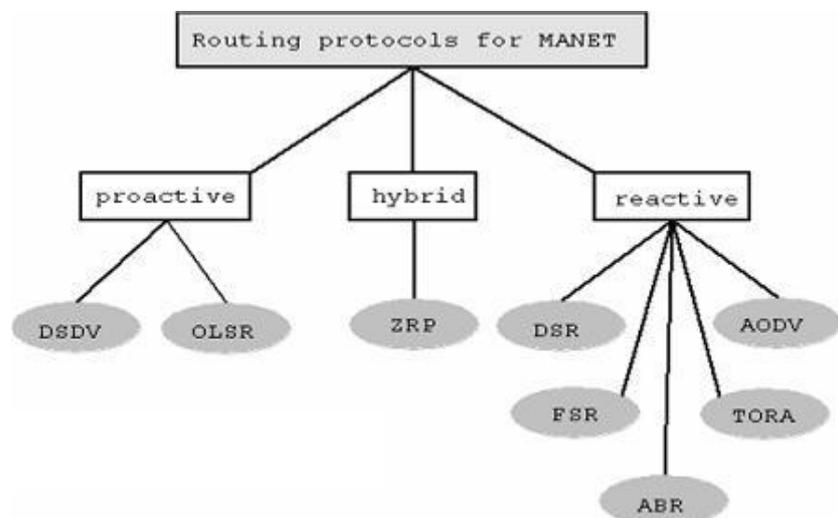
Transportation networks model facilities for fast movement on the plane. A transportation network, together with its underlying distance, induces a new distance. Previously, only the Euclidean and the L 1 distances have been considered as such underlying distances. However, this paper first considers distances induced by general distances and transportation networks, and present a unifying approach to compute Voronoi diagrams under such a general setting. With this approach, we show that an algorithm for convex distances can be easily obtained.

**Keywords:** Transportation networks, Euclidean Framework, Voronoi diagrams

### Introduction

Transportation networks model facilities for fast movement on the plane. They consist of roads and nodes; roads are assumed to be segments along which one can move at a certain fixed speed and nodes are endpoints of roads. We assume that there are no crossings among roads but roads can share their endpoints as nodes. We thus define a transportation network as a plane graph whose vertices are nodes and whose edges are roads with speeds assigned. Also, we assume that one can access or leave a road through any point on the road. The problem distinguishable from and more difficult than those in other similar settings such as the airlift distance. In the presence of a transportation network, the distance between two points is defined to be the shortest elapsed time among all possible paths joining the two points using the roads of the network.

We call such an induced distance a transportation distance. More precisely, a transportation distance is induced on the plane by a transportation network and its underlying distance that measures the distance between two points without roads. Since early considerations for roads fundamental geometric problems, such as shortest paths and Voronoi diagrams, under transportation distances have been receiving much attention recently.



**Fig 1:** Routing Protocols in Manet

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Since transportation distances have quite different properties depending on their underlying distances, there has not been a common approach to extend such problems to more general underlying distances. This paper thus considers geometric problems, in particular, shortest paths and Voronoi diagrams under transportation distances induced from general distances.

This paper presents, to the best of our knowledge, the first result that studies general underlying distances and gives algorithms for computing Voronoi diagrams with transportation networks. More precisely, we classify transportable distance functions where transportation networks and transportation distances are well-defined. Transportable distances include asymmetric convex distances, nice metrics, and even transportation distances.

In this general setting, a unifying approach to compute Voronoi diagrams is presented. As a special case of transportable distances, we take the convex distances into account. Based on the approach together with geometric and algorithmic observations on convex distances, we first obtain an efficient and practical algorithm that computes the Voronoi diagram with a transportation network under a convex distance.

### Proposed Methodology

The scheme described in the previous section; a bundle of properties have to be shown: how to compute needles produced from a needle, how to check the effectiveness of needles, how many needles and events to handle, how to compute the Voronoi diagram for needles, and some technical lemmas to reduce the complexity. A convex distance is defined by a compact and convex body  $C$  containing the origin, or the center, and is measured as the factor that  $C$  centered at the source should be expanded or contracted for its boundary to touch the destination. Note that a convex distance is symmetric, i.e. being a metric, if and only if  $C$  is symmetric at its center.

Consider the convex  $C$  as a black box which supports some kinds of elementary operations. These are finding the Euclidean distance from the center to the boundary in a given direction, finding two lines which meet at a given point and are tangent to  $C$ , finding the foot point for a needle and a road, and computing the bisecting curve between two sites under the convex distance based on  $C$ . Here, we assume that these operations consume reasonable time bounds.

### Proposed Schemes:

#### Voronoi Diagrams under Transportation Distances

As noted in the previous section, a transportable distance  $d$  and a transportation network  $G$  induce a new distance  $dG$  and  $dG$ -straight paths. The structure of any  $dG$ -straight path can be represented by a string of  $\{S, T\}$ , where  $S$  denotes a  $d$ -straight path without using any road and  $T$  denotes that along a road.

Let us consider a single road  $e$  as a simpler case. Given a transportation network  $G$  with only one road  $e$ , a  $dG$ -straight path is of the form  $STS$  or its substring except for  $SS$ . This is quite immediate; paths represented by longer strings than  $STS$  can be reduced since a road is  $d$ -straight and  $d$  satisfies the triangle inequality.

Thus, any  $dG$ -straight path  $P$  from  $p$  to  $q$  using a road  $e$  can be represented as  $P = (p, p_0, q_0, q)$ , where  $p_0$  is the entering point to  $e$  and  $q_0$  is the exiting point to  $q$ . We then call  $q_0$  a foot point of  $q$  one.

### Computing Effective Needles

In this subsection, we present an algorithm for computing the Voronoi diagram  $V_{dG}(S)$  for a given set  $S$  of sites under  $dG$ . The algorithm consists of three phases; it first computes the set  $S$  of needles from  $G$  and  $S$ , secondly, the Voronoi diagram  $V_d(S)$  for  $S$  under  $d$  is constructed, and the Voronoi diagram  $V_{dG}(S)$  for  $S$  under  $dG$  is finally obtained from  $V_d(S)$ .

The second phase, computing  $V_d(S)$ , would be solved by several techniques and general approaches to compute Voronoi diagrams, such as the abstract Voronoi diagram. Also, the third phase can be done by Corollary 5. We therefore focus on the first phase, computing  $S$ —in fact, its finite subset—from  $G$  and  $S$ . Recall that  $S$  is defined as all the needles recursively produced from given sites and contains infinitely many useless needles, that is, needles that do not constitute the Voronoi diagram  $V_d(S)$ . We call a needle  $p \in S$  effective with respect to  $S$  if the Voronoi region of  $p$  in  $V_d(S)$  is not empty. Let  $S^* \subseteq S$  be the largest set of effective needles with respect to  $S$ , i.e.,  $V_d(S^*) = V_d(S)$ .

### Transportation Networks under Convex Distances

In this section, we deal with convex distances as a special case of transportable distances. We thus investigate geometric and algorithmic properties of the induced distance by a convex distance and a transportation network, and construct algorithms that compute the Voronoi diagram for given sites under the induced distance.

In order to devise such an algorithm, we apply the abstract scheme described in the previous section; a bundle of properties have to be shown: how to compute needles produced from a needle, how to check the effectiveness of needles, how many needles and events to handle, how to compute the Voronoi diagram for needles, and some technical lemmas to reduce the complexity.

A convex distance is defined by a compact and convex body  $C$  containing the origin, or the center, and is measured as the factor that  $C$  centered at the source should be expanded or contracted for its boundary to touch the destination.

We consider the convex  $C$  as a black box which supports some kinds of elementary operations. These are finding the Euclidean distance from the center to the boundary in a given direction, finding two lines which meet at a given point and are tangent to  $C$ , finding the foot point for a needle and a road, and computing the bisecting curve between two sites under the convex distance based on  $C$ . Here, we assume that these operations consume reasonable time bounds. Throughout this section, for a convex body  $C$ , we denote by  $C + p$  a translation of  $C$  by a vector  $p$  and by  $\lambda C$  expansion or a contraction of  $C$  by a factor  $\lambda$ .

### Voronoi Diagrams for Needles

In general, bisectors between two needles under a convex distance can be parted into two connected components. However, it will be shown that  $S^*$  can be replaced by such nice needles, so called non-piercing, that the Voronoi diagram for them is an abstract Voronoi diagram which can be computed in the optimal time and space. Computing the Voronoi diagrams for non-piercing needles.

The abstract Voronoi diagram is a unifying approach to define and compute general Voronoi diagrams, introduced by Klein. In this model, we deal with not a distance but bisecting curves  $J(p, q)$  defined in an abstract fashion between two sites  $p$  and  $q$ .

A system  $(S, \{J(p, q) | p, q \in S, p \neq q\})$  of bisecting curves for  $S$  is called admissible. In fact, the first three conditions are enough to handle abstract Voronoi diagrams theoretically but the fourth one is necessary in a technical sense. Though all convex distances satisfy the first three ones, there exist convex distances violating the fourth one. We guarantee the fourth condition by postulating that  $\partial C$  is semi algebraic. We also note that two-dimensional bisectors can be avoided by a total order on given sites.

**Experimental results**

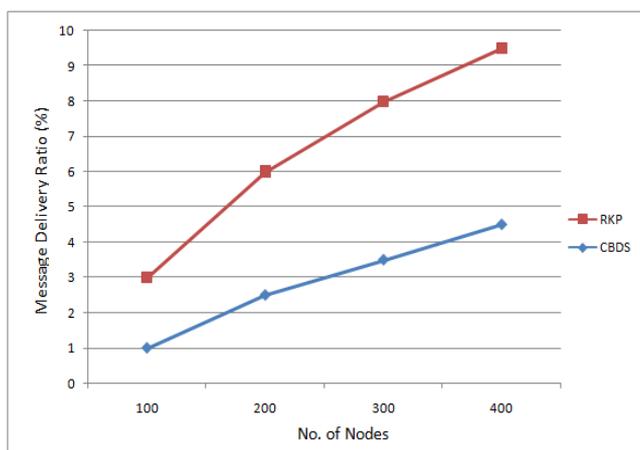
Network Simulator (NS2) is a discrete event driven simulator developed at UC Berkeley. It is part of the VINT project. The goal of NS2 is to support networking research and education. It is suitable for designing new protocols, comparing different protocols and traffic evaluations.

This study used ns-2 as the network simulator and conducted numerous simulations to evaluate the network performance. All sensor nodes are randomly scattered with a uniform distribution. Randomly select one of the deployed nodes as the source node. The location of the sink is randomly determined. This study evaluates the routing performance under scenarios with different numbers of sensor nodes.

This study evaluates the following main performance metrics:

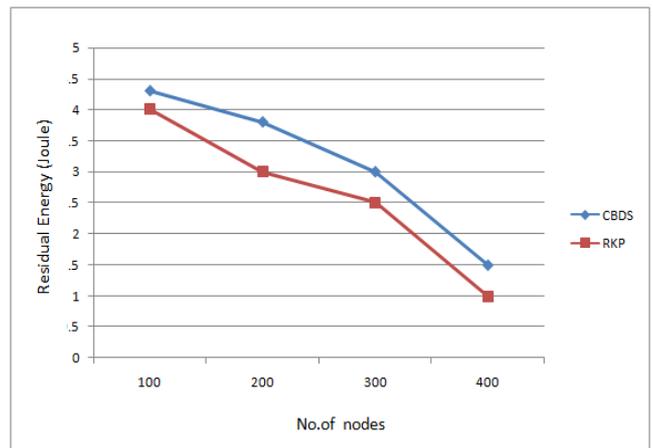
- Message delivery ratio: is the ratio of the number of report messages the sink receives to the total number of report messages the source node sends.
- Residual energy: measures the mean value of the residual energy of all alive sensor nodes when simulation terminates.
- Delivery latency: means the time delay experienced by the source node while transmitting a report message to the sink.

**Message Delivery Ratio**



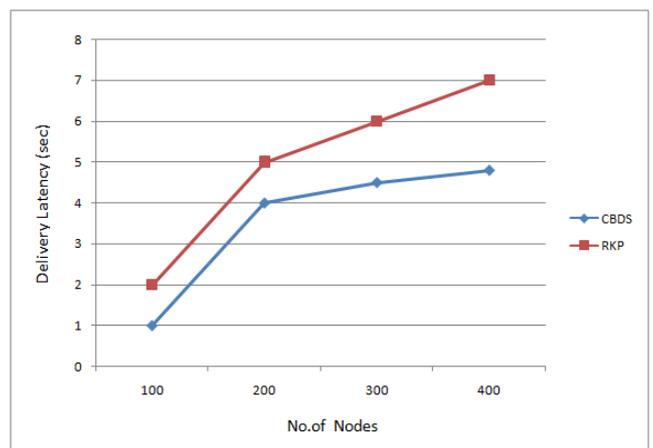
Above graph compares the simulation results of message delivery ratios of this mechanisms decreases, as  $N_s$  increases. Because increasing  $N_s$  increases the number of packets in the network, the probability of packet collisions also increases. This reconstruction may lead to additional energy consumption of sensor nodes, thereby decreasing the packet delivery ratio.

**Residual Energy**



Following graph provides a comparison of the energy consumption results of four clustering mechanisms under scenarios with different nodes. Nodes have to consume additional battery power to transmit the increased number of report messages. This leads to a reduction of the residual energy of the nodes in the network.

**Delivery Latency**



Above graph shows the average delivery latency of proposed mechanisms under scenario with different  $N_s$  and  $N_{req}$ . As  $N_s$  increases, more data are generated and the length of the discovered routing path also increases. This leads to along delivery latency, as illustrated in above this may cause reconstruction to determine a new path, thereby increasing the delivery latency. The reconstruction and retransmission generate along message latency.

**Conclusion**

The abstract Voronoi diagram is a unifying approach to define and compute general Voronoi diagrams. In this model, we deal with not a distance but bisecting curves defined in an abstract fashion between two sites. The results which are obtained for the given example shows that Voronoi cell Algorithm is very effective tool to find the path with lowest cost from node to node.

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