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Dominator coloring and total dominator coloring of certains graphs

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Abstract
Let G be a graph, then the dominator coloring of G is a proper coloring, in which every vertex of G dominates every vertex of at least one color class. And the Total dominator coloring of graph is a proper coloring with extra property that every vertex in the graph properly dominates an entire color class.
In this paper, the dominator coloring and total dominator coloring of Prism, Sunlet and Sparse graphs have been presented. Also the relation between their chromatic number, dominator chromatic number and total dominator chromatic number have been discussed.

Keywords: Dominator coloring, total dominator coloring, chromatic number, dominator chromatic number, total dominator chromatic number

1. Introduction
A graph G is a triple consisting of a vertex set V(G), an edge set E(G) and a relation that associates with each edge two vertices called its endpoints. The chromatic number of G, denoted by $\chi(G)$ is the smallest number of colors required for coloring the graph G. The proper coloring of a graph G is coloring of graph in which no two adjacent vertices must have same color. A color class is a set of vertices which have the same colors. The minimum number of colors needed for dominator coloring is known as dominator chromatic number $\psi_d(G)$. In this paper we have estimated the relation between the dominator chromatic number $\psi_d(Y_n)$ and chromatic number $\psi(Y_n)$ of prism graph, sunlet graph and total dominator chromatic number $\psi_{td}(G)$ of sparse graph have been discussed.

2. Definitions
1. A Prism graph Y_n is a graph having its own prism as its skeleton, it has 2n vertices and 3n edges.
2. A n-sunlet graph is a graph in which n-pendent edges are attached to cycle C_n, it is of 2n vertices.
3. A graph having edges lesser than the possible number of edges is known as Sparse graph.

3. Dominator Coloring of Prism and Sunlet graphs
Theorem 3.1
For a prism graph Y_n, n=5, the dominator coloring is 6.

Proof
This prism graph consists of vertex set V=v_1, v_2, v_3, v_4,….v_{10} and edge set $E_1 = v_1v_2 \cup v_2v_3 \cup v_3v_4 \cup v_4v_5$, $E_2 = v_5v_6 \cup v_6v_7 \cup v_7v_8 \cup v_8v_9 \cup v_9v_{10}$, $E_3 = v_1v_7 \cup v_2v_8 \cup v_3v_9 \cup v_4v_{10}$. Then the colors are assigned to the vertices.

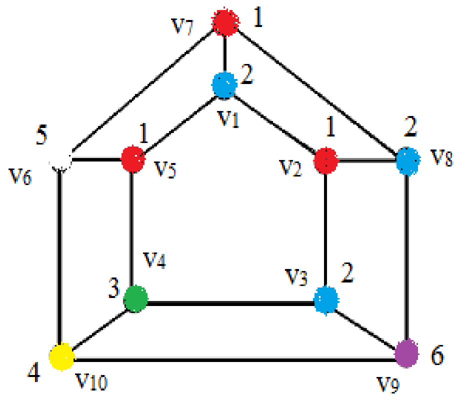


Fig 3.1

The vertices $v_k, \{1 \le k \le 5\}$, dominate color class k. The vertices $v_6 v_7$ dominate color class 5 and the vertices $v_{5+k}, 3 \le k \le 5$ dominate color class 6. Hence the dominator coloring of Y_5 is 6.

Theorem 3.2
Let the prism graph Y_n for n=3 be$\psi_d(Y_n) = n$

Proof
This prism graph consists of vertices $V = v_1, v_2, v_3$and the edges be
$E = e_1 \cup e_2 \cup e_3$, where$e_1 = v_1v_2 \cup v_1v_5 \cup v_3v_1, e_2 = v_2v_6 \cup v_2v_3 \cup v_3v_4, e_3 = v_4v_5 \cup v_5v_6 \cup v_4v_6$. Then the colors are assigned to the vertices.
The vertices v_1, v_4 are assigned color 3, vertices v_2, v_5 are assigned color 1 and vertices v_3, v_6 are assigned color 2.

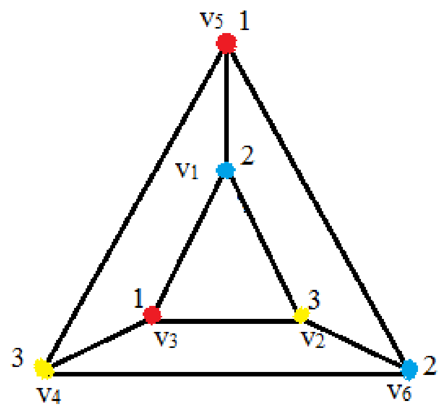


Fig 3.2

Note that v_1, v_6 dominate color class 1, v_2, v_4 dominate color class 2 and v_3, v_5 dominate color class 3.

Theorem 3.3
Let the prism graph be Y_n, n=4 then $\psi_d(Y_n) = n$

Proof
This prism graph consists of vertices $v_i = \{1 \le i \le 4\}$ and edge set
$E = e_i \cup e_j \cup e_k \cup e_l$, where $e_i = v_1v_4 \cup v_1v_2 \cup v_1v_6, e_j = v_2v_3 \cup v_2v_7$, $e_k = v_3v_8 \cup v_3v_4 \cup v_4v_5, e_l = v_5v_6 \cup v_6v_7 \cup v_7v_8 \cup v_5v_8$. The colors are assigned to the vertices.
Color 1 is assigned to the vertices v_4, v_6, v_8; color 2 is assigned to the vertices v_2, v_5 ; color 3 is assigned to the vertices v_1, v_3 ; color 4 is assigned to the v_7.

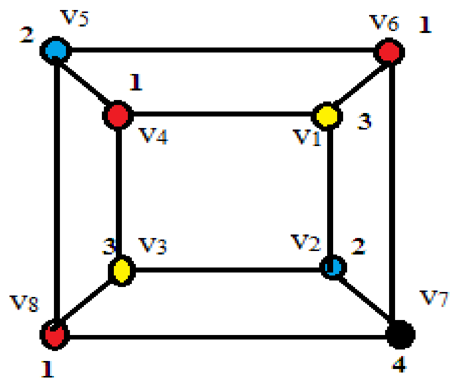


Fig 3.3

From the figure we note that the color class 1,2 is dominated by the vertices v_1, v_2. Likewise the color class 1 is dominated by vertices v_3, v_5 ; the color class 3 is dominated by the vertices v_6, v_7, v_8.
Hence the dominator chromatic number for $\psi_d(Y_4) = 4$.

Theorem 3.4
Let the prism graph Y_n, $n \le 6$ then $\psi_d(Y_n) = n$

Proof
This prism graph consists of vertex set, V= 1,2…….12 and edge set E be the union of e_1, e_2, e_3, e_4 i.e, $e_1 = \{v_r v_{r+1}; 1 \le r \le 6\}, e_2 = v_1 v_n$, $e_3 = \{v_{r+1} v_{r+2}; 6 \le r \le 10\}, e_4 = v_{2n} v_{n+1}$. Now the vertices v_1, v_3, v_9 are assigned color 1, v_2, v_6, v_8 are assigned color 2, v_4, v_{12} are assigned color 3, v_5, v_{11} are assigned color 4, the vertex $v_7 \& v_{10}$ are assigned colors 6 & 5 respectively.

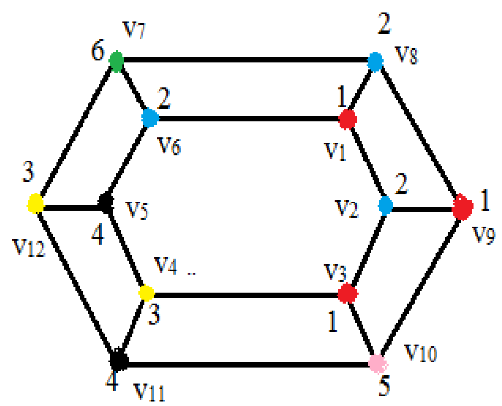


Fig 3.4

The vertices v_1, v_2 dominate color class 2,1 respectively. Likewise the vertices v_3, v_9, v_{10}, v_{11}dominate color class 5, v_4, v_{12}dominate color class 4, v_5 dominate color class 3, v_6, v_7, v_8dominate color class 6.
Hence the dominator chromatic number for $\psi_d(Y_n) = n$. i.e, $\psi_d(Y_6) = 6$.

Theorem 3.5
Let the prism graph Y_n for n=9 be$\psi_d(Y_n) = n + 1$

Proof
This Y_9 graph consists of vertices $v_k = \{1 \le k \le 18\}$ and edge set $E_1 = v_r v_{r+1} \cup v_{r+1} v_{r+2} \cup v_{r+2} v_{r+3 \ldots\ldots\ldots\ldots} \cup v_{r+16} v_{r+17} \{r = 1\}, E_2 = v_1 v_9 \cup v_1 v_{11}$,

$E_3 = v_2v_{12} \cup v_3v_{13} \cup v_4v_{14}$, $E_4 = v_5v_{15} \cup v_6v_{16} \cup v_7v_{17}$ and $E_5 = v_8v_{18}$. Now colors are assigned to the vertices. Color 1 is assigned to the vertices v_2, v_9, v_{11}; color 2 to v_1, v_{12}, v_3; color 3 to v_4; color 4 to v_5, v_{14}; color 5 to v_6, v_{15}; color 6 to v_7, v_{16} ; color 7 to v_8, v_{17} ; color 8 to v_{18}; color 9 to v_{10} and color 10 to v_{13}.

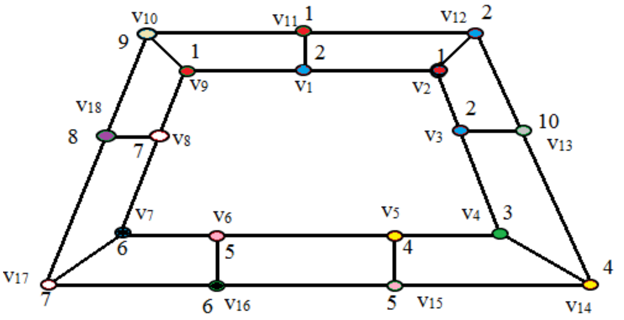


Fig 3.5

We notice that $v_k\{1 \le k \le 9\}$ dominate color class k. Then v_{10}, v_{11}dominate the color class 9; v_{12}, v_{13}, v_{14} dominate color class 10 and $v_{n+6}, v_{n+7}, v_{n+8}, v_{n+9}\{9 \ge n \ge 6\}$ dominate color class r $\{4 \le r \le 7\}$.
Hence the dominator chromatic number for $\psi_d(Y_9) = 10$.

Theorem 3.6
Let a prism graph Y_n, n=10, $\psi_d(Y_n) = n + 1$

Proof
This Y_{10} graph consists of vertex set $V = v_k\{1 \le k \le 20\}$ and edge set$E_1 = v_rv_{r+1} \cup v_{r+1}v_{r+2} \cup v_{r+2}v_{r+3}$.............. $\cup$ $v_{r+18}v_{r+19}\{r = 1\}$,
$E_2 = v_1v_{12} \cup v_2v_{13} \cup v_3v_{14} \cup v_4v_{15}$, $E_3 = v_5v_{16} \cup v_6v_{17} \cup v_7v_{18} \cup v_8v_{19} \cup v_9v_{20}$. Now colors are assigned to the vertices. Color 1 is assigned to the vertices v_2, v_{12}; color 2 to v_1, v_3, v_{13} ; color 3 to v_4 ; color 4 to v_5, v_{15} ; color 5 to v_6, v_{16} ; color 6 to v_7, v_{17} ; color 7 to v_8, v_{87} ; color 8 to v_9, v_{19} ; color 9 to v_{10}, v_{20} ; color 10 to v_{11} and color 11 to v_{14}.

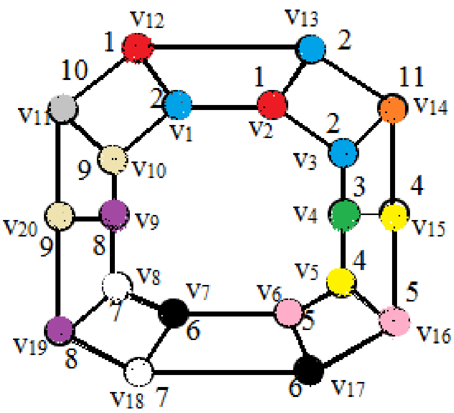


Fig 3.6

We notice that $v_k\{1 \le k \le 10\}$ dominate color class k. Then v_{13}, v_{14}, v_{15} dominate the color class 11; v_{12} dominate color class 2 and $v_k\{1 \ge k \ge 10\}$ dominate color class r $\{4 \le r \le 8\}$.

Hence the dominator chromatic number for $\psi_d(Y_{10}) = 11$.

Theorem 3.7
Let a prism graph $Y_n, n \ge 3$ then $\psi_d(Y_n) > \gamma(Y_n)$

Proof
The dominator chromatic number of a prism Y_n is

$$Y_n = \begin{cases} n+1 \; for \; n = 9 \;\&\; n \ge 9 \\ n \; for \; n \ne 5, n = 3,4 \end{cases}$$

The domination number of a prism Y_n is

$$\gamma(Y_n) = \begin{cases} \frac{n}{2} \; if \; n \equiv -4(mod4) \\ \frac{n}{2} + 1 \; if \; n \equiv -2(mod4) \\ \frac{n+1}{2} \; if \; n \equiv -1(mod2) \end{cases}$$

Hence $\psi_d(Y_n) > \gamma(Y_n), n \ge 3$.

Theorem 3.8
Let a prism graph $Y_n, n > 3$, n = 10 then $\psi_d(Y_n) > \chi(Y_n)$

Proof
The dominator chromatic number of a prism Y_n is

$$Y_n = \begin{cases} n+1 \; for \; n = 9 \;\&\; n = 10 \\ n \; for \; n = 4, 3 \le n \le 8 \end{cases}$$

The domination number of a prism graph Y_n is

$$\chi(Y_n) = \begin{cases} 2 \; when \; n \; is \; even \\ 3 \; when \; n \; is \; odd \end{cases}$$

Hence $n > 3$, then $\psi_d(Y_n) > \psi(Y_n)$.

Theorem 3.9
Let a prism graph $Y_n, n = 3$, then $\psi_d(Y_n) = \chi(Y_n)$

Proof
The dominator chromatic number of a prism Y_n is

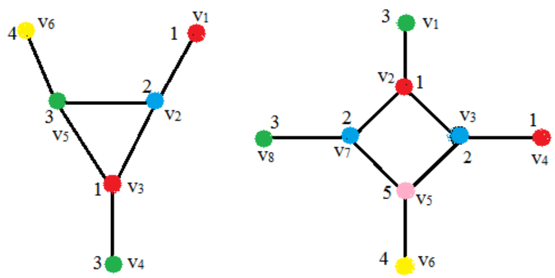
$$Y_n = \begin{cases} n+1 \; for \; n = 9 \;\&\; n = 10 \\ n \; for \; n = 3 \end{cases}$$

The chromatic number of a prism graph Y_n is

$$\chi(Y_n) = \begin{cases} 2 \; when \; n \; is \; even \\ 3 \; when \; n \; is \; odd \end{cases}$$

Hence $n = 3$, then $\psi_d(Y_n) = \chi(Y_n)$.

Sunlet graph



$S_n=3$ $S_n=4$

Fig 3.7

Theorem 3.10
For a sunlet graph $S_n, n = 3, \psi_d(S_n) = n + 1$

Proof
The Vertex set of the sunlet graph is given by $V = v_1, v_2, v_3, v_4, v_5, v_6$ and edge set is given by $E = v_1v_2Uv_2v_3Uv_3v_4Uv_3v_5Uv_5v_6Uv_2v_5$. Now some colors are

assigned to the vertices. Color 1 is assigned to vertices $v_1 \& v_2$, Color 2 to v_2, Color 3 to $v_5 \& v_4$ and Color 4 to v_6. We notice that the vertices v_2, v_4 dominate color class 1; v_1, v_5dominate color class 2; $v_3 \& v_6$ dominate color class 3 & 4 respectively. Hence $\psi_d(S_3) = 4$.

Theorem 3.11
For a sunlet graph $S_n, n = 4, \psi_d(S_n) = n + 1$

Proof
The Vertex set of the sunlet graph is given by $V = v_i \{1 \leq i \leq 8\}$ and edge set
$E = e_1 \, U \, e_2 \, U \, e_3$where $e_1 = v_1v_2 \, U \, v_2v_3 \, U \, v_3v_4, e_2 = v_3v_5 \, U \, v_5v_6 \, U \, v_5v_7, e_3 = v_7v_8 \, U \, v_7v_2$. Now some colors are assigned to the vertices. Color 1 is assigned to the vertices v_2, v_4; Color 2 is assigned to v_3, v_7; Color 3 is assigned to v_1, v_8, Color 4 to v_6 and Color 5 to v_5 .
We notice that the vertices v_1, v_3 dominate color class 1; v_2, v_4, v_5 dominate color class 2; v_7, v_8 dominate color class 5 & 3; $v_6$dominate color class 4. Hence $\psi_d(S_4) = 5$.

Theorem 3.12
For a sunlet graph $S_n, n < 6$,then, $\psi_d(S_n) > \chi(S_n)$.

Proof
The dominator chromatic number of sunlet graph S_n is
$S_n = \{ n + 1$ for $n < 6$
The chromatic number of sunlet graph is $\chi(S_n) = 3$
Hence $\psi_d(S_n) > \chi(S_n)$.

4. Total dominator coloring of Sparse graph

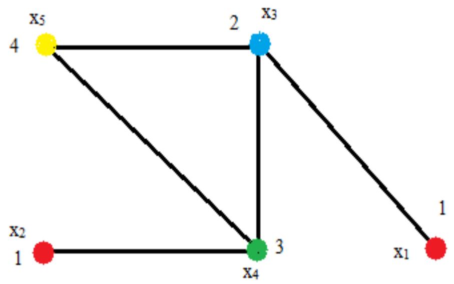


Fig 4.1

Case 1
G is a fig 4.1 sparse graph with $\chi(G) = 3$. The vertices x_1, x_2 are given same color 1. Then the vertices x_3, x_4 are given different color which are not repeated namely 2, 3 and the vertex x_5 is assigned color 4, all the colors assigned are non repeated. Clearly the vertices $\{x_3, x_4\}$ properly dominates x_5. Thus every vertex in G properly dominates an entire color class. Hence this graph G needed at most 4 colors, $\psi_{td}(G) \leq 4$.

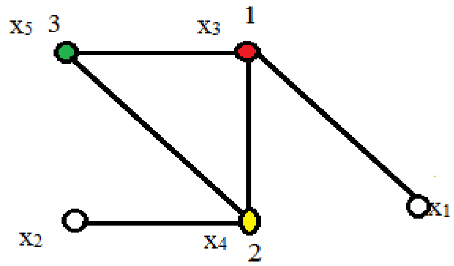


Fig 4.2

Case 2
Suppose $\psi_{td}(G) \leq 3$, then the vertex x_3 is only dominated by the pendant vertex x_1. Hence x_3 is a color class and is non repeated color. Likewise $x_4$is also non repeated color. Vertices $x_3 \& x_4$ are given color 1 & 2, then x_5 receives color 3. The vertices x_3, x_4 has to dominate 3. Hence the end vertices requires one more color, So $\psi_{td}(G) = 4$.

5. Conclusion
In this paper, the dominator coloring $\psi_d(G)$, total dominator coloring $\psi_{td}(G)$ of Prism graph, Sunlet graph and Sparse graph have been discussed. The chromatic number, the dominator chromatic number and the total dominator chromatic number of prism graph, sunlet graph and sparse graph have been examined.
Also the relation between the chromatic number, the dominator chromatic number and the total dominator chromatic number of above mentioned graphs were discussed.

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