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On eccentric connectivity index of certain graphs

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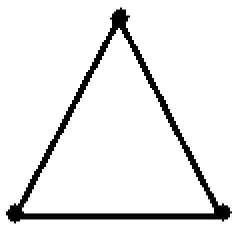
Abstract
Let G be a Connected graph, then the eccentric connectivity index of G is defined as $\xi^{c}(G) = \sum \deg(v)\,.\,ecc(v)$, where deg(v) and ecc(v) denotes the vertex degree and eccentricity respectively.
In this paper, the explicit formula for the values of the eccentric connectivity index for cycle graph have been presented. Next, the eccentric connectivity index of subdivision, subdivision-related graph of Line graph for cycle graph have been calculated.

Keywords: Eccentric connectivity index, subdivision graph, subdivision-related graph

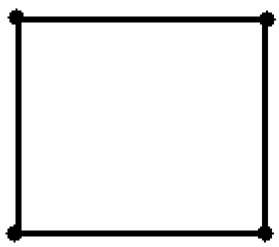
1. Introduction
A Graph G = (V, E) is a finite nonempty set of objects called vertices of G called edges. The vertex set is denoted by V(G) and the edge set is denoted by E(G). The degree of a vertex v in a graph G is the number of edges of graph G incident with v, which is denoted by $deg_G v$ or deg v. In a directed graph, the distance d(u, v) between two vertices u and v and it is defined as the length of a shortest directed path from u to v. The eccentric connectivity ecc(v) of a vertex v ϵ V(G) is the maximum distance u and any other vertex in G. The eccentric connectivity index of a graph G, $\xi^{(c)}(G)$ is defines as $\sum \deg(v).ecc(v)$ where deg(v) is the degree vertex of v and ecc(v) is eccentricity.

2. Cycle Graph
In this dissertation, We consider the cycle graph and their Line graph of cycle graph.

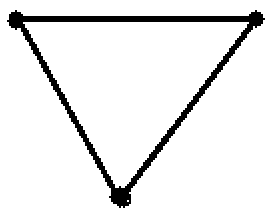
Example
The cycle graph of C_3, C_4, $C_5, and\ C_6$ and Line graph of cycle graphs L[C_3], L[C_4], L[C_5], and L[C_6] are shown in the below figure.



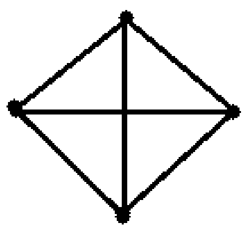
C_3



C_4



L [C_3]



L [C_4]

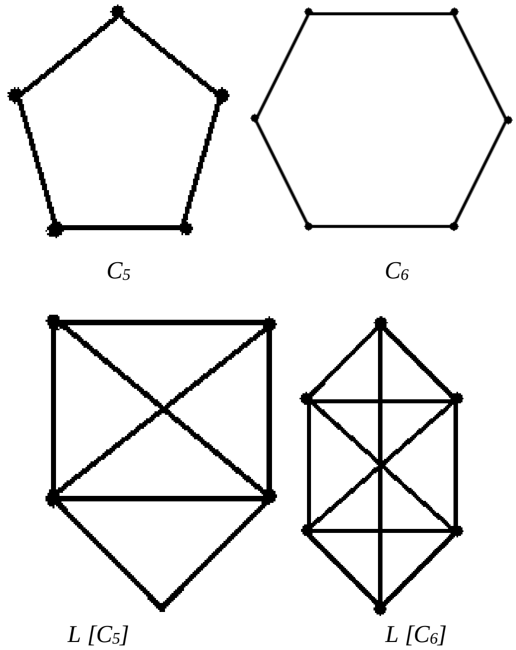


Fig 1: Cycle graphs and their Line graphs

3. Main Results

In this section, we derived an expression for the eccentric connectivity index of the cycle graph and their line graph of subdivision, subdivision-related graph of Line graph for cycle graph have been calculated.

Notations

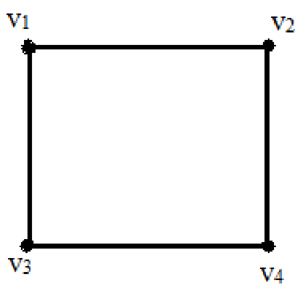
- $\xi^c[(C_n)]$ = Eccentric connectivity index of cycle graph
- $\xi^c[L(C_n)]$ = Eccentric connectivity index of the line graph of cycle Graph
- $\xi^c[S(C_n)]$ = Eccentric connectivity index of the subdivision graph of the cycle graph
- $\xi^c S[L(C_n)]$ = Eccentric connectivity index of the subdivision graph of the Line graph of cycle graph
- $\xi^c[R(C_n)]$ = Eccentric connectivity index of the subdivision-related Graph of cycle graph
- $\xi^c R[L(C_n)]$ = Eccentric connectivity index of the subdivision-related Graph of the line graph of cycle graph

Theorem 3.1: The eccentric connectivity index of the cycle graph is $\xi^c[(C_n)] = 2(n)(n-2)$ for $n \geq 3$

Proof

The cardinality of the vertex set of the cycle graph (C_n) is n. All vertices of the cycle graph have the degree 2, vertices n and eccentric connectivity n-2. The eccentric connectivity index of the cycle graph is obtained by.

$\xi^c(C_n) = 2(n)(n-2)$



Cycle graph C_4

Theorem 3.2: The eccentric connectivity index of a line graph of cycle graph is $\xi^c[L(C_n)] = 4(n^2 - 6n + 11)$ for $n \geq 5$.

Proof

Case (i)

When n = 3 and n = 4

The cardinality of the vertex set of the line graph of cycle graph $[L(C_n)]$ is n.

The degree (n - 1) is of the vertices n have eccentricity 1.

Therefore

$\xi^c[L(C_n)] = (n-1)(n)(1) = n^2 - n$

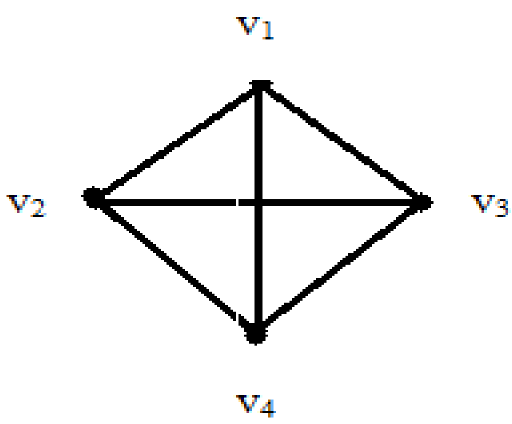
Case (ii)

The cardinality of the vertex set of the line graph of cycle graph $[L(C_n)]$ is n. $(n-2)$ Vertices is of the degree $2(n-4)$ and eccentricity 2.

The degree $\frac{3n}{n}$ of the vertices $\frac{2n}{n}$ and have eccentricity 2.

Hence

$$\xi^c[L(C_n)] = 2(n-4)(n-2)(2) + \left(\frac{3n}{n}\right)\left(\frac{2n}{n}\right)(2)$$
$$= 4n^2 - 8n - 16n + 32 + 12$$
$$= 4n^2 - 24n + 44$$
$$= 4(n^2 - 6n + 11)$$



The Line graph of Cycle graph C_4

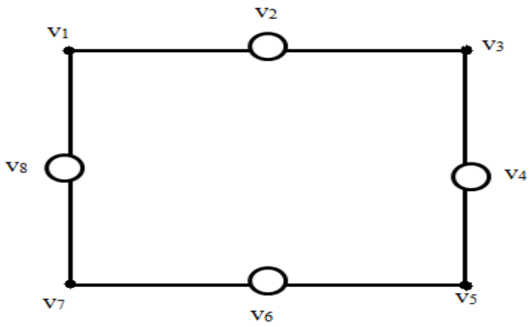
Theorem 3.3: The eccentric connectivity index of the subdivision graph of the cycle graph $S(C_n)$ is $\xi^c[S(C_n)] = 8n^2 - 4n^2$ for $n \geq 4$

Proof

The cardinality of the vertex set of the wheel graph $S(C_n)$ is 2n.

The number of the vertices 2n is of the degree $\frac{2n}{n}$ and its eccentricity is $2n - n$.

Hence $\xi^c[S(C_n)] = 2(n)\left(\frac{2n}{n}\right)(2n-n) = 8n^2 - 4n^2 = 4n^2$



The Subdivision graph of C_4

Theorem 3.4: The eccentric connectivity index of the subdivision graph of line graph of the cycle graph $S(C_n)$ is $\xi^c S[L(C_n)] = 7n^2 - 11n + 2$ for $n \geq 5$

Proof

Case (i)

When n = 3 and n = 4

The cardinality of the vertex set is $(4n - 6)$.

The degree (n-1) of n vertices and its eccentricity 3 and the remaining $3(n-2)$ vertex of the degree 2 and it have eccentricity n.

$\xi^c S[L(C_n)] = (n-1)(n)(3) + 3(n-2)(2)(n)$

$= 3n^2 - 3n + 6n^2 - 12n$

$= 9n^2 - 15n$

Case (ii)

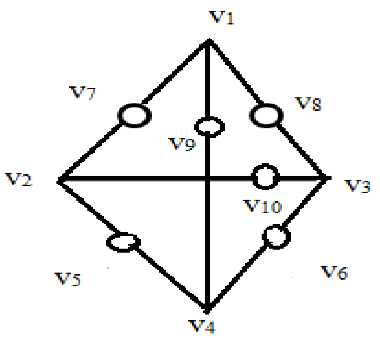
The cardinality of the vertex set is n.

The degree $(n-2)$ of the vertices is 4 and it's have the eccentricity is $2(n-2)$ Vertices of degree 2 and have the eccentricity 4.

The degree $n-3$ have the vertices $n-4$ and its eccentricity 5 and the remaining $n-3$ vertices are of degree 2 and eccentricity is $(n-1)$.

Therefore

$\xi^c S[L(C_n)] = (n-2)(4)(4) + 2(2)(n-2)(4) + (n-3)(n-4)(5) + 2(n-3)(n-1) = 7n^2 - 11n + 2$



The Subdivision graph of Line graph of C_4

Theorem 3.5: The eccentric connectivity index of the subdivision-related graph of the cycle graph $R(C_n)$ is $\xi^c[R(C_n)] = 6n^2 - 10n$ for $n \geq 4$

Proof

Case (i)

When n = 3

The n vertices of degree 4 and have the eccentricity 2.

The number of n vertices of degree 2 and eccentricity 2.

Therefore

$\xi^c[R(C_n)] = 4(n)(2) + (n)(2)(2) = 12n$

Case (ii)

The cardinality of the vertex set of the subdivision-related graph $[R(C_n)]$ is 2n.

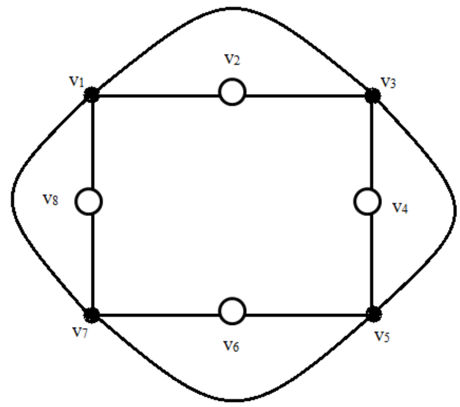
The n vertices are of degree 4 and have eccentricity $(n-2)$.

Now, we compute by considering eccentricity of the vertex set of $[R(C_n)]$.

The n vertices of degree 2 and have eccentricity $(n-1)$.

Hence

$\xi^c[R(C_n)] = 4(n)(n-2) + 2(n)(n-1) = 4n^2 - 8n + 2n^2 - 2n = 6n^2 - 10n$



The Subdivision – Related graph of C_4

Theorem 3.6: The eccentric connectivity index of the subdivision-related graph of line graph of the cycle graph $R(C_n)$ is $\xi^c R[L(C_n)] = 6n^2 - 16n + 46$ for $n \geq 4$

Proof

Case (i)

When n = 3 and n = 4

The cardinality of the vertex set of the subdivision related graph $R[L(C_n)]$ is $(4n - 6)$.

The degree $(2n - 2)$ of the vertices n and the eccentricity is 2.

The eccentricity $(n-1)$ of the vertices $\frac{n(n-1)}{2}$ and have degree 2.

Therefore

$\xi^c[R(C_n)] = (2n-2)(n)(2) + \frac{2n(n-1)}{2}(n-1)$

$= n^3 - n^2 - n^2 + n + 4n^2 - 4n = n^3 + 2n^2 - 3n$

Case (ii)

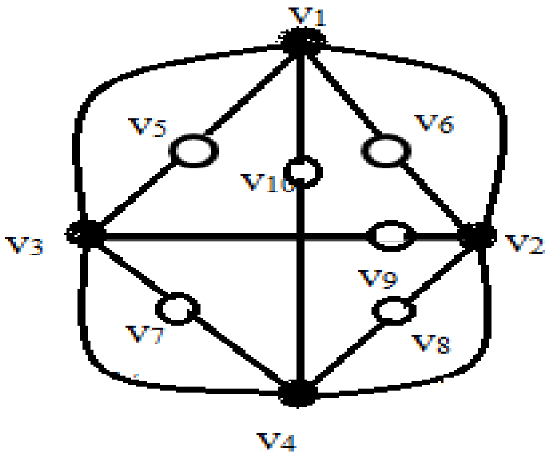
For $n \geq 5$

The cardinality of the vertex set of the subdivision-related graph $R[L(C_n)]$ is $(4n - 7)$.

The degree $(2n - 4)$ of the vertices 2 have its eccentricity as 2.

The vertices 2 of the degree $\frac{8n}{n}$ have the eccentricity 2.
The eccentricity is 3 for $n-4$ vertices of degree is $2n-6$.
The vertices $\frac{4n}{n}$ of the graph of degree 2 have the eccentricity 3.
The vertices $2n-8$ of the degree 2 have the eccentricity 3.
The degree 2 of the vertices $n-3$ have its eccentricity as 3.
Hence we have

$\xi^c\, R[L(C_n)] = (2n-4)(2)(2) + \frac{8n}{2}(2)(2) + (2n-6)(n-4)(3) + 2\left(\frac{4n}{n}\right)(3) + 2(2n-8)(3) + 2(n-3)(3)$
$= 6n^2 - 16n + 46$



The Subdivision – Related graph of Line graph of C_4

4. Conclusion

In this dissertation, the eccentric connectivity index and modified eccentric connectivity index have been calculated for some classes of graph, such as cycle graph. Subdivision graph and subdivision-related graph for cycle graph have been calculated. Also Line graph, subdivision graph of line graph and subdivision-related graph of line graph for cycle graph have been calculated.

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