International Journal of Applied Research 2017; 3(12): 243-249

ISSN Print: 2394-7500
ISSN Online: 2394-5869
Impact Factor: 5.2
IJAR 2017; 3(12): 243-249
www.allresearchjournal.com
Received: 09-10-2017
Accepted: 10-11-2017

S Lakshmi
Assistant Professor,
Department of Mathematical (PG&SF) PSGR Krishnammal College for Women Coimbatore, Tamil Nadu, India

S Priya
Research Scholar,
Department of Mathematics PSGR Krishnammal College For Women Coimbatore, Tamil Nadu, India

Correspondence
S Lakshmi
Assistant Professor,
Department of Mathematical (PG&SF) PSGR Krishnammal College for Women Coimbatore, Tamil Nadu, India

Elegant labeling of some special line graph

S Lakshmi and S Priya

Abstract

An elegant labeling g of graph G with ‘v’ edges an injective function from the vertices of G to the set {0,1,2,....... v} such that when each edge (e=uv) is assigned the label {(g(x)+g(y)) mod (v+1)} the resulting edge labels are distinct and non-zero. In this paper it is shown to be certain families of line graphs are elegant graphs are elegant graphs.

Keywords: Path graph, p2n, comb graph, Hnn, Bnn

1. Introduction

We consider all graphs are finite, simple and undirected. The G has ‘V” vertice and ‘E’ edges. A Graph labeling is an assignment of integers to the vertices or edges or both subject to certain condition. We refer to survey on graph labelling by Gallian [5]. Chang, Rogers and Hsu introduced elegant labeling (1981).The elegant labeling is a variation of harmonious labeling, Balakrishnan and Sampathkumar Gallian, Lee *et al* are found harmonious labeling and felicitious labeling of graphs.

The elegantness is also possible if n value is even found by Balakrishnan selvan, Yengnanaryan. They are apply this result on Hnn, Bnn. Recently Pn2, PnK have shown also elegant graphs by V Laxmi, Alias Gomathi, N murugan and A. Nagrajan.

2. Main Results

Theorem 2.1

The line graph of (P^2n -e) is an elegant graph if n=1 (mod 2) n ≥3 and e = n-4

Proof:

Let G=(P^2n) be a graph.The graph contains u vertices and v edges. The line graph of Pn^2 is denoted by L(P^2n) {x1,x2,x3,................xn;y1,y2,y3,...........yn}be the (e1,e2,e3........en}

We define the function g: V(G)→{0,1,2,........v}

g(x1)=0 g(x2)=1

g(x2i+1)=5i for 1≤i≤(v-u) / 2 g(x2i+2)=5i+1 for 1≤i≤(v-u) / 2 g(y1)=2

g(y2i)=4i+j for 1≤i≤(v-u)/2 and 0≤j≤((v-u)/2)-1 g(y2i+1)=7i-j for 1≤i≤(v-u)/2 and j=0,2,4,......(n-5)

In this labeling easy to verify that all the vertex labels are different. We get the edge label in the forn {1,2,.......V}. No zero integers and edge labels are not repeated.

Hence L{P^2n-e} is a elegant labeling graph

Example 1

L(P24) is a elegant labeling graphs Here n=4 e=n-4

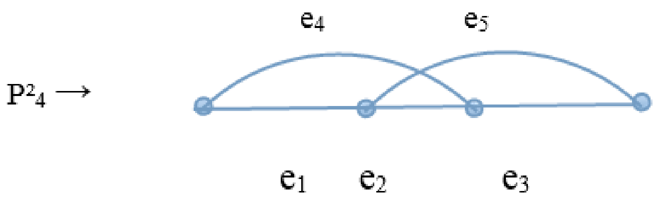

Fig 2.1

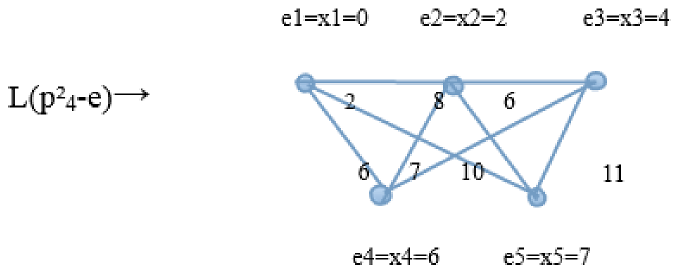

Fig 2.2

Example 2

L (P²5) is a elegant labeling graph Here n=5 e=n-6

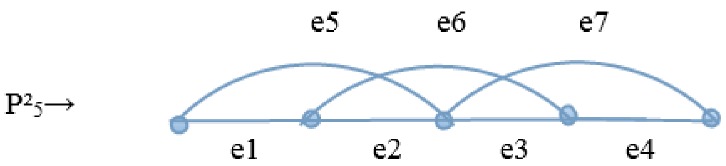

Fig 2.3

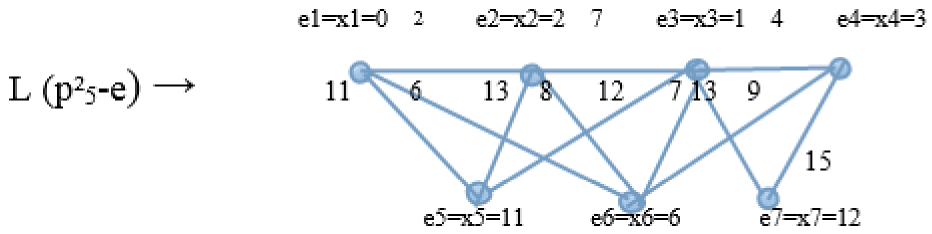

Fig 2.4

Theorem 2.2

The line graph of (P2n-e) is an elegant graph if n=0(mod 2), n ≥ 4 and (e=n-4)

Proof

Let G=(P2n) be a graph with u vertices and v edges and its line. The line graph of P2n is denoted by L(P2n). The vertices are {x1,x2,......xn;y1,y2,.....yn}and the edge set is{e1,e2......en}

The labeling function is defined by

g=V(G)→{0,1,2,..........V}

$$g(x1+2i)=5i \quad \text{for} \quad 0\le i\le(n/2-1)$$
$$g(x2+2i)=5i+3 \quad \text{for} \quad 0\le i\le(n/2-1)$$
$$g(y1+2i)=5i+1 \quad \text{for} \quad 0\le i\le(n/2-1)$$
$$g(y2+2i)=5i+2 \quad \text{for} \quad 0\le i\le(n/2-2)$$

In this defined labeling we can easy to verify that all the vertex labels are distinct values. We get the edge label in the forn {1,2,3,.....V} non zero integers and the edge labels are not repeated Hence L(P2n-e) is a elegant labeling graph

Example 1.

L(p²5) is a elegant labeling graph Here n=5,e=n-4,e=1

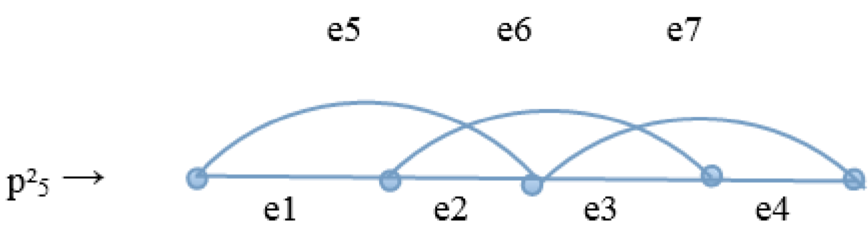

Fig 2.5

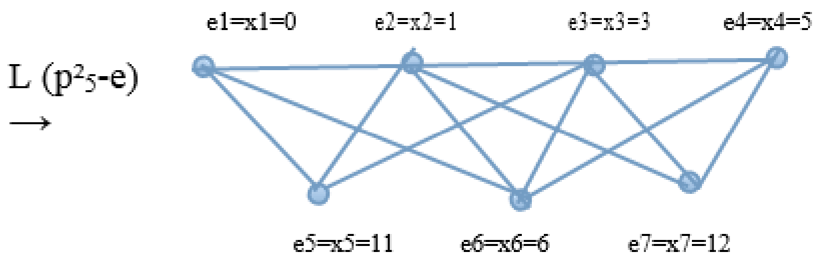

Fig 2.6

Example 2
$L(p^2 7)$ is a elegant labeling graph Here n=7,e=n-4,e=3

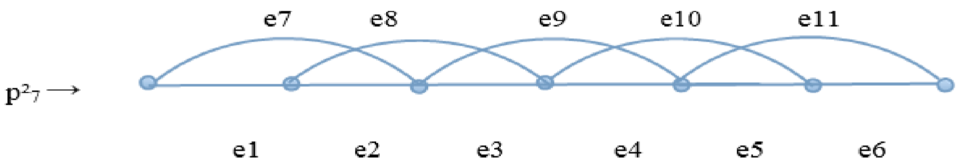

Fig 2.7

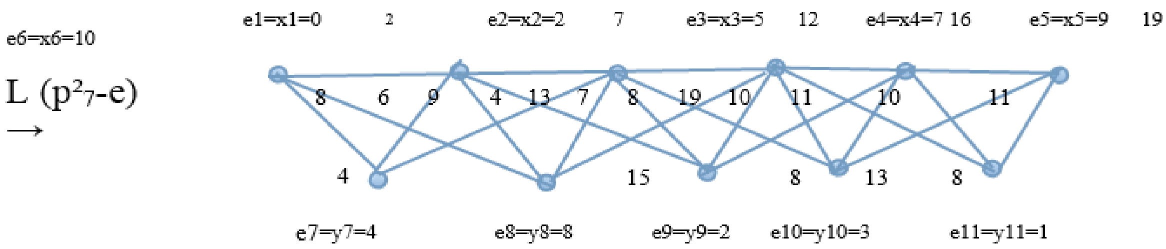

Fig 2.8

Example 3
$L(p^2 8)$ is a elegant labeling graph Here n=8,e=n-4,e=4

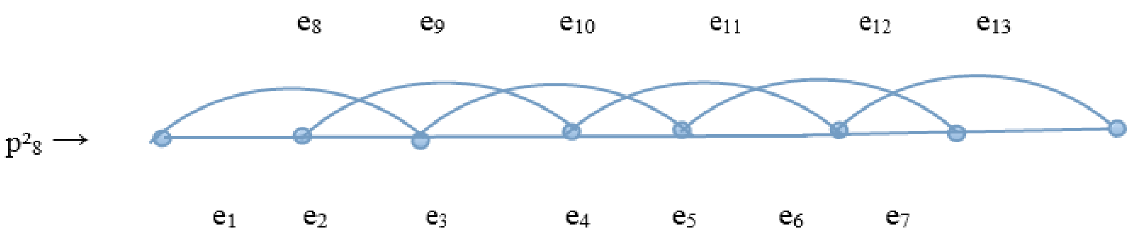

Fig 2.9

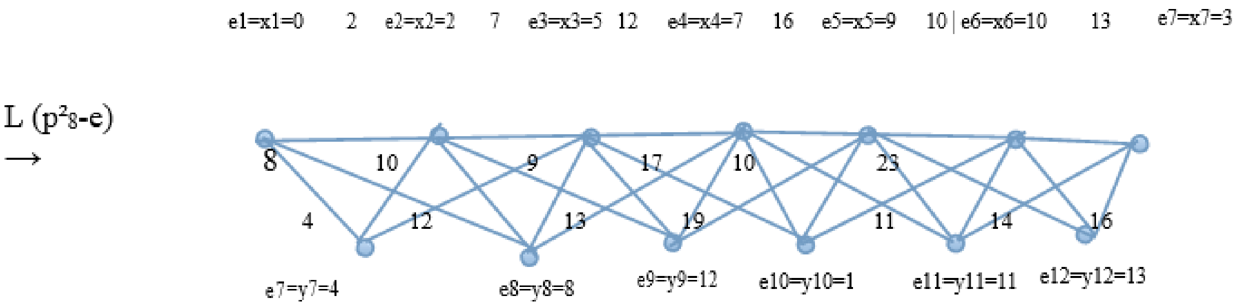

Fig 2.10

Theorem 2.3
The line graph of(PnK1) is an elegant graph if $n \leq 5$
Proof:
Let G=(Pn K1)be a comb graph with 'u' vertices and v edges.
The line graph of Pn K1 is denoted by L (Pn K1).
{x0,x1,x2,..............xn,y0,y1,.......yn}and edge set {e1,e2,,...........en}
The labeling function is as follows g=V(G)→{0,1,2,3......q}

Case 1:n=2
g(x1)=0 g(y1)=1 g(y2)=2
Case 2:n=3
g(xj)=j for 0≤ j ≤n-2 g(yi)=j+1 for 0≤ j ≤n
Case 3: n=4
g(x1+2j)=5i+1 for j=0,1 g(x2)=2
g(yj+2j)=5j for j=0,1 g(y2+2j)=3j+4 for j=0,1
Case 4: n=5
g(xj)=j for 0≤ j ≤n-2
g(y1+2j)=(n-1)+j(j+1)+i for 0≤ j ≤(n-1)/2 g(y2j)=(n-1)+2j for j=1,2

The vertex labeling pattern above which covers all the vertices with distinct values The edge label in the form {1,2,3.....V}non zero + integers and the edge labels are not repeated
Hence L(PnK1) is an elegant labeling.

Example 1
L(P4 K1) is a elegant labeling graph Here n=4 P4 K1

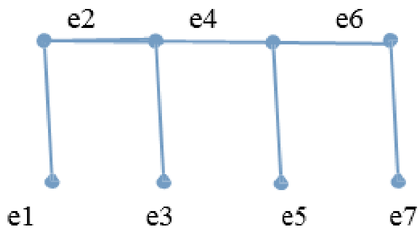

Fig 2.11

L (P4 K1)

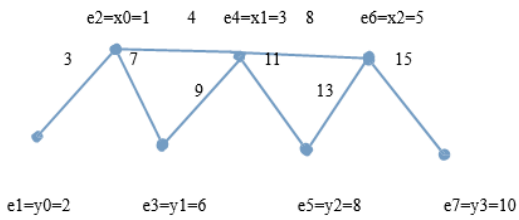

Fig 2.12

Example 2
L (P3 K1) is a elegant labeling graph Here n=3e2 e4 P3k1→

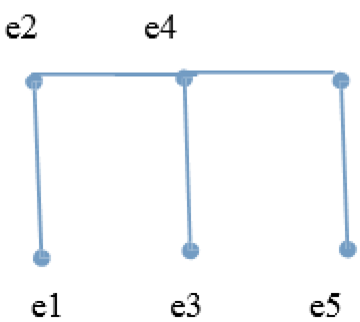

Fig 2.13

L (P3 K1) →

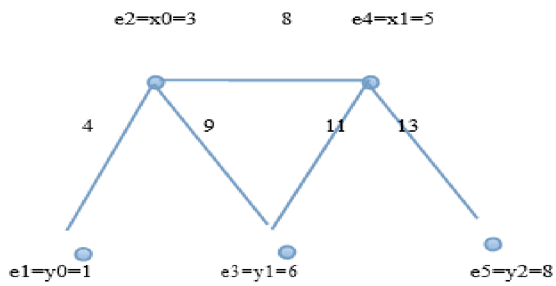

Fig 2.14

Theorem 2.4
The line graph of (Hn,n) is a elegant graph if n ≤ 3

Proof:
Let G be a (Hn,n) graph
The graph G have u vertices and v edges The line graph of (Hn,n) is denoted by L(Hn,n)
The vertex set is {x1,x2,x3....xn} vertices set and edge set is{e1,e2,....en}
The labeling function is defined as follows g:V(G)→{0,1,2.......n-1}
Case 1:n=2
g(x1)=0 g(x2)=1 g(x3)=2
Case 2: n=3
g(x1)=6 g(x2)=0
g(x2+j)=j for j=1,2,3,4
Such that the vertex labels are distinct
We get edge label in the forn {1,2,3.....V}non zero integers and the edge label is not repeated
Hence L(Hn,n)is an elegant labeling graph

Example: 1
L(H2,2)is a elegant labeling graph Here n=2
(H2,2)→

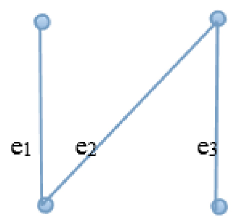

Fig 2.15

L (H2, 2) →

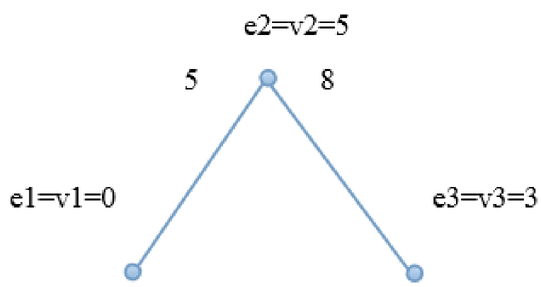

Fig 2.16

Theorem 2.4
The line graph of Pn is an elegant graph if n=0 (mod 2)
Proof:
The line graph of (Pn) be is denoted by L(Pn) The line graph L(Pn) has u vertices and (V-1)edges We define the labeling function g=V(G)→{0,1,2......V}as follows
g(xj)=(u-1/2 +j) nod (v+1) for 1≤j≤u
The vertex labeling pattern defined above which covers all the vertices with distinct numbers.
The label vertices fron right to left side of the path The edge label in the forn of (1,2,3....v}

Example 1

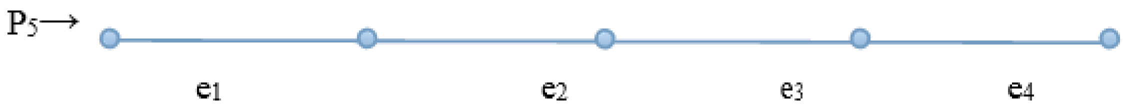

Fig 2.17

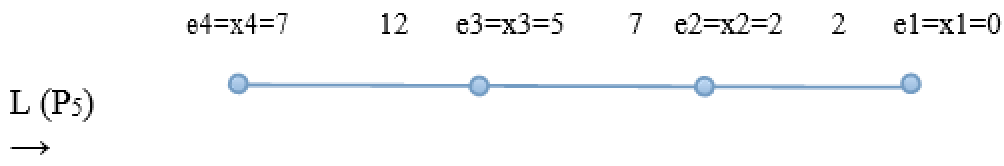

Fig 2.18

Example: 2

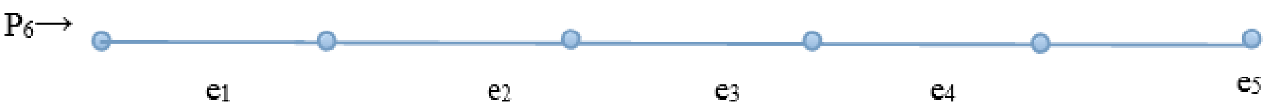

Fig 2.19

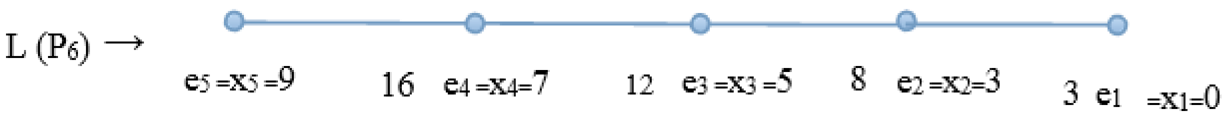

Fig 2.20

Theorem 2.5
The line graph of Bn,n is a elegant graph if n=2

Proof:
Let G be a bistargraph with x1, x2,................xn be the vertices and edges e1,e2..................en The line graph of G is denoted by L(Bn, n)
It has V vertices and E edge
The labeling function is defined as
g:V(G)------------.>{0,1,2.....v} as follows,
g(xj)=0
g(vj+1)=3j+1 for j=0,1 g(vj+1)=3j+3 for j=0,1
We can label the vertices with different values and we get the edge label in the forn{1,2,3,....v}non zero integers and the edge label is not repeated.
Hence L(Bn,n) is a elegant labeling graph

Example: 1

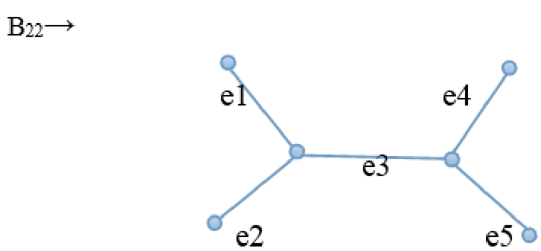

Fig 2.21

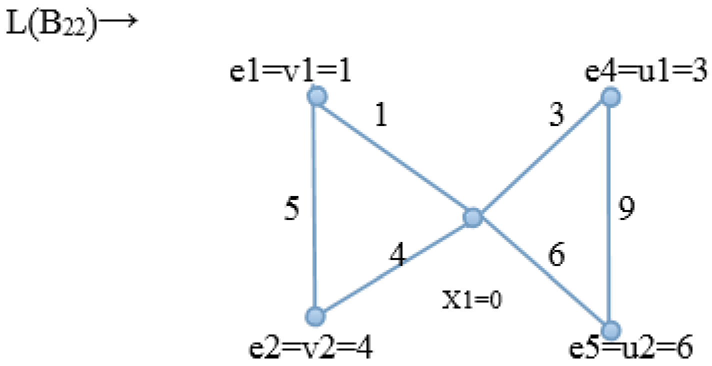

Fig 2.22

3. Conclusion

In this paper we have shown that line graphs of Path graph, p2n**,** comb graph, Hnn, Bnn are elegant graphs.

4. Reference

1. Chang CJ, Hsu DF, Rogers DG. Congressus numeratium. 1991; 32:81-97
2. Lee SM, Schmeihel E, Shee SC. Discrete Math. 1991; 93:201-09.
3. Elumalai A, Sethuraman G. Elegant labeled graphs, Jinform. Math. Sci. 2010; 2:45-49.
4. Balakrishnan R, Sampathkumar R. utilitas Mathematics. 1994; 46:97-102.
5. Balakrishnan A, Selvan, Yegnarayanan V. Some result on elegant graphs, Ars combin. 2002; 65:177-197.