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**P Sathishmohan**
Assistant Professor of Mathematics, Kongunadu Arts and Science College, Coimbatore, Tamil Nadu, India

**V Rajendran**
Assistant Professor of Mathematics, Kongunadu Arts and Science College, Coimbatore, Tamil Nadu, India

**A Devika**
Associate Professor of Mathematics, PSG College of Arts and Science, Coimbatore, Tamil Nadu, India

**R Vani**
Assistant Professor of Mathematics with CA, PSG College of Arts and Science, Coimbatore, Tamil Nadu, India

**Correspondence**
**P Sathishmohan**
Assistant Professor of Mathematics, Kongunadu Arts and Science College, Coimbatore, Tamil Nadu, India

# On nano semi-continuity and nano pre-continuity

**P Sathishmohan, V Rajendran, A Devika and R Vani**

**Abstract**
The purpose of this paper to propose a new class of functions called nano semi-continuous functions and nano semi-continuous functions and also derive their characterizations in terms of nano semi-closed sets, nano semi-closure and nano semi-interior. There is also an attempt to define nano semi-open maps, nano semi-closed maps and nano semi-homeomorphism.

**Keywords:** Nano semi-open sets, nano semi-closed sets, nano semi-interior, nano semi-closure, nano semi-continuous functions, nano pre-continuous functions, nano semi-open maps, nano semi-closed maps, nano semi- homeomorphism.
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## 1. Introduction
Continuity of functions is one of the core concepts of topology. In general, a continuous function is one, for which small changes in the input result in small changes in the output. The notion of Nano topology was introduced by Lellis Thivagar [2, 3], which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it. He has also defined nano closed sets, nano-interior, nano closure and nano continuity. In this paper we have introduced a new class of functions on nano topological spaces called nano semi-continuous functions and nano pre-continuous functions and also derived their characterizations in terms of nano semi-closed sets, nano semi-closure and nano semi-interior. We have also established nano semi-open maps, nano semi-closed maps and nano semi-homeomorphisms and their representations in terms of nano semi-closure and nano semi-interior.

## 2. Preliminaries
**Definition 2.1** [4] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let X $\subseteq$ U.

1. The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R$ (X). That is, $L_R$ (X) = $\bigcup_{x\in U}\{R(x): R(x)\subseteq X\}$, where R(x) denotes the X ∈U equivalence class determined by x ∈ U.
2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R$(X). That is, $U_R$(X) = $\bigcup_{x\in U}\{R(x): R(x)\cap X \neq \phi\}$.
3. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not-X with respect to R and it is denoted by $B_R$(X). That is, $B_R(X) = U_R(X) - L_R(X)$.

**Property 2.2** [4] If (U, R) is an approximation space and X, Y⊆U, then
i) $L_R(X) \subseteq X \subseteq U_R(X)$
ii) $L_R(\varphi) = U_R(\varphi) = \varphi$
iii) $L_R(U) = U_R(U) = U$
iv) $U_R(X\cup Y) = U_R(X) \cup U_R(Y)$

v) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$
vi) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$
vii) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
viii) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$
ix) $U_R(X^C) = [L_R(X)]^C$ and $L_R(X^C) = [U_R(X)]^C$
x) $U_R(U_R(X)) = L_R(U_R(X)) = U_R(X)$
xi) $L_R(L_R(X)) = U_R(L_R(X)) = L_R(X)$

**Definition 2.3** [2] Let U be a non-empty, finite universe of objects and R be an equivalence relation on U. Let $X \subseteq U$. Let $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$. Then $\tau_R(X)$ is a topology on U, called as the nano topology with respect to X. Elements of the nano topology are known as the nano – open sets in U and $(U, \tau_R(X))$ is called topological space. $[\tau_R(X)]^C$ is called as the dual nano topology of $\tau_R(X)$. Elements of $[\tau_R (X)]^C$ are called as nano closed sets.

**Remark 2.4** [2] The basis for the nano topology $\tau_R(X)$ with respect to X is given by $\beta_R(X) =\{U, L_R(X), B_R(X)\}$.

**Definition 2.5** [2] If $(U,\tau_R(X))$ is a nano topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then the nano interior of A is defined as the union of all nano-open subsets of A and it is denoted by NInt(A). That is, NInt(A) is the largest nano-open subset of A. The nano closure of A is defined as the intersection of all nano closed sets containing A and it is denoted by NCl(A). That is, NCl(A) is the smallest nano closed set containing A.

**Definition 2.6** [3] Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be nano topological spaces. Then a mapping $f:(U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y)$, is nano continuous on U if the inverse image of every nano-open set in V is nano open set in U.

**Definition 2.7** [1] If $(U, \tau_R(X))$ is a nano topological space and $A \subseteq U$. Then A is said to be
i) nano semi-open if $A \subseteq Ncl(Nint(A))$
ii) nano pre-open if $A \subseteq Nint(Ncl(A))$
iii) nano semi pre-open if $A \subseteq Ncl(Nint(Ncl(A)))$

Throughout this paper, U and V are non-empty, finite universes;
$X \subseteq U$ and $Y \subseteq V$; U/R and V/R'denote the families of equivalence classes by equivalence relations R and R' respectively on U and V. $(U, \tau_R (X))$ and
$(V, \tau_R(Y))$ are the nano topological spaces with respect to X and Y respectively.

### 3. Nano semi-continuity and Nano pre-continuity

**Definition 3.1** Let $(U,\tau_R(X))$ and $(V,\tau_{R'}(Y))$ be nano topological spaces. Then a mapping $f:(U,\tau_R(X)) \rightarrow (V,\tau_{R'}(Y)$, is

1. Nano semi-continuous, $f^{-1}(A)$ is nano semi-open on U for every nano open set in V.
2. Nano pre-continuous, $f^{-1}(A)$ is nano pre-open on U for every nano open set in V.

**Definition 3.2** Let U = {1, 2, 3, 4} with U/R = {{1}, {4}, {2, 3}}. Let X = {1, 3} $\subseteq$ U. Then $\tau_R(X)$ = {U, ∅, {1}, {2,3}, {1,2,3}} and semi open $\tau_R(X)$ = {U, ∅, {1}, {4}, {2,3}, {1,2,3},{2,3,4}}. Let V = {a,b,c,d} with V/R' = {{a}, {c}, {b,d}} and Y = {b,d}. Then $\tau_{R'}$ (Y)={V, ∅,{b,d}}. Define f: U → V f as f(1) = a, f(2) = b, f(3) = d, f(4) = c. Then $f^{-1}$ ({b, d}) = {2, 3}. That is, the inverse image of every nano-open set in V is nano semi-open in U. Therefore, f is nano semi-continuous.

The following theorem characterizes nano semi-continuous functions in terms of nano closed sets.

**Theorem 3.3** A function $f : (U,\tau_R(X)) \rightarrow (V,\tau_{R'}(Y))$ is nano semi-continuous if and only if the inverse image of every nano closed set in V is nano semi-closed in U.

**Proof:** Let f be nano semi-continuous and F be nano closed in V. That is, V - F is nano-open in V Since f is nano semi-continuous, $f^{-1}(V- F)$ is nano semi-open in U. That is, U - $f^{-1}(F)$ is nano semi-open in U. Therefore, $f^{-1}(F)$ is nano semi-closed in U. Thus, the inverse image of every nano closed set in V is nano semi-closed in U, if f is nano semi-continuous on U. Conversely, let the inverse image of every nano closed set be nano semi-closed. Let G be nano-open in V. Then V - G is nano closed in V. Then, $f^{-1}(V- G)$ is nano semi-closed in U. That is, U - $f^{-1}(G)$ is nano semi-closed in U. Therefore, $f^{-1}(G)$ is nano semi-open in U. Thus, the inverse image of every nano-open set in V is nano semi-open in U. That is, f is nano semi-continuous on U.

In the following theorem, we establish a characterization of nano semi-continuous functions in terms of nano semi-closure.

**Theorem 3.4** A function $f: (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is nano semi-continuous if and only if $f(NClNint(A)) \subseteq NClNint(f(A))$ for every subset A of U.

**Proof:** Let f be nano semi-continuous and $A \subseteq U$. Then $f(A) \subseteq V$. NClNint (f(A)) is nano closed in V. Since f is nano semi-continuous, $f^{-1}(NClNint(f (A)))$ is nano semi-closed in U. Since $f(A) \subseteq NClNint(f (A))$, $A \subseteq f^{-1}(NClNint(f (A)))$. Thus $f^{-1}(NClNint(f(A))$ is a nano semi-closed set containing A. But, NClNint(A) is the smallest nano semi-closed set containing A. Therefore $NClNint(A) \subseteq f^{-1}(NClNint(A))$. That is, $f(NClNint(A)) \subseteq NClNint(f(A))$. Conversely, let $f(NClNint(A)) \subseteq NClNint(f(A))$ for every subset A of U. If F is nano closed in V, since $f^{-1}(F) \subseteq U$, $f(NClNint(f^{-1}(F))) \subseteq NClNint(f(f^{-1}(F))) \subseteq NClNint(F)$. That is, $NClNint(f^{-1}(F)) \subseteq f^{-1}(NClNint(F)) = f^{-1}(F))$, since F is nano closed.Thus $NClNint(f^{-1}(F) \subseteq f^{-1}(F)$. But $f^{-1}(F) \subseteq NClNint(f^{-1}(F))$.
Therefore, $NClNint(f^{-1}(F)) = f^{-1}(F)$. Therefore, $f^{-1}(F)$ is nano semi-closed in U for every nano closed set F in V. That is, f is nano semi-continuous.

**Remark 3.5** If $f: (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is nano semi-continuous, then f (NClNint(A)) is not necessary equal to NClNint(f(A)) where $A \subseteq U$.

**Example 3.6** In Example 3.2, Let A = {1, 2}$\subseteq$U. Then f (NClNint(A)) = f ({1,2}) = f(U) = V. But, NClNint(f (A)) = NClNint(f{1,2}) = NClNint{a,d} = {a}. Thus, f (NClNint(A) ≠NClNint( f (A)), even though f is nano semi-continuous. That is, equality does not hold in the previous theorem when f is nano semi-continuous.

**Theorem 3.7** Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be two nano topological spaces where $X \subseteq U$ and $Y \subseteq V$. Then $\tau_{R'}(Y)$ = {V, ∅, $L_{R'}(Y)$, $U_{R'}(Y)$, $B_{R'}(Y)$} and its basis is given by $B_{R'}$

$=\{V, L_{R'}(Y), B_{R'}(Y)\}$. A function f: $(U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is nano semi-continuous if and only if the inverse image of every member of $B_{R'}$ is nano semi-open in U.

**Proof:** Let f be nano semi-continuous on U. Let $B \in B_{R'}$. Then B is nano-open in V. That is, $B \in \tau_{R'}(Y)$. Since f is nano semi-continuous, $f^{-1}(B) \in \tau_R(X)$. That is, the inverse image of every member of $B_{R'}$ is nano semi-open in U. Conversely, let the inverse image of every member of $B_{R'}$ be nano-open in U. Let G be a nano-open in V. Then $G = \cup\{B: B \in B_1\}$ where $B_1 \subset B_{R'}$. Then $f^{-1}(G) = f^{-1}(\cup\{B : B \in B_1\}) = \cup\{f^{-1}(B) : B \in B_1\}$, where each $f^{-1}(B)$ is nano semi-open in U and hence their union, which is $f^{-1}(G)$ is nano semi-open in U. Thus f is nano semi-continuous on U.

The above theorem characterizes nano semi-continuous functions in terms of basic elements. In the following theorem, we characterize nano semi-continuous functions in terms of inverse image of nano semi-closure.

**Theorem 3.8** A function $f:(U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is nano semi-continuous if and only if $NClNint(f^{-1}(B)) \subseteq f^{-1}(NClNint(B))$ for every subset B of V.

**Proof:** If f is nano semi-continuous and $B \subseteq V$, NClNint(B) is nano closed in V and hence $f^{-1}(NClNint(B))$ is nano semi-closed in U. Therefore, $NClNint[f^{-1}(NClNint(B))] = f^{-1}(NClNint(B))$. Since $B \subseteq NClNint(B)$, $f^{-1}(B) \subseteq f^{-1}(NClNint(B))$. Therefore, $NClNint(f^{-1}(B)) \subseteq NClNint(f^{-1}(NClNint(B))) = f^{-1}(NClNint(B))$. That is, $NClNint(f^{-1}(B)) \subseteq f^{-1}(NClNint(B))$. Conversely, let $NClNint(f^{-1}(B)) \subseteq f^{-1}(NClNint(B))$ for every $B \subseteq V$. Let B be nano closed in V. Then NCl(B) = B. By assumption, $NClNintf^{-1}(B) \subseteq f^{-1}(NCl(B)) = f^{-1}(B)$. Thus, $NClNintf^{-1}(B) \subseteq f^{-1}(B)$. But $f^{-1}(B) \subseteq NClNint(f^{-1}(B))$. Therefore, $NClNint(f^{-1}(B)) = f^{-1}(B)$. That is, $f^{-1}(B)$ is nano semi-closed in U for every nano closed set B in V. Therefore, f is nano semi-continuous on U.

The following theorem establishes a criteria for nano pre-continuous functions in terms of inverse image of nano interior of a subset of V.

**Theorem 3.9** A function $f:(U,\tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is nano pre-continuous on U if and only if $f^{-1}(NIntNcl(B)) \subseteq NIntNcl(f^{-1}(B))$ for every subset B of V.

**Proof:** Let f be nano pre-continuous and $B \subseteq V$. Then NIntNcl(B) is nano-open in $(V,\tau_{R'}(Y))$. Therefore $f^{-1}(NIntNcl(B))$ is nano pre-open in $(U,\tau_R(X))$. That is, $f^{-1}(NIntNcl(B)) = NInt[f^{-1}(NIntNcl(B))]$. Also, $NIntNcl(B) \subseteq B$ implies that $f^{-1}(NIntNcl(B)) \subseteq f^{-1}(B)$. Therefore $NIntNcl[f^{-1}(NIntNcl(B))] \subseteq NIntNcl(f^{-1}(B))$.That is, $f^{-1}(NIntNcl(B)) \subseteq NIntNcl(f^{-1}(B))$. Conversely, Let $f^{-1}(NIntNcl(B)) \subseteq NIntNcl(f^{-1}(B))$ for every subset B of V. If B is nano-open in V, NIntNcl (B) = B. Also, $f^{-1}(NIntNcl(B)) \subseteq NIntNcl(f^{-1}(B))$. That is, $f^{-1}(B) \subseteq NIntNcl(f^{-1}(B))$. But $NIntNcl(f^{-1}(B)) \subseteq f^{-1}(B)$.Therefore, $f^{-1}(B) = NIntNcl(f^{-1}(B))$. Thus, $f^{-1}(B)$ is nano pre-open in U for every nano-open set B in V. Therefore, f is nano pre-continuous.

**Example 3.10** Let U = {a, b, c, d}with U/R = {{a, d}, {b}, {c}}. Let X = {a, c}⊆U. Then the nano topology, $\tau_R(X)$ with respect to X is given by {U,∅,{c},{a,c,d},{a,d}} and hence the nano closed sets in U are U,∅,{a,b,d},{b} and {b,c}. Let V = {x, y, z, w} with V/R′ = {{x}, {y}, {z}, {w}}. Let Y = {x, w} ⊆V. Then the nano topology on V with respect to Y is given by $\tau_{R'}(Y)$ = {V,∅, {x,w}}, and the nano closed sets in V are V,∅ and {y, z}. Define f:U→V as f (a) = x, f (b) = y, f (c) = z and f (d) = w. Then f is nano continuous on U, since inverse image of every nano-open set in V is nano-open in U. Let B = {xyz} ⊂V. Then $f^{-1}(NClNint(B)) = f^{-1}(\{y, z\})$ = {b,c} and $NClNint(f^{-1}(B))$ = {∅}. Thus, $NClNint(f^{-1}(B)) \neq f^{-1}(NClNint(B)$. Also when A = {z} ⊆V, $f^{-1}(NIntNcl (A)) = f^{-1}(\{y,z\})$ = {b,c} but $NIntNcl(f^{-1}(A))$ = NIntNcl({c}) = {c}. That is, $f^{-1}(NIntNcl (A)) \neq NIntNcl(f^{-1}(A))$. Thus, equality does not hold in Theorems 3.8 and 3.9 when f is nano semi-continuous and nano pre-continuous.

**Theorem 3.11** If $(U, \tau_R(X))$ and $(V,\tau_{R'}(Y))$ are nano topological spaces with respect to X⊆U and Y ⊆V respectively, then for any function f :U→V, the following are equivalent:

1. f is nano semi-continuous.
2. The inverse image of every nano closed set in V is nano semi-closed in U.
3. $f(NClNint(A)) \subset NClNint(f (A))$ for every subset A of V.
4. The inverse image of every member of the basis $B_{R'}$ of $\tau_R$ (Y) is nano semi-open in U.
5. NClNint $(f^{-1}(B)) \subseteq f^{-1}(NClNint(B))$ for every subset B of V.
6. Proof of the theorem follows from Theorems 3.3 to 3.9.

## 4. Nano semi-open maps, Nano semi-closed maps and Nano semi-homeomorphism

**Definition 4.1** A function $f:(U,\tau_R(X)) \rightarrow (V,\tau_{R'}(Y))$ is a nano semi-open map if the image of every nano semi-open set in U is nano-open in V. The mapping f is said to be a nano semi-closed map if the image of every nano semi-closed set in U is nano closed in V.

**Theorem 4.2** A mapping $f:(U,\tau_R(X)) \rightarrow (V,\tau_{R'}(Y))$ is nano semi-closed map if and only if $NClNint(f(A)) \subseteq f(NClNint(A))$, for every subset A of U.

**Proof:** If f is nano semi-closed, f(NClNint(A)) is nano closed in V, since NClNint(A) is nano semi-closed in U. Since $A \subseteq NClNint(A)$, $f(A) \subseteq f(NClNint(A))$. Thus f(NClNint(A)) is a nano closed set containing f(A). Therefore, $NClNint(f(A)) \subseteq f(NClNint(A))$. Conversely, if $NClNint(f(A)) \subseteq f(NClNint(A))$ for every subset A of U and if F is nano semi-closed in U, then NClNint(F) = F and hence $f(F) \subseteq NClNint(f(F)) \subseteq f(NClNint(F)) \subseteq f(Ncl(F)) = f(F)$. Thus, f (F) = NCl(f(F)). That is, f (F) is nano closed in V. Therefore, f is a nano semi-closed map.

**Theorem 4.3** A mapping $f:(U,\tau_R(X)) \rightarrow (V,\tau_{R'}(Y))$ is nano semi-open map if and only if $f(NIntNcl(A)) \subseteq NIntNcl(f(A))$, for every subset A⊆U.

Proof is similar to that of Theorem 4.2.

**Definition 4.4** A function $f:(U,\tau_R(X)) \rightarrow (V,\tau_{R'}(Y))$ is said to be a nano semi-homeomorphism if

1. f is 1-1 and onto
2. f is nano semi-continuous and
3. f is nano semi-open

**Theorem 4.5** Let f:$(U,\tau_R(X))\rightarrow(V,\tau_{R'}(Y))$ be a one-one onto mapping. Then f is a nano semi-homeomorphism if and only if f is nano semi-closed and nano semi-continuous.

**Proof:** Let f be a nano semi-homeomorphism. Then f is nano semi-continuous. Let F be an arbitrary nano semi-closed set in $(U,\tau_R(X))$. Then U -F is nano semi-open. Since f is nano semi-open, f (U− F) is nano- open in V. That is, V −f(F) is nano-open in V. Therefore, f(F) is nano-closed in V. Thus, the image of every nano semi-closed set in U is nano closed in V. That is, f is nano semi-closed. Conversely, let f be nano semi-closed and nano semi-continuous. Let G be nano semi-open in $(U,\tau_R(X))$. Then U-G is nano semi-closed in U. Since f is nano semi-closed, f(U-G)=V-f(G) is nano semi-closed in V and hence nano-closed in V. Therefore f(G) is nano semi-open in V. Thus, f is nano semi-open and hence f is a nano semi-homeomorphism.

**Theorem 4.6** A one-one map f of $(U,\tau_R(X))$ onto $(V,\tau_{R'}(Y))$ is a nano semi-homeomorphism iff f(NClNint(A))=NClNint[f (A)] for every subset A of U.

**Proof:** If f is a nano semi-homeomorphism, f is nano semi-continuous and nano semi-closed. If A⊆U, f(NClNint(A)) ⊆NClNint(f(A)), since f is nano semi-continuous. Since NClNint(A) is nano semi-closed in U and f is nano semi-closed, f(NClNint(A)) is nano semi-closed in V. NClNint(f(NClNint(A)))=f(NClNint(A)). Since A⊆NClNint(A), f (A) ⊆f (NClNint(A)) and hence NClNint(f(A)) ⊆ NClNint[f(NClNint(A))] = f(NClNint(A)). Therefore, NClNint(f(A))⊆f(NClNint(A)). Thus, f(NClNint(A))=NClNint(f(A)) if f is a nano semi-homeomorphism. Conversely, if f (NClNint(A))=NClNint(f(A)) for every subset A of U, then f is nano semi-continuous. If A is nano semi-closed in U, NClNint(A)=A which implies f(NClNint(A))=f(A). Therefore, NClNint(f(A))=f (A). Thus, f(A) is nano closed in V, for every nano semi-closed set A in U. That is f is nano semi-closed. Also f is nano semi-continuous. Thus, f is a nano semi-homeomorphism.

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