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On Soft SGB-Closed sets in soft topological spaces

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Abstract
This paper focuses on soft sgb-closed sets and soft sgb-open sets in soft topological spaces and to investigate its properties. Further soft sgb-$T_{1/2}$ space is introduced and its basic properties are discussed.

Keywords: Soft closed, soft generalized closed, soft sgb-closed, soft sgb-$T_{1/2}$ space, soft topological spaces.

1. Introduction
In 1999, D. Molodtsov [12] introduced the notion of soft set and applied the soft theory in several fields such as smoothness of functions, game theory, probability, Perron integration, Riemann integration and theory of measurement. Lashin *et al*. [7] introduced generalized rough set theory in the framework of topological spaces. Levine [8] introduced g-closed sets in general topology. Kannan [6] introduced soft g-closed sets in soft topological spaces. They have also studied some of the basic concepts of soft topological spaces. Muhammad Shabir and Munazza Naz [14] introduced soft topological spaces and the notions of soft open sets, soft closed sets, soft closure, soft interior points, soft neighborhood of a point and soft separation axioms. J. Mahanta, P. K. Das [9] studied soft topological space via semiopen and semiclosed soft sets. Maji, Biswas and Roy [10] made a theoretical study of soft set theory. Tridev Jyoti Naog [15] studied a new approach to the theory of soft sets. Soft semi open sets, Soft g closed sets, soft gβ and soft regular generalized closed sets have been introduced and studied by the researchers. [1, 2, 5, 6, 16]
In this paper, we define a new class of sets called soft sgb-closed sets and study the relationships with other soft sets. Also we introduce soft sgb-$T_{1/2}$ and study its basic properties.

2. Preliminaries
Let U be an initial universe set and E be a collection of all possible parameters with respect to U, where parameters are the characteristics or properties of objects in U.Let P(U) denote the power set of U, and let A⊆E.

Definition 2.1. [11] A pair (F, A) is called a soft set over U, where F is a mapping given by F:A→P(U). In other words, a soft set over U is a parameterized family of subsets of the universe U. For a particular e ∈ A, F (e) may be considered the set of e-approximate elements of the soft set (F, A).

Definition 2.2: [3] For two soft sets (F, A) and (G, B) over a common universe U, we say that (F,A) is a soft subset of (G,B) if
(i) A ⊆B and
(ii)∀ e ∈ A, F(e) ⊆G(e).
We write (F, A) ⊆(G,B). (F, A) is said to be a soft super set of (G, B), if (G, B) is a soft subset of (F,A) and is denoted by (F,A) ⊇ (G,B).

Definition 2.3: [10] For two soft sets (F, A) and (G, B) over a common universe U, union of two soft sets of (F,A) and (G,B) is the soft set (H,C),where C = A ∪ B and ∀e ∈ C,

$$H(e) = \begin{cases} F(e) \text{ if } e \in A - B \\ G(e) \text{ if } e \in B - A \\ F(e) \cup G(e) \text{ if } e \in A \cap B \end{cases}$$

We write (F, A) ∪ (G,B)=(H,C).

Definition 2.4: [3] The Intersection (H,C) of two soft sets (F,A) and (G,B) over a common universe U denoted by (F,A) ⋂(G,B) is defined as C= A⋂B and H(e) = F(e)⋂G(e) for all e∈C.

Definition 2.5: [14] Let τ be the collection of soft sets over X, then τ is called a soft topology on X if τ satisfies the following axioms:

1) ϕ,$\tilde{X}$ belong to τ

2) The union of any number of soft sets in τ belongs to τ.

3) The intersection of any two soft sets in τ belongs to τ.

The triplet (X, τ, E) is called a soft topological space over X. For simplicity, throughout the work we denote the soft topological space (X, τ, E) as X.

Definition 2.6: [14] Let (X, τ, E) be soft space over X. A soft set (F, E) over X is said to be soft closed in X, if its relative complement (F,E)′ belongs to τ. The relative complement is a mapping F′:E→P(X) defined by F′(e)= X-F(e) for all e ∈A.

Definition 2.7: [6] Let X be an initial universe set, E be the set of parameters and τ={ ϕ,$\tilde{X}$}. Then τ is called the soft indiscrete topology on X and (X, τ, E) is said to be a soft indiscrete space over X. If τ is the collection of all soft sets which can be defined over X, then τ is called the soft discrete topology on X and (X, τ,E) is said to be a soft discrete space over X.

Definition 2.8: [6] Let (X, τ, E) be a soft topological space over X and the soft interior of (F, E) denoted by Int(F,E) is the union of all soft open subsets of (F,E). Clearly, (F,E) is the largest soft open set over X which is contained in (F,E).The soft closure of (F,E) denoted by Cl(F,E) is the intersection of all closed sets containing (F,E). Clearly, (F, E) is the smallest soft closed set containing (F, E).

Int (F, E) = ∪ {(O, E): (O, E) is soft open and (O, E) ⊆ (F,E)}.

Cl(F,E) = ⋂{ (O,E): (O,E) is soft closed and (F,E) ⊆ (O,E)}.

Definition 2.9: [6] Let U be the common universe set and E be the set of all parameters. Let (F, A) and (G, B) be soft sets over the common universe set U and A,B ⊆ E. Then (F, A) is a subset of (G, B), denoted by (F, A) ⊆ (G,B). (F,A) equals (G,B), denoted by (F,A)=(G,B) if (F,A) ⊆(G,B) and (G,B) ⊆(F,A).

Definition 2.10: A soft subset (A, E) of X is called

(i)a soft generalized closed (soft g-closed) [6] if Cl(A,E)⊆(U,E)whenever (A,E)⊆(U,E)and(U,E)is soft open in X

(ii) a soft semi open [2] if (A,E) ⊆ Cl(Int(A,E)).

(iii) a soft regular open [5] if (A,E)= Int(Cl(A,E)).

(iv) a soft α-open [5] if (A,E) ⊆Int(Cl(Int(A,E))).

(v) a soft b-open [5] if (A,E) ⊆ Cl(Int(A,E))∪ Int(Cl(A,E)).

(vi) a soft pre-open [5] set if (A,E) ⊆ Int (Cl(A,E)).

(vii) a soft semi-generalized closed (sg-closed) [5] if scl (A, E) ⊆ (U,E) whenever (A,E) ⊆ (U,E) and (U,E) is semi-open in (X, τ, E).

(viii) a soft β-open [1] set if (A,E) ⊆ Cl(Int (Cl(A,E))).

(ix) a soft generalized β closed (Soft gβ-closed) [1] in a soft topological space (X,τ,E) if βCl(A,E) ⊆ (U,E) whenever (A,E)⊆(U,E) and (U,E) is soft open in X.

(x) a soft generalized-semi closed (gs-closed) [5] if scl(A,E) ⊆ (U,E) whenever (A, E) ⊆ (U, E) and (U,E) is open in (X, τ, E).

The complement of the soft semi open, soft regular open, soft α-open, soft b-open, soft pre-open sets are their respective soft semi closed, soft regular closed, soft α-closed, soft b-closed and soft pre-closed sets.

Definition 2.11: [6] A soft topological space X is called a soft $T_{1/2}$-space if every soft g-closed set is soft closed in X.

Definition 2.12: [5] The soft regular closure of (A, E) is the intersection of all soft regular closed sets containing (A, E). That is the smallest soft regular closed set containing (A, E) and is denoted by srcl(A, E).

The soft regular interior of (A, E) is the union of all soft regular open sets contained in (A, E) and is denoted by srint (A, E).

Similarly, we define soft α-closure, soft pre-closure, soft semi closure and soft b-closure of the soft set (A,E) of a topological space X and are denoted by sαcl(A,E), spcl (A,E), sscl(A,E) and sbcl(A,E) respectively.

Proposition 2.13: [4] Let (X, τ,E) be a soft topological space over X and (F,E) and (G,E) be a soft set over X. Then

(1) int (int (F,E)) = int (F,E)

(2) (F,E) ⊆(G,E) implies int (F,E) ⊆ int(G,E)

(3) cl (cl (F,E)) = cl (F,E)

(4) (F,E) ⊆ (G,E) implies cl (F,E) ⊆ cl(G,E).

Definition 2.14: [8] A subset A in a topological space is defined to be a Q-set if int(cl(A)=cl(int(A))

3. Soft sgb-Closed Sets

In this section, we define new class of sets called soft sgb-closed sets and establish its relationship with other soft sets is discussed.

Definition 3.1: A soft subset (A, E) of a soft topological space X is called soft sgb-closed in X if sbcl(A,E) ⊆ (U,E) whenever (A,E) ⊆ (U,E) and (U,E) is soft semi -open in X.

Theorem 3.2

1. Every soft closed set is soft sgb-closed.
2. Every soft g-closed is soft sgb-closed.
3. Every soft α-closed set is soft sgb-closed.
4. Every soft sg-closed set is soft sgb –closed.
5. Every soft gs-closed set is soft sgb –closed.
6. Every soft semi-closed set is soft sgb-closed.
7. Every soft sgb-closed set is soft gβ-closed.

Remark 3.3: The converse of the above theorem is not true as seen in the following example.

Example 3.4: Let X = {a, b, c, d}, E = {e_1,e_2}.Let F_1, F_2, F_3, F_4, F_5, F_6 functions from E to p (X) and are defined as follows:$F_1(e_1)$ = {a},$F_1(e_2)$ = {c},$F_2(e_1)$ = {b},$F_2(e_2)$ = {d},$F_3(e_1)$ = {a,b},$F_3(e_2)$ = {c,d},$F_4(e_1)$ = {b,d},$F_4(e_2)$ = {a,d},$F_5(e_1)$ = {b,c,d},$F_5(e_2)$ = {a,b,c},$F_6(e_1)$ = {a,b,d, $F_6(e_2)$ ={a,c,d}.Then τ = { ϕ, X, (F_1, E), (F_2, E), (F_3, E), (F_4, E),

$(F_5, E),(F_6, E)\}$ is a soft topology and elements in τ are soft open sets.

1. The soft set (A, E) = {{b}, {a, c}} is a sgb-closed but not soft closed.
2. The soft set (A, E) = {{a}, {b, d}} is a sgb-closed but not soft g-closed.
3. The soft set (A, E) = {{b, c},{c}} is a sgb-closed but not soft α-closed.
4. The soft set (A,E) = {{b},{c}} is a sgb-closed but not soft sg-closed.
5. The soft set (A, E) = {{b,c},{b,c}} is a sgb-closed but not soft gs-closed.
6. The soft set (A, E) = {{b,c},{c,a}} is a sgb-closed but not soft semi closed.
7. The soft set (A, E) = {{a},{c,d}} is a soft gβ-closed but not soft sgb- closed.

Remark 3.5
We depict the above discussions in the following diagram.

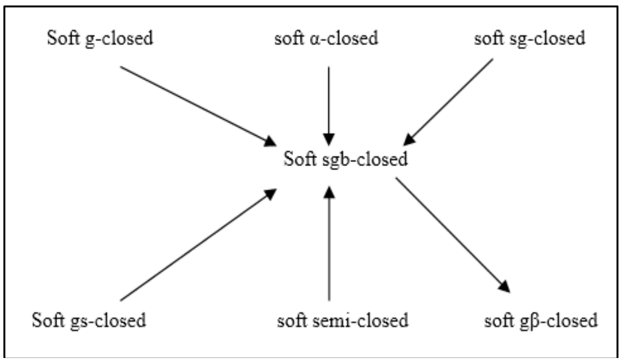

Fig 1

Theorem 3.6: If (A, E) is soft semi-open and soft sgb-closed, then (A, E) is soft b-closed.

Proof: Let (A, E) be soft semi-open and soft sgb-closed. Let (A, E) ⊆ (A, E) where (A, E) is soft semi-open. Since (A, E) is soft sgb-closed, we have sbcl(A,E)⊆(A,E). Then (A, E) = sbcl(A,E).Hence (A,E) is soft b-closed.

Theorem 3.7: Let (A, E) be soft sgb-closed in X. Then sbcl (A, E) - (A,E) does not contain any non-empty soft semi-closed set.
Proof: Let (F,E) be a non-empty soft semi-closed set such that(F,E)⊆ sbcl(A,E) ⊆(A,E).Since (A,E) is soft sgb-closed, (A,E) ⊆ X-(F,E) where X-(F,E) is soft semi-open implies sbcl(A,E) ⊆ X-(F,E).Hence (F,E) ⊆X-sbcl(A,E). Now, (F,E) ⊆(sbcl (A,E) - (A,E)) ⋂(X-sbcl (A,E)) implies (F,E) =ϕ. Which is a contradiction.
Therefore, sbcl(A,E) -(A,E) does not contain any non-empty soft semi- closed set.

Corollary 3.8: Let (A, E) be soft sgb-closed in X. Then (A, E) is soft b-closed if and only if sbcl (A,E) - (A,E) is soft semi-closed.
Proof: Let (A, E) be soft b-closed. Then sbcl (A,E) = (A,E).This implies sbcl (A,E)-(A,E)= ϕ which is soft semi-closed. Assume that sbcl (A,E)-(A,E) is soft semi-closed. Then sbcl(A,E) - (A,E) = ϕ. Hence, sbcl(A,E)=(A,E).
Theorem 3.1.7: For a soft subset (A, E) of X, the following statements are equivalent:
(1) (A, E) is soft semi-open and soft sgb- closed.
(2) (A,E) is soft regular open.

Proof: (1) ⇒ (2): Let (A,E) be a soft semi-open and soft sgb-closed subset of X.
Then sbcl(A,E) ⊆ (A,E). Hence intcl(A,E) ⊆ (A,E). Since (A, E) is soft open, we have (A,E) is soft pre-open and thus (A, E) ⊆ intcl(A,E). Therefore, we have intcl(A,E) = (A,E), which shows that (A,E) is soft regular open.
(2)⇒ (1): Since every soft regular open set is soft semi-open then sbcl(A,E) = (A,E) and sbcl(A,E) ⊆ (A,E). Hence, (A, E) is soft sgb-closed.

Remark 3.8: Finite union of soft sgb-closed sets need not be sgb-closed.

Example 3.9: Let the two soft sets are $G(e_1)$ = {a}, $G(e_2)$ = {b,d} and
$H(e_1)$ = {b,c},$H(e_2)$ = {b,c}.Then (G, E) and (H, E) are soft sgb-closed sets over X. But their union
(A, E) ={{a,b,c},{d,b,c}} is not soft sgb-closed.

Remark 3.10: Finite intersection of soft sgb-closed sets need not be soft sgb-closed.
Example 3.11: Let the two soft sets be $G(e_1)$ = {b,c}, $G(e_2)$ = {c} and $H(e_1)$={b},$H(e_2)$ = {c}. Then (G, E) and (H,E) are soft sgb-closed sets over X. But their intersection (A,E) ={{b},{c}} is not soft sgb-closed.
Definition 3.12:A soft topological space X is said to be soft hyperconnected if the closure of every soft open subset is X.

Theorem 3.13: Let X be a soft hyperconnected space. Then every soft sgb-closed subset of X is soft sgs- closed.
Proof: Assume that X is soft hyperconnected.Let (A,E) be soft closed and let (U,E) be an soft semi-open set containing (A,E). Then sbcl(A,E)=(A,E)⋃[int(cl((A,E))⋂cl(int(A,E))]=(A,E)⋃int(cl (A,E))=sscl((A,E)). Since sbcl ((A,E)) = sscl((A,E)), sscl((A,E)) ⊆ (U,E). Hence, (A, E) is soft sgs-closed.

Theorem 3.14: Let (A, E) be a soft sgb-closed set and soft dense in X. Then (A, E) is soft sgp-closed.
Proof: Suppose that (A, E) be soft sgb-closed set and soft dense in X. Let (U, E) be a soft semi-open set containing (A, E). Since sbcl(A, E) = (A,E) ⋃ [int(cl(A,E)) ⋂ cl(int(A,E))] and soft dense, we obtain bcl((A,E))=(A,E)⋃cl(int((A,E)))=pcl((A,E)) ⊆ (U,E). Therefore (A, E) is sgp-closed.

4. Soft sgb-Open Sets

Definition 4.1: A soft subset (A, E) ⊆ X is called soft sgb-open if its relative complement is soft sgb-closed.
Lemma 4.2: Let (F, A) be a soft subset of a topological space X, then sbcl(X-(F,A)) = (X-sbint(F,A)).
Proof: Let x∈X- sbint((F, A)). Then x∉ sbint((F,A)). That is every soft b-open set (G, A) containing x is such that (G,A) ⊈ (F,A). Hence every soft b-open set (G, A) containing x intersects X-(F, A). This implies x∈ sbcl (X-(F, A)). Hence X-sbint((F, A)) ⊆ sbcl (X-(F,A)).
Conversely, Let x∈ sbcl (X-(F, A)). Thus every soft b-open set (H, A) containing x intersects (X-(F,A)). That is every b-open set (H, A) containing x is such that (H, A) ⊈ (F,A).This implies x∉ sbint((F, A)). Thus sbcl(X-(F,A))⊆X-sbint ((F,A)). Hence sbcl(X-(F, A)) = (X-sbint (F, A)).

Theorem 4.3: The soft subset (A,E) of X is soft sgb-open if and only if F ⊆ sbint(A,E) whenever (F,E) is soft semi-closed and (F,E) ⊆ (A,E).

Proof: Necessity:Let (A,E) be soft sgb-open. Let (F,E) be soft semi-closed and (F,E) ⊂ (A,E). Then X-(A,E) ⊆ X-(F,E) where X-(F,E) is soft semi-open. By assumption, sbcl(X-(A,E)) ⊂ X-(F,E). By Lemma 4.2, X-sbint (A,E) ⊆ X-(F,E). Thus (F,E) ⊆ sbint(A,E).

Sufficiency: Suppose (F, E) is soft semi-closed and (F, E) ⊆ (A, E) such that (F,E) ⊆ sbint(A,E). Let X-(A,E) ⊆ (U,E) where (U,E) is soft semi-open.Then X-(U,E) ⊆ (A,E) where X-(U,E) is soft semi-closed. By hypothesis, X-(U,E) ⊆sbint(A,E).That is X-sbint(A,E) ⊆ (U,E). Hence sbcl(X-(A, E)) ⊆ (U, E). Thus X-(A, E) is soft sgb-closed and (A, E) is soft sgb-open.

Theorem 4.4: If sbint(A,E) ⊆ (B,E) ⊆ (A,E) and (A,E) is soft sgb-open then (B,E) is soft sgb-open.

Proof: Let sbint(A,E) ⊆ (B,E) ⊆ (A,E). Thus X-(A, E) ⊆ X-(B,E) ⊆ X-sbint(A,E). That is X-(A, E) ⊆ X-(B,E) ⊆ sbcl(X-(A,E)) by Lemma 4.2. Since X-(A,E) is soft sgb-closed, by Theorem 4.3, (X-(A,E)) ⊆ (X-B) ⊆ sbcl (X-(A,E)) implies (X-(B,E)) is soft sgb-closed. Hence (B,E) is soft sgb-open.

5. Soft sgb-$T_{1/2}$ Spaces

Definition 5.1:A soft topological space X is called

(i) soft sgb-$T_{1/2}$ space if every soft sgb- closed set is soft b-closed.

(ii) soft sgb- space if every soft sgb- closed is soft closed.

(iii) soft T_{sgb} -space if every soft sgb-closed set is soft sg-closed.

Theorem 5.2: For a soft topological space (X, τ, E) the following are equivalent

(i) X is soft sgb-$T_{1/2}$ space.

(ii) Every singleton set is either soft semi-closed or soft b-open.

Proof: To prove (i) ⇒ (ii): Let X be a soft sgb-$T_{1/2}$ space. Let (A,E) be a soft singleton set in X and assume that (A,E) is not soft semi-closed. Then clearly X- (A,E) is not soft semi-open. Hence X- (A,E) is trivially a soft sgb- closed. Since X is soft sgb- $T_{1/2}$ space, X-(A, E) is soft b-closed. Therefore (A, E) is soft b-open.

To prove (ii)⇒ (i):Assume that every singleton of X is either soft semi-closed or soft b-open. Let (A,E) be a sgb-closed set. Let (A, E) ∈ sbcl (A, E).

Case (i): Let the singleton set (F, E) be soft semi-closed. Suppose (F, E) does not belong to (A, E). Then (F, E) ∈ bcl (A, E) - (A, E). By Theorem 3.7, (F, E)∈(A,E). Hence sbcl (A, E) ⊆(A,E).

Case (ii): Let the singleton set (F, E) be soft b-open. Since (F, E) ∈ sbcl (A,E), we have (F,E) ∩ (A,E) ≠ ϕ implies that (F,E) ∈ (A,E). In both the cases sbcl (A, E) ⊆ (A, E) or equivalently (A, E) is soft b-closed.

Theorem 5.3: For a soft topological space (X, τ, E) the following are equivalent

(i) X is soft sgb-$T_{1/2}$ space.

(ii) For every soft subset (A, E) of X, (A, E) is soft sgb-open if and only if (A, E) is soft b-open.

Proof: (i) ⇒ (ii): Let the soft topological space X be soft sgb-$T_{1/2}$ and let (A,E) be a soft sgb-open soft subset of X. Then X- (A, E) is soft sgb-closed and so X- (A,E) is soft b-closed. Hence (A, E) is soft b-open.

Conversely, Let (A, E) be a soft b-open subset of X. Thus X- (A, E) is soft b-closed. Since every soft b-closed set is soft sgb-closed then X- (A, E) is soft sgb-closed. Hence (A, E) is soft sgb-open.

(ii)⇒ (i): Let (A,E) be a soft sgb-closed subset of X. Then X-(A, E) is soft sgb-open. By the hypothesis, X- (A, E) is soft b-open. Thus (A, E) is soft b-closed. Since every soft sgb-closed is soft b-closed, X is soft sgb-$T_{1/2}$ space.

Theorem 5.4: SBO(X, τ, E) ⊆ SSGBO(X, τ, E).

Proof: Let (A, E) be soft b-open, then X- (A, E) is soft b-closed so X- (A, E) is soft sgb-closed. Thus (A, E) is soft sgb-open. Hence SBO (τ) ⊆ SSGBO(τ).

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