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Cordial labelings on middle graph of extended duplicate graph of path

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Abstract
A graph labeling is a mapping that carries a set of graph elements onto a set of numbers called labels (usually the set of integers). In this paper we prove the existence of graph labeling such as cordial, total cordial, product cordial, total product cordial, E- cordial, total E- cordial, product E-cordial and total product E-cordial labeling for the middle graph of extended duplicate graph of path P_m, by presenting algorithms, where m represents the length of the path.
AMS Subject Classification: 05C78

Keywords: Graph labeling, Path, Duplicate graph, Middle graph

1. Introduction
The concept of graph labeling was introduced by Rosa in 1967. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. One of the most famous and productive labeling in graph theory is cordial labeling which was introduced by I. Cahit in the year 1987. It is proved that, every tree is cordial; K_n is cordial iff $n \leq 3$; $K_{m,n}$ is cordial for all m and n.
Sundaram, Ponraj and Somasundaram [8, 9] have introduced the concept of product, total product and prime cordial labeling. They proved that the following graphs are product cordial: trees, unicyclic graphs of odd order; $K_{m,n}$ (m,n > 2), $P_m \times P_n$ (m,n > 2) and wheels are not product cordial.
In 1997, Yilmaz and Cahit [11] introduced the concept of E-cordial. They proved that the following graphs are E- cordial: trees with n vertices if and only if $n \not\equiv 2 \pmod 4$; regular graphs of degree 1 on 2n vertices iff n is even; friendship graphs $C_3^{(n)}$ for all n.
Middle graph was introduced by J. Akiyama, T. Hamada, I. Yoshimura in 1975 [1]. The concept of extended duplicate graph was introduced by K. Thirusangu, *et al* in [10] and they proved that $EDG(P_m)$ is cordial and also proved that the extended duplicate graph of a path P_m admits cordial, total cordial, product cordial and total product cordial labeling.
In this paper, we prove that, the middle graph of extended duplicate graph of path P_m is cordial, total cordial, product cordial, total product cordial, E- cordial, total E- cordial, product E-cordial and total product E-cordial by presenting algorithms.

2 Preliminaries
In this section, we give basic notions relevant to this paper.

Definition 2.1
Let G (V,E) be a simple graph. A Duplicate graph of G is $DG=(V_1,E_1)$ where the vertex set $V_1 = V \cup V'$ and $V \cap V' = \varphi$ and f: $V \rightarrow V'$ is bijective (for $v \in V$, we write $f(v) = v'$ for convenience) and the edge set E_1 of DG is defined as follows: The edge v_iv_j is in E if and only if both v_iv_j' and $v_i'v_j$ are edges in E_1. Clearly duplicate graph of a path is disconnected.

Definition 2.2
Let $DG = (V_1,E_1)$ be a duplicate graph of a path G(V,E). The Extended Duplicate graph of the path P_m, denoted by $EDG(P_m)$ is obtained by adding an edge $(v_2\, v_2')$ in DG, where $v_2 \in V$ and $v_2' \in V'$.

Definition 2.3

The middle graph of G(V,E) is defined with the vertex V(G) ∪ E(G) where two vertices are adjacent if they are either adjacent edges in G or one is a vertex and the other is an edge incident with it and it is denoted by M(G).

Definition 2.4

A function *f*: V → {0,1} is said to be cordial labeling if each edge *uv* receives the label $|f(u) - f(v)|$ such that the number of vertices labeled with '0' and the number of vertices labeled with '1' differ by at most one, and the number of edges labeled with '0' and the number of edges labeled with '1' differ by at most one. Moreover, it admits total cordial labeling if the number of vertices and edges labeled with '0' and the number of vertices and edges labeled with '1'differ by at most one.

Definition 2.5

A function *f* : V → {0,1} such that each edge *uv* receives the label $f(u) \times f(v)$ is said to be product cordial labeling if the number of vertices labeled with '0' and the number of vertices labeled with '1' differ by at most one, and the number of edges labeled with '0' and the number of edges labeled with '1'differ by at most one. Moreover, it is said to be total product cordial labeling if the number of vertices and edges labeled with '0' and the number of vertices and edges labeled with '1' differ by at most one.

Definition 2.6

A graph G(V,E) is said to admit E-cordial labeling if there exists a function

f : E → {0,1} such that the induced function *f** on V defined by $f^*(v_i) = \{\sum f(v_i\ v_j) \mid v_i\ v_j \in E\} (\text{mod } 2)$ satisfies the property that the number of edges labeled with '0' and the number of edges labeled with '1' differ by at most one and number of vertices labeled with '0' and the number of vertices labeled with '1' differ by at most one. A graph that admits E-cordial labeling is called E-cordial. It admits total E-cordial labeling if it satisfies the property that the number of edges and vertices labeled with '0' and the number of edges and vertices labeled with '1' differ by at most one.

Definition 2.7

A Graph G(V,E) is said to admit product E-cordial labeling if there exists a function *f* : E → {0,1} such that the induced function *f** on V defined by $f^*(v_i) = \{\prod f(v_i\ v_j) / v_i\ v_j \in E\}$ satisfies the property that the number of edges labeled with '0' and the number of edges labeled with '1' differ by at most one and the number of vertices labeled with '0' and the number of vertices labeled with '1' differ by at most one. A graph that admits product E-cordial labeling is called product E-cordial. It is said to admit total product E-cordial labeling if it satisfies the property that the number of edges and vertices labeled with '0' and the number of edges and vertices labeled with '1' differ by at most one.

3. Main Result

In this section we prove the existence of cordial, total cordial, product cordial, total product cordial, E-cordial, total E- cordial, product E- cordial and total product E-cordial labeling for the middle graph of extended duplicate graph of a path by presenting algorithms.

Definition 3.1

The structure of middle graph of extended duplicate graph is defined as follows : Let EDG(P_m) be a graph with 2m+2 vertices $\{v_1, v_2 \ldots\ldots v_{m+1}, v_1', v_2' \ldots.. v'_{m+1}\}$ and 2m+1 edges where 'm' represents the length of the path P_m. The middle graph of EDG(P_m) is obtained by introducing a new vertex w_i on each edge as follows:

v_iv_{i+1}' ⟵ w_i ; $1 \le i \le m$
v_iv_{i-1}' ⟵ w_{i+m-1} ; $2 \le i \le m+1$
v_2v_2' ⟵ w_{2m+1}

Thus MEDG(P_m), the middle graph of EDG(P_m) is a (V,E) graph where $V=\{v_i \cup v_i' \cup w_k \ / \ 1 \le i \le m+1 \text{ and } 1 \le k \le 2m+1\}$ and $E= \{v_iw_i, w_iv_{i+1}' \text{ for } 1 \le i \le m\} \cup \{v_iw_{i+m-1}, w_{i+m-1}v_{i-1}' \text{ for } 2 \le i \le m+1\} \cup \{v_2w_{2m+1}, w_{2m+1}v_2'\} \cup \{w_iw_{i+m+1} \text{ for } 1 \le i < m\} \cup \{w_iw_{im+1} \text{for } m<i<2m\} \cup \{w_iw_{2m+1} \text{ for } 1 \le i \le 2 \text{ and } i=m+1, i=m+2\}$.

Thus MEDG(P_m) has 4m+3 vertices and 6m+4 edges.

Algorithm 3.1

Procedure: cordial labeling for MEDG (P_m), m ≥ 4
//assignment of labels to the vertices.
$w_1, w_2, w_{m+2}, w_{m+3} \leftarrow 1$
$w_3, w_{m+1}, w_{2m+1} \leftarrow 0$
for i = 1 to m + 1 do

{$v_i \leftarrow 0$
$v_i' \leftarrow 1$}

end for
for i = 4 to m do
{
$w_i \leftarrow \begin{cases} 0 \text{ if } i = 4l+1, l \in N \\ 1 \ otherwise \end{cases}$
$w_{i+m} \leftarrow \begin{cases} 1 \text{ if } i = 4l+3, l \in N \\ 0 \ otherwise \end{cases}$ }
end for
end procedure
Output: Vertex labeled MEDG (P_m).

Theorem 3.1

Middle graph of extended duplicate graph of path MEDG (P_m), m ≥ 4 is cordial.

Proof

From the construction of Middle graph of extended duplicate graph of path P_m, it is clear that MEDG (P_m) has 4m + 3 vertices and 6m + 4 edges.
In order to label the vertices, define a function f : V → {0, 1} as given in the algorithm 3.1. The vertices have the labels as follows:

Case 1: If m ∈ {(4i) ∪ (4i + 3) | i ∈ N}, the number of vertices labeled '1' is 2m + 2 and the number of vertices labeled '0' is 2m + 1.

Case 2: If m ∈ {(4i + 1) ∪ (4i + 2) | i ∈ N}, the number of vertices labeled '1' is 2m + 1 and the number of vertices labeled '0' is 2m + 2.
In both cases the number of vertices labeled '0' and '1' is differ by atmost 1.

In order to get the edge labels, define the induced map $f^*: E \rightarrow N$ such that $f^*(v_i v_j) = [f(v_i) + f(v_j)] \pmod 2$.
Thus the edge labels are obtained as follows:
$f^*(v_1 w_1) = f^*(v_2 w_2) = f^*(v_3 w_{m+2}) = f^*(v_1' w_{m+1}) = f^*(v_2' w_{2m+1}) = f^*(w_1 w_{2m+1}) = f^*(w_2 w_{m+1}) = f^*(w_2 w_{2m+1}) = f^*(w_3 w_{m+2}) = f^*(w_{m+2} w_{2m+1}) = f^*(w_3 v_4') = f^*(v_4 w_{m+3}) = 1$
$f^*(v_2 w_{m+1}) = f^*(v_2 w_{2m+1}) = f^*(v_3 w_3) = f^*(v_2' w_1) = f^*(v_2' w_{m+2}) = f^*(v_3' w_2) = f^*(v_3' w_{m+3}) = f^*(w_1 w_{m+2}) = f^*(w_2 w_{m+3}) = f^*(w_{m+1} w_{2m+1}) = 0$

For $4 \leq i \leq m$

$$f^*(v_i w_i) = \begin{cases} 0 \text{ if } i = 4l+1, l \in N \\ 1 \text{ otherwise} \end{cases}$$

$$f^*(v_i' w_{m+i}) = \begin{cases} 0 \text{ if } i = 4l+3, l \in N \\ 1 \text{ otherwise} \end{cases}$$

$$f^*(w_i v_{i+1}') = \begin{cases} 1 \text{ if } i = 4l+1, l \in N \\ 0 \text{ otherwise} \end{cases}$$

$$f^*(w_{i+m} v_{i+1}) = \begin{cases} 1 \text{ if } i = 4l+3, l \in N \\ 0 \text{ otherwise} \end{cases}$$

$$f^*(w_i w_{m+i-1}) = \begin{cases} 0 \text{ if} \{i \in (4l) \cup (4l+1), l \in N\} \\ 1 \text{ otherwise} \end{cases}$$

For $3 \leq i \leq m-1$

$$f^*(w_i w_{m+i+1}) = \begin{cases} 1 \text{ if} \{(4l) \cup (4l+3), l \in N\} \\ 0 \text{ otherwise} \end{cases}$$

Thus the number of edges labeled with '1' $=12+3m-10=3m+2$
The number of edges labeled with '0' $=10+3m-8=3m+2$
Hence the number of edges labeled with '1' and '0' differ by atmost one.
Therefore MEDG (P_m) is cordial.

Theorem 3.2
Middle graph of extended duplicate graph of path MEDG (P_m), $m \geq 4$ is total cordial.

Proof
By theorem 3.1, we have that the number of vertices labeled with '1' and '0' are $2m + 2$ and $2m + 1$ in case (1) and $2m + 1$ and $2m + 2$ in case (2) respectively. The number of edges labeled with '1' and '0' are same as $3m + 2$. Thus the total number of vertices and edges labled with '1' is $5m+4$ in case (1) and $5m+3$ in case (2) and the total number of vertices and edges labled with '0' is $5m+3$ in case (1) and $5m+4$ in case (2). Clearly the total number of vertices and edges labeled with '1' and the total number of vertices and edges labeled with '0' differ by atmost one.
Hence the middle graph of extended duplicate graph of path P_m is total cordial.

Example: 3.1
Cordial labeling for MEDG(P_6) and MEDG(P_4) is shown in figure 3.1(a) and 3.1(b) respectively.

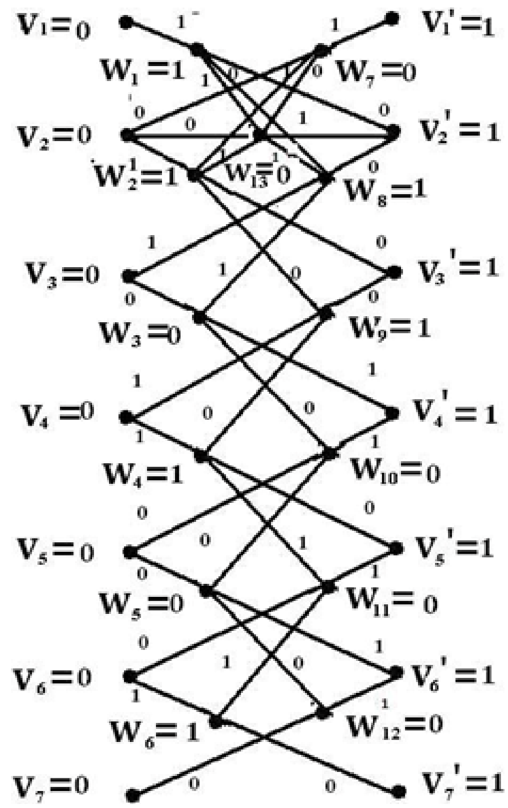

Fig 1(a)

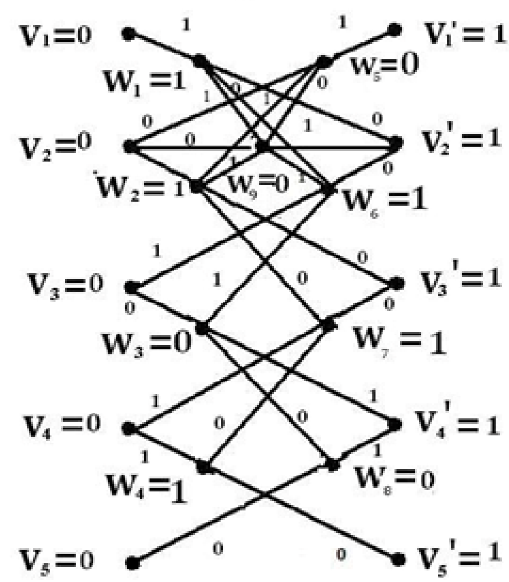

Fig 1(b)

Algorithm 3.2
Procedure: Product cordial labeling for MEDG (P_m), $m \geq 2$
//assignment of labels to the vertices:

$w_{2m+1} \leftarrow 1$

for $i = 1$ to $m + 1$ do

$$\{v_i \leftarrow \begin{cases} 0 \text{ if } i \equiv 1 \pmod 2 \\ 1 \text{ otherwise} \end{cases}$$

$$v_i' \leftarrow \begin{cases} 1 \text{ if } i \equiv 1 (\text{mod } 2) \\ 0 \text{ } otherwise \end{cases} \}$$

end for

for i = 1 to m do

$$\{w_i \leftarrow \begin{cases} 0 \text{ if } i \equiv 1 (\text{mod } 2) \\ 1 \text{ } otherwise \end{cases}$$

$$w_{m+i} \leftarrow \begin{cases} 1 \text{ if } i \equiv 1 (\text{mod } 2) \\ 0 \text{ } otherwise \end{cases} \}$$

end for

end procedure

Output: Vertex labeled MEDG (P_m).

Theorem 3.3

Middle graph of extended duplicate graph of path MEDG (P_m), $m \geq 2$ is product cordial.

Proof

Clearly, MEDG (P_m) has 4m + 3 vertices and 6m + 4 edges. In order to label the vertices, define a function $f : V \rightarrow \{0, 1\}$, using algorithm 3.2.

Thus the number of vertices labeled with '0' = $\frac{m+1}{2} + \frac{m+1}{2} + \frac{m}{2} + \frac{m}{2}$

= 2m + 1

Number of vertices labeled with '1' = $\frac{m+1}{2} + \frac{m+1}{2} + \frac{m}{2} + \frac{m}{2} + 1$

= 2m + 2

Hence the number of vertices labeled with '0' and '1' is differ by atmost one. In order to get the edge labels, define the induced map $f^*: E \rightarrow N$ such that $f^*[v_iv_j] = [f(v_i) \times f(v_j)]$

Thus the edge labels are obtained as follows:

$f^*(v_2 w_{2m+1}) = f^*(w_2 w_{2m+1}) = f^*(w_{m+1} w_{2m+1}) = 1$

$f^*(v_2' w_{2m+1}) = f^*(w_1 w_{2m+1}) = f^*(w_{m+2} w_{2m+1}) = 0$

For $1 \leq i \leq m$

$$f^*(v_iw_i) = f^*(w_iv_{i+1}') = \begin{cases} 1 \text{ if } i = 2n, n \in N \\ 0 \text{ } otherwise \end{cases}$$

For $1 \leq i < m$

$$f^*(w_i w_{m+i+1}) = \begin{cases} 1 \text{ if } i = 2n, n \in N \\ 0 \text{ } otherwise \end{cases}$$

For $2 \leq i \leq m$

$$f^*(w_i w_{m+i-1}) = \begin{cases} 1 \text{ if } i = 2n, n \in N \\ 0 \text{ } otherwise \end{cases}$$

For $2 \leq i \leq m + 1$

$$f^*(v_i w_{m+i-1}) = f^*(w_{m+i-1} v_{i-1}') = \begin{cases} 1 \text{ if } i = 2n, n \in N \\ 0 \text{ } otherwise \end{cases}$$

Thus the number of edges labeled with '0' = number of edges labeled with '1'

$= 3 + \frac{m}{2} + \frac{m}{2} + \frac{m-1}{2} + \frac{m-1}{2} + \frac{m}{2} + \frac{m}{2}$

= 3m + 2

Hence the number of edges labeled with '0' and '1' differ by atmost one.

Therefore MEDG (P_m) is product cordial.

Theorem 3.4

Middle graph of extended duplicate graph of path MEDG (P_m), $m \geq 2$ is total product cordial.

Proof

By theorem 3.3, we have the number of vertices labeled with '0' and '1' are 2m + 1 and 2m + 2 respectively. The number of edges labeled with '0' and '1' are same as 3m+2.

Thus the total number of vertices and edges labeled with '0' is 5m + 3 and the total number of vertices and edges labeled with '1' is 5m + 4.

Hence the total number of vertices and edges labeled with '0' and '1' differ by atmost one.

Therefore MEDG (P_m) is total product cordial.

Example 3.2

Product cordial labeling for MEDG (P_6) is shown in figure 3.2.

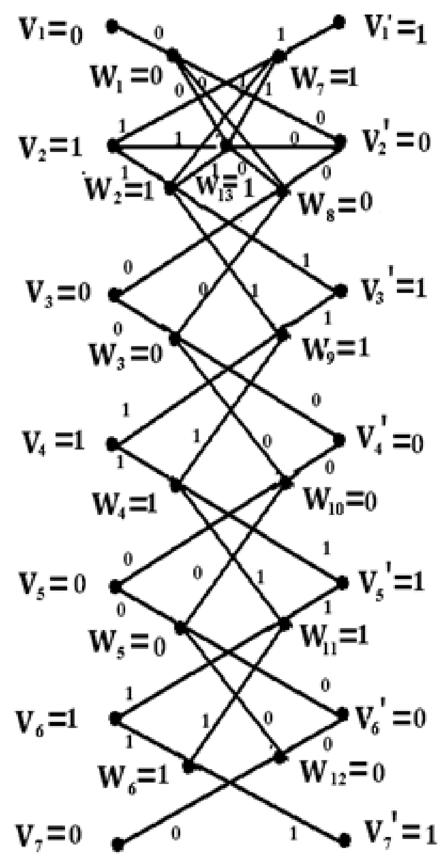

Fig 2

Algorithm: 3.3

Procedure: E-Cordial labeling for MEDG (P_m), $m \geq 2$

//assignment of labels to the edges.

$w_1 w_{2m+1}$; $w_{m+1} w_{2m+1}$; $w_{2m} v_{m+1}$ $\leftarrow 1$

$v_2 w_{2m+1}$; $v_2' w_{2m+1}$; $w_2 w_{2m+1}$; $w_m v_{m+1}'$; $w_{m+2} w_{2m+1}$ $\leftarrow 0$

for i = 1 to m do

$\{v_i w_i$; $v_i' w_{m+i} \leftarrow 1\}$

end for

for i = 2 to m do

$\{w_i w_{m+i-1} \leftarrow 1$

$v_i\, w_{m+i-1}$; $v_i'\, w_{i-1}$ $\leftarrow 0\}$
end for
for i = 1 to m - 1 do
{$w_i\, w_{m+i+1}$ $\leftarrow 0$ }
end for

end procedure
Output: Edge labeled MEDG (P_m).

Theorem 3.5
Middle graph of extended duplicate graph of path MEDG (P_m), $m \geq 2$ is E-cordial.

Proof
From the construction of Middle graph of extended duplicate graph of path P_m, it is clear that MEDG (P_m) has $4m + 3$ vertices and $6m + 4$ edges.
In order to label the edges, define a function $f : E \rightarrow \{0, 1\}$ using algorithm 3.3.
Clearly number of edges labeled with '1' is $3m + 2$ and number of edges labeled with '0' is $3m + 2$.
Thus the number of edges labeled with '0' and '1' is differ by atmost one.
For vertices, define the induced map $f^* : V \rightarrow N$ such that $f^*[v] = [\sum_{u \in N(V)} f(uv)] \pmod 2$
Thus the vertex labels are obtained as follows:
$f^*(v_{m+1}) = f^*(w_{m+1}) = 1$
$f^*(v_{m+1}') = f^*(w_1) = f^*(w_{2m+1}) = 0$
For $1 \leq i \leq m$, $f^*(v_i) = f^*(v_i') = 1$
For $2 \leq i \leq m$, $f^*(w_i) = f^*(w_{m+i}) = 0$
Thus the number of vertices labeled with '1' $= 1 + 1 + m + m$
$= 2m + 2$
The number of vertices labeled with '0' $= 3 + m - 1 + m - 1$
$= 2m + 1$
Hence the number of vertices labeled with '1' and '0' differ by atmost one.
Therefore MEDG (P_m) is E-cordial.

Theorem 3.6
Middle graph of extended duplicate graph of path MEDG (P_m), $m \geq 2$ is total E-cordial.

Proof
By theorem 3.5, we have the number of edges labeled with '1' and '0' are same as $3m + 2$. The number of vertices labeled with '1' and '0' are $2m + 2$ and $2m + 1$ respectively.
Thus the total number of vertices and edges labeled with '1' is $5m+4$ and '0' is $5m+3$ respectively. Hence the total number of vertices and edges labeled with '1' and the total number of vertices and edges labeled with '0' are differ by atmost one.
Therefore the middle graph of extended duplicate graph of path P_m is total E-cordial.

Example 3.3
E-cordial labeling for MEDG (P_6) is given in figure 3.3.

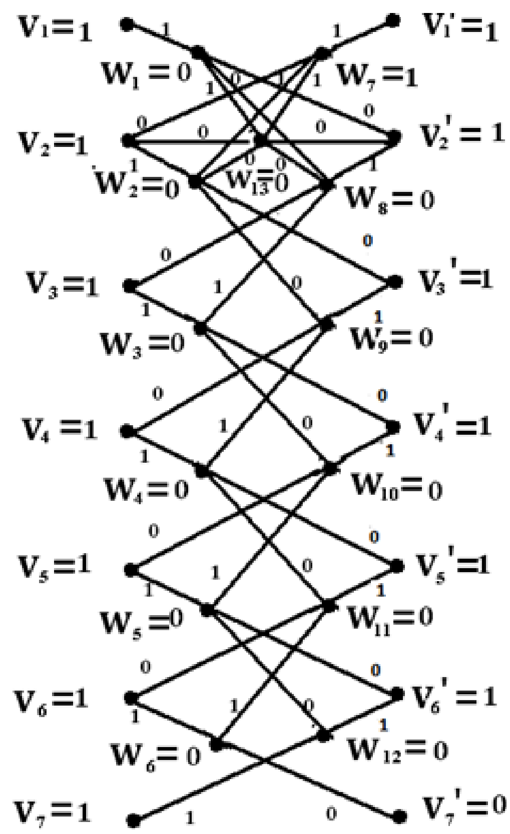

Fig 3

Algorithm 3.4
Procedure: product E-cordial labeling for MEDG (P_m), $m \geq 2$
//assignment of labels to the edges.
$v_2\, w_{2m+1}$; $w_2\, w_{2m+1}$; $w_{m+1}\, w_{2m+1}$ $\leftarrow 1$
$v_2'\, w_{2m+1}$; $w_1\, w_{2m+1}$; $w_{m+2}\, w_{2m+1}$ $\leftarrow 0$

$$w_m\, v_{m+1}' \leftarrow \begin{cases} 1 \text{ if } m = 2n, n \in N \\ 0 \text{ } otherwise \end{cases}$$

$$w_{2m}\, v_{m+1} \leftarrow \begin{cases} 0 \text{ if } m = 2n, n \in N \\ 1 \text{ } otherwise \end{cases}$$

for i = 1 to m do

$$\{v_i\, w_i ; w_i\, w_{m+i+1} \leftarrow \begin{cases} 0 \text{ if } i \equiv 1 \pmod 2 \\ 1 \text{ } otherwise \end{cases}$$

$$v_i'\, w_{m+i} \leftarrow \begin{cases} 1 \text{ if } i \equiv 1 \pmod 2 \\ 0 \text{ } otherwise \end{cases} \}$$

end for
for i = 2 to m do

$$\{v_i\, w_{m+i-1} ; w_i\, w_{m+i-1} \leftarrow \begin{cases} 0 \text{ if } i \equiv 1 \pmod 2 \\ 1 \text{ } otherwise \end{cases}$$

$$v_i'\, w_{i-1} \leftarrow \begin{cases} 1 \text{ if } i \equiv 1 \pmod 2 \\ 0 \text{ } otherwise \end{cases} \}$$

end for
end procedure

Output: Edge labeled MEDG (P_m).

Theorem 3.7
Middle graph of extended duplicate graph of path MEDG (P_m), $m \geq 2$ is product
E-cordial.

Proof
From the construction of Middle graph of extended duplicate graph of path P_m, it is clear that MEDG (P_m) has 4m + 3 vertices and 6m + 4 edges.
In order to label the edges, define a function $f : E \to \{0, 1\}$, using algorithm 3.4.

Case 1: If $m = 2n, n \in N$
Number of edges labeled with '1' $= 3 + 1 + \left(\frac{m}{2} - 1\right) + \frac{m}{2} + \frac{m}{2} + \frac{m}{2} + \frac{m}{2} - 1$

$= \frac{6m}{2} + 4 - 2$
$= 3m + 2$

Number of edges labeled with '0' $= 3 + 1 + \frac{m}{2} + \frac{m}{2} + \frac{m}{2} + \frac{m}{2} - 1 + \frac{m}{2} - 1 + \frac{m}{2}$

$= 3m + 2$

Case 2: If $m = 2n + 1, n \in N$
Number of edges labeled with '1' $= 3 + 1 + \frac{m-1}{2} + \frac{m+1}{2} + \frac{m-1}{2} + \frac{m-1}{2} + \frac{m-1}{2} + \frac{m-1}{2}$

$= \frac{6m-4}{2} + 4$
$= 3m + 2$

Number of edges labeled with '0' $= 3 + 1 + \frac{m+1}{2} + \frac{m-1}{2} + \frac{m-1}{2} + \frac{m-1}{2} + \frac{m-1}{2} + \frac{m-1}{2}$

$= 3m + 2$

Thus the number of edges labeled with '1' and '0' differ by atmost one.
In order to get the labels for vertices, define the induced map $f^* : V \to N$ such that
$f^*(v) = \left[\prod_{u \in N(v)} f(uv)\right]$
Thus the vertex labels are obtained as follows:
$f^*(w_{2m+1}) = 0$

For $1 \leq i \leq m + 1$

$$f^*(v_i) = \begin{cases} 1 \text{ if } i \equiv 0 (\text{mod } 2) \\ 0 \text{ otherwise} \end{cases}$$

$$f^*(v_i') = \begin{cases} 1 \text{ if } i \equiv 1 (\text{mod } 2) \\ 0 \text{ otherwise} \end{cases}$$

For $1 \leq i \leq m$

$$f^*(w_i) = \begin{cases} 1 \text{ if } i \equiv 0 (\text{mod } 2) \\ 0 \text{ otherwise} \end{cases}$$

$$f^*(w_{m+i}) = \begin{cases} 1 \text{ if } i \equiv 1 (\text{mod } 2) \\ 0 \text{ otherwise} \end{cases}$$

Case 1: If $m = 2n, n \in N$
Number of vertices labeled with '1' $= \frac{m}{2} + \left(\frac{m}{2} + 1\right) + \frac{m}{2} + \frac{m}{2}$

$= 2m + 1$

Number of vertices labeled with '0' $= 1 + \left(\frac{m}{2} + 1\right) + \frac{m}{2} + \frac{m}{2} + \frac{m}{2}$

$= 2m + 2$

Case 2: If $m = 2n + 1, n \in N$
Number of vertices labeled '1' $= \frac{m+1}{2} + \frac{m+1}{2} + \frac{m-1}{2} + \frac{m+1}{2}$
$= 2m + 1$

Number of vertices labeled '0' $= 1 + \frac{m+1}{2} + \frac{m+1}{2} + \frac{m+1}{2} + \frac{m-1}{2}$

$= 2m + 2$

Thus the number of vertices labeled with '1' is 2m + 1 and '0' is 2m + 2 respectively.
Hence the number of vertices labeled with '1' and '0' differ by atmost one.
Therefore MEDG (P_m) is product E-cordial.

Theorem 3.8
Middle graph of extended duplicate graph of path MEDG (P_m), $m \geq 2$ is total product
E-cordial.

Proof
By theorem 3.7, we have the number of edges labeled with '1' and '0' are same as 3m + 2. The number of vertices labeled with '1' and '0' are 2m + 1 and 2m + 2 respectively.
Thus the total number of vertices and edges labeled with '1' is 5m+3 and '0' is 5m+4 respectively.
Hence the total number of vertices and edges labeled with '1' and the total number of vertices and edges labeled with '0' differ by atmost one.
Therefore the middle graph of extended duplicate graph of path P_m is total product E- cordial.

Example 3.4
Product E-cordial labeling for MEDG (P_6) and MEDG(P_5) is given in figure.3.3(a) and 3.3(b) respectively.

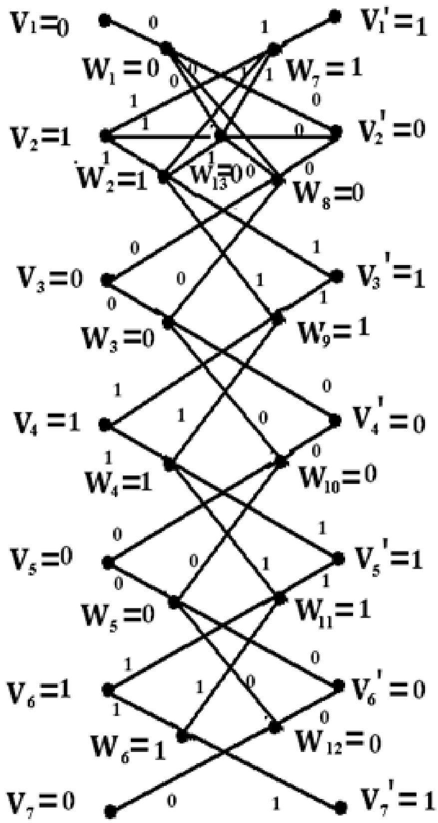

Fig 4(a)

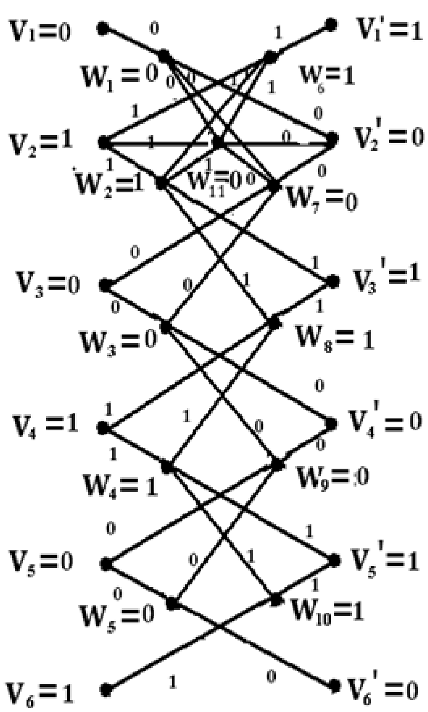

Fig 4(b)

Conclusion

In this paper we have presented algorithms and proved that the middle graph of extended duplicate graph of path admits cordial, total cordial, product cordial, total product cordial, E-cordial, total E-cordial, product E-cordial and total product E-cordial labeling.

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