International Journal of Applied Research 2017; 3(3): 220-223

ISSN Print: 2394-7500
ISSN Online: 2394-5869
Impact Factor: 5.2
IJAR 2017; 3(3): 220-223
www.allresearchjournal.com
Received: 12-01-2017
Accepted: 13-02-2017

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Edge magic labeling in triplication of graphs

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Abstract
In this paper, we investigate the existence of Z_3-edge magic labeling, total Z_3-edge magic labeling and *n* – edge magic labeling for the extended triplicate graph of twig by presenting algorithms.

Keywords: Graph labeling, Z_3-edge magic labeling, *n*-Edge Magic Labeling

1. Introduction
The origin of the study of graph labeling as a major area of graph theory can be traced to a research paper by Rosa [9].
Labeling is a 1-1 map that carries a set of graph elements, usually the vertices and/or edges, into a set of numbers, usually integers, called labels. If the domain is the vertex set or the edge set, then the labelings are respectively called vertex labeling or edge labeling. If the domain is both the vertex set and the edge set, then the labeling is called total labeling. J.A.Gallian has given a detailed survey on different kinds of graph labeling [5].
Sedlacek has introduced the concept of A-magic graph [10]. A graph is called a*A-magic graph* if the edges have distinct non-negative labels which satisfies the condition that the sum of the labels of the edges incident to a particular vertex is the same for all vertices. Chou and Lee investigated the concept of Z_3-magic graphs [4]. A graph G admits Z_3-magiclabeling, if there exists a function *f* from E to [1, 2] such that the sum of the labels on the edges incident at each vertex v ∈ V is a constant.
Bala and Thirusangu introduced the concept of extended triplicate graph of a path and proved the existence of E-Cordial and Z_3 –magic labelings [2].
Jayapriya and Thirusangu introduced and proved the existence of 0-Edge magic labeling for some class of graphs [6].
Motivated by the study, in this paper we prove the existence of Z_3 – edge magic labeling, total Z_3-edge magic labelingand *n*-edge magic labeling for the extended triplicate graph of a twig.

2. Preliminaries
In this section we will provide brief summary of definitions and other information which are prerequisites for the present investigations.

Definition 2.1
The vertex-weight of a vertex *v* in *G* under an edge labeling is the sum of edge labels corresponding to all edges incident with *v*. Under a total labeling, vertex-weight of *v* is defined as the sum of the label of *v* and the edge labels corresponding to the entire edges incident with *v*. If all vertices in *G* have the same weight *k*, we call the labeling vertex-magic edge labeling or vertex-magic total labeling respectively and we call *k* a magic constant. If all vertices in G have different weights, then the labeling is called vertex-antimagic edge labeling or vertex-antimagic total labeling.

Definition 2.2
The edge-weight of an edge *e* under a vertex labeling is defined as the sum of the vertex labels corresponding to every vertex incident with *e* under a total labeling, we also add the label of *e*. Using edge weight, we derive edge-magic vertex or edge-magic total labeling and edge-antimagic vertex or edge-antimagictotal labeling.

Definition 2.3
A graph G(V, E) is said to admit Z_3 edge magic labeling if there exists a function f : V → {1, 2} such that the induced function f* : E → {0, 1} defined as f* (v_iv_j) = {f (v_i) + f (v_j) + f (v_iv_j)} (mod 3) = k, a constant for all edges $v_iv_j \in$ E and v_j is adjacent with v_i.

Definition 2.4
A graph G(V, E) is said to admit n - edge magic labeling if there exists a function f : V → {-1, n+1} such that the induced function f* : E → {n} defined as f* (v_iv_j) = {f (v_i) + f (v_j)} = n, a constant for all edges $v_iv_j \in$ E and v_j is adjacent with v_i.

3. Structure of the extended triplicate graph of twig
In this section, we discuss about the structure of the extended triplicate graph of twig by presenting algorithm.

Definition 3.1: (Twig graph)
A graph *G (V, E)* obtained from a path by attaching exactly two pendent edges to each internal vertices of the path is called a Twig graph. A twig obtained from a path with m-internal vertices is denoted as T_m. If n is even then $T_m$is called an even twig. T_m is said to be odd if m is odd. Generally, a twig T_m has 3m+1 edges and 3m+2 vertices.

Algorithm 3.1
Input: Twig T_m
Procedure triplicate of twig graph T_m
fori = 1 to $m+2$ do $V_1 \leftarrow \{ v_i \cup v_i' \cup v_i'' \}$
End for fori = 2 to $m+1$ do
$V_2 \leftarrow \{ u_i \cup u_i' \cup u_i'' \}u_i$; $V_3 \leftarrow \{ w_i \cup w_i' \cup w_i'' \}$
End for V ← $V_1 \cup V_2 \cup V_3$
fori = 1 to $m+1$ do
$E_1 \leftarrow (v_i v_{i+1}') \cup (v_i' v_{i+1}) \cup (v_i'' v_{i+1}') \cup (v_i' v_{i+1}'')$
end for fori = 2 to $m+1$ do $E_2 \leftarrow (v_i' u_i) \cup (v_i u_i') \cup (v_i' w_i) \cup (v_i w_i') \cup (v_i' u_i'') \cup (v_i'' u_i') \cup (v_i' w_i'') \cup (v_i'' w_i')$ end for E ← $E_1 \cup E_2$

End procedure
Output: Triplicate graph of twig T_m
From the above algorithm 3.1, the triplicate graph of a twig denoted by TG(T_m) is a disconnected graph with 9m + 6 vertices and 12m + 4 edges. To make it as a connected graph, for convenience, we include an edge v_1v_1' to the edge set E as defined in the above algorithm. Thus the graph so obtained is called an extended triplicate graph of twig graph T_m and is denoted by ETG(T_m). By the construction, it is clear that, the graph ETG(T_m) has 9m + 6 vertices and 12m + 5 edges.

Illustration 3.1
The structutre of extended triplicate graph of twig ETG (T_3) is given in figure 1.

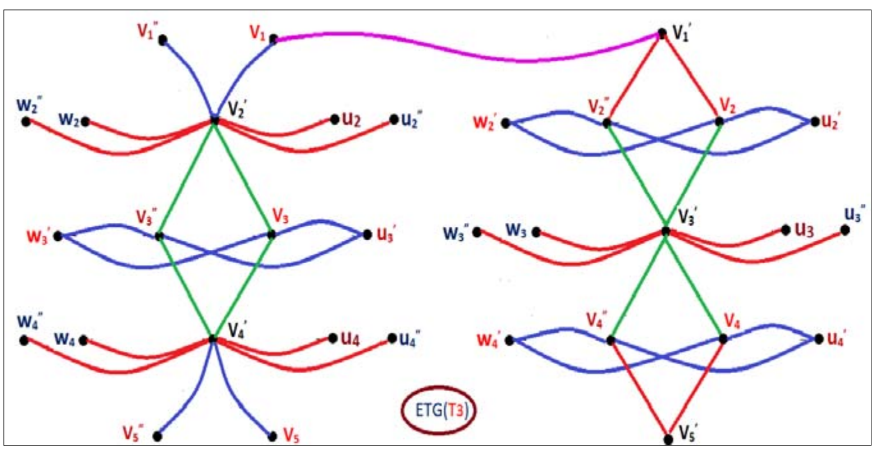

Fig 1: 4z_3–Edge Magic Labeling and Total Z_3-Edge Magic Labeling

In this section we prove the existence of Z_3 - edge magic labeling and total Z_3 - edge magic labeling for ETG(T_m) by presenting algorithms.

Algorithm 4.1
Input: Extended triplicate graph of twig T_m
Procedure (Z_3 - edge magic labeling for ETG(T_m))
V ←{v_1, v_2,..v_{n+2}, u_2, u_3,…,$u_{n+1,}$ w_1, w_2,…w_{n+1}}
fori = 1 to m+2 do
$v_i \leftarrow v_i'' \leftarrow 2$
$v_i' \leftarrow 1$
end for
fori = 2 to m+1 do
$u_i \leftarrow u_i'' \leftarrow w_i \leftarrow w_i'' \leftarrow 2$
$u_i' \leftarrow w_i' \leftarrow 1$
end for
end procedure

Output: *labelled vertices of ETG(T_m)*
Theorem 4.1: ETG(T_m) admits Z_3-edge magic labeling.
Proof:
We know that, the extended triplicate graph of a twig has 9m + 6 vertices and 12m + 5 edges.The vertices are labeled by defining a function f : V→{1, 2} as given in algorithm 4.1
In order to obtain the labels for the edges, define the induced map f* : E→{0, 1, 2} such that for any vertex v_i, f*(v_iv_j) ={ f(v_i) + f(v_j) }(mod 3) where v_j is adjacent with v_i.
Thus for all m, the induced function yields a constant '0'.
Hence the extended triplicate graph of twigETG(T_m) admits Z_3-edge magic labeling.

Illustration 4.1: ETG (T_5) and its Z_3-edge magic labeling is shown in figure 2.

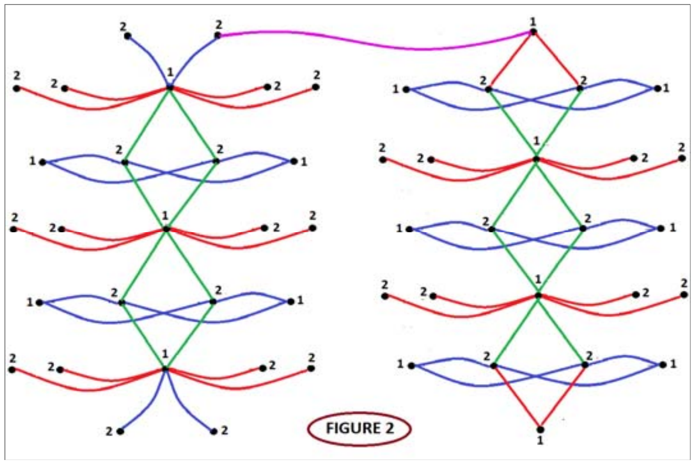

Table 2

Algorithm 4.2
Input: Extended triplicate graph of twig T_m
Procedure (total Z_3 - edge magic labeling for ETG(T_m))
V ←{v_1, v_2,..v_{m+2}, u_2, u_3,…,u_{m+1}, w_1, w_2,…w_{m+1}}
fori = 1 to m+2 do
$v_i \leftarrow v_i'' \leftarrow 2$; $v_i' \leftarrow 1$
end for
fori = 2 to m+1 do
$u_i \leftarrow u_i'' \leftarrow w_i \leftarrow w_i'' \leftarrow 2$
$u_i' \leftarrow w_i' \leftarrow 1$
end for
for i = 1 to m + 1
$v_i v_{i+1}' \leftarrow v_i' v_{i+1} \leftarrow v_i' v_{i+1}'' \leftarrow v_i'' v_{i+1}' \leftarrow 2$
end for
for i = 2 to m + 1 do
$v_i' u_i \leftarrow v_i' u_i'' \leftarrow v_i u_i' \leftarrow v_i w_i' \leftarrow v_i' w_i \leftarrow v_i' w_i'' \leftarrow v_i'' u_i' \leftarrow v_i'' w_i' \leftarrow 2$
end for
$v_1 v_1' = 2$
end procedure

Output: *labeled vertices and edges of ETG(T_m)*
Theorem 4.2: ETG(T_m) admits total Z_3-edge magic labeling.

Proof: From the structure, we know that the extended triplicate graph of twig has 9*m*+6 vertices and 12*m*+ 5 edges. The vertices and edges are labeled by defining a function f: V∪E→ {1, 2} as
Given in algorithm 4.2.
Define the induced function f* : E→{0, 1, 2} such that for any vertex v_i, f*($v_i v_j$) ={ f(v_i) + f(v_j) + f($v_i v_j$) }(mod 3) where v_j is adjacent with v_i.
Thus, for all $v_i v_j$∈ E, the induced function yields the constant '2'.
ETG(T_m) admits total Z_3- edge magic labeling.

Illustration 4.2: ETG (T_5) and its total Z_3-edge magic labeling is shown in figure 3.

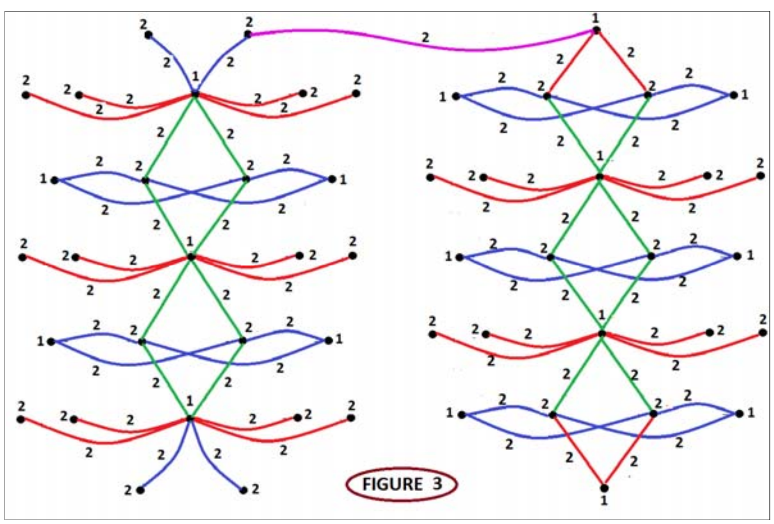

Table 3

5 n - edge magic labeling
In this section we prove the existence of *n* - edge magic labeling for ETG(T_m) by presenting algorithms.

Algorithm 5.1
Input: Extended triplicate graph of twig T_m
Procedure (*n* - edge magic labeling for ETG(T_m))
V ←{v_1, v_2,..v_{m+2}, u_2, u_3,…,u_{m+1}, w_1, w_2,…w_{m+1}}
fori = 1 to m+2 do

$v_i \leftarrow v_i'' \leftarrow$ n+1 ;$v_i' \leftarrow$ - 1
end for
fori = 2 to m+1 do
$u_i \leftarrow u_i'' \leftarrow w_i \leftarrow w_i'' \leftarrow$n+1
$u_i' \leftarrow w_i' \leftarrow$ - 1
end for
end procedure

Output: labelled vertices of ETG(T_m)

Theorem 5.1: ETG(T_m) admits n-edge magic labeling.

Proof: We know that, the extended triplicate graph of a twig has 9m + 6 vertices and 12m + 5 edges. Vertices are labeled by defining the function f: V → {-1, n+1} as given in algorithm 5.1.

In order to obtain the labels for the edges, define the induced map f* : E→{ n } such that for any vertex v_i, f*(v_iv_j) ={ f(v_i) + f(v_j) }(mod 3) where v_j is adjacent with v_i.

Thus for all *m*, the induced function yields a constant '*n*'.

Hence the extended triplicate graphof twig ETG (T_n) admits *n*-edge magic labeling.

Illustration 5.1

ETG (T_3) and its n-edge magic labeling is shown in figure 4.

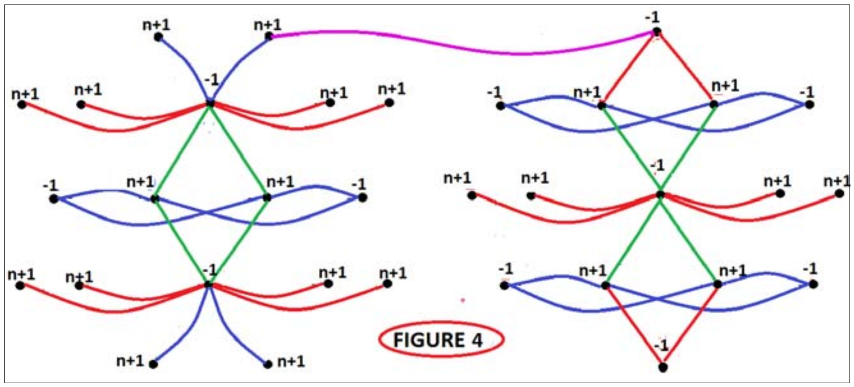

Table 4

6. Conclusion

In this paper, we have proved the existence of Z_3-edge magic labeling, total Z_3-edge magic labeling and *n* – edge magic labeling for the extended triplicate graph of twig by presenting algorithms.

7. References

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