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Sylow p-subgroups and its applications

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Abstract
Let p be a prime st. p^n devides order of a group G and p^{n+l} does not divide it G. then a subgroup H of G s.t. O (H) = p^n is called a sylow p-subgroup of G. there are three postulates of Sylow p-subgroups first theorem shows the existence of a sylow *p*-subgroup of G for every prime *p* dividing order of G while. Second theorem shows that any two sylow *p*-subgroups of G are conjugate and the third theorem says about the number of sylow *p*-subgroups of G.

Keywords: Conjugate *p*-subgroups O(G), O (H).

Introduction

Let *p* be a prime and m, a+ve integer St. p^n /O(G). Then a subgroup H of G. St O (H) = p^n.
Any two sylow *p*-subgroups of a finite group G are conjugate to one another.
Let *p* be sylow *p*-subgroups of G is of the from 1+kp whon 1+kp/O(G), K being a non-negative integer Let *p* be a sylow *p*-subgroups of G. Then the number of sylow *p*-subgroups of G is eqal to $\frac{O(G)}{O[N(P)]}$. This is also a matter to discuss that the number of sylow *p*-subgroups of G is of the form l+kp where 1+kp/O(G), k being a none negative integer.

Note: If O (G) = $p^n q$, (p, q)=1 then the number of sylow *p*-subgroups is 1+kp where 1+kp/$p^n q$.

Example: Let G be a group of order 231. Show that N–Sylon subgroup of G is contained in the centre of G.

Soln: Let O(G) = 231 = 3×7×11
The number of sylow 11–subgroup of G is 1+11k and (1+11k)/21. Clearly then it is possible if k=0 so, sylon 11-subgroup H of G is normal in G.
The number of sylo 7–subgroup of G is 1+7k' and (1+7k')/33. So k'=0, thus, Sylo 7-subgroup k of G is normal in G O(H) = 11, O(k) = 7

Now O$\left(\frac{G}{k}\right)$ = $\frac{3\times7\times11}{7}$=33 = 3×11 and 3× (11–1), Thus $\frac{G}{k}$ is cyclic, [If O(G) = pq, where p, q are distinct primes, p<q, p×q–1, then G is cyclic]. Thus $\frac{G}{k}$ is cyclic and so $\frac{G}{k}$ is abelian. But G' is the smallest subgroup of G such that G/G' is abelian (G' denote the commutator subgroup of G)
$\therefore$ G'$\subseteq$ k $\Rightarrow$ O (G') = 1 or 7 if O (G) = 1 Then G' {e} $\Rightarrow$ $x^{-1}y^{-1}xy = e \Rightarrow xy = yx$ for all x, y $\in$ G $\Rightarrow$ G is abelian $\Rightarrow$ G = Z (G) $\Rightarrow$ H$\subseteq$ Z (G). Let O (G) = 7 $\Rightarrow$ G' = K. Clearly H $\cap$k = {e} as O (H$\cap$k) divides O (H) = 11 and O (k) = 7. Let x $\in$ H, Y $\in$ G Then x^{-1} $y^{-1}xy$ $\in$G' = k. also x^{-1} $y^{-1}xy$ =$x^{-1}(y^{-1}xy)$ $\in$ H as H is normal is G
$\therefore$ x^{-1} $y^{-1}xy$ $\in$ H$\cap$k = {e} $\Rightarrow$ xy = yx, y$\in$G, x$\in$H $\Rightarrow$ H$\subseteq$z(G).
If p is sylow p-subgroup of G Let X $\in$ N (p) st O (x) = p^i, then X $\in$ p. If every p-subgroup of a finite group is contained in some sylow p-subgroup of G.
$\Rightarrow$ 1+kp/q as (1+kp, p^n) = 1

of *p* is the only sylow *p*-subgroups of G then p is normal in G and if p is normal G then p is only sylow *p*-subgroups of G. If p is sylow *p*-subgroups of G to very *p*-Subgroup of a finte group is contained in some sylow *p*-subgroups of G.

Aim

In a finite group G no sylow *p*-subgroups can be properly contained in a p-subgroup. If let G be a finite group and p be a *p*-subgroup of G, then *p* is sylow *p*-subgroups of G if and only of no p-subgroup of G properly contains *p*. If O(G) = p, q, where p, q are distinct prime, p<q, p×q-1 then it is cyclie. Using sylow's theorem we can find

¬p-1=1 mod p for every prime p. Let p we a prime dividing O(G) and if $(ab)^p = a^p b^p$ for all a, b ∈ G. then sylow *p*-subgroups p is normal in G and \ a normal subgroup N of G St. P∩N={e} and G = PN together with G has non trivial centre.

Conclusion

Let G be a finite group and H≤G suppose p is a prime dividing O(G). Let p be a sylow *p*-subgroups of H contained in some sylow *p*-subgroups of G. then p = S∩H. Let G be a finite group and let H be normal in G. If p be a prime dividing O(G). If ([G:H],p)=1 Then H contains every sylow *p*-subgroups of G for finite group G and p being a prime dividing O(H) where H≤G then the number of sylow *p*-subgroups of H is less than or equal to the number of sylow *p*-subgroups of G.

Let p be a prime dividing O(G) St. if K is normal in G and p is a sylow *p*-subgroups of G. Then p∩K is a sylow *p*-subgroups of G and $\frac{pk}{K}$ is a sylow *p*-subgroups of G/K. Every sylow *p*-subgroups of $\frac{G}{K}$ is of the form $\frac{pk}{K}$ where p is a sylow *p*-subgroup of G. Let G be a group of order p q r, p<q< being a prime. In this situation some sylow *p*-subgroup of G is normal in G and then G is can't be simple.

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