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A mathematical modeling on two phase systolic blood flow pulmonary arterioles during emphysema due to smoking

Arti Tripathi, V Upadhyay, AK Agrawal and PN Pandey

Abstract
In the present paper, we will discuss the pulmonary blood flow in lungs. We have collected clinical data in case of COPD for Hematocrit v/s Blood pressure. Agrawal and Upadhyay have considered the blood flow of two phase, one of which is red blood cell and other is plasma. They have also applied the Herschel-Bulkley Non-Newtonian model in Bio-fluid mechanical setup. The graphical presentation for particular parametric value in much closed to the clinical observation. The overall presentation is in tensorial form and solution technique adopted is analytical as well as numerical.

Keywords: Pulmonary, Herschel Bulkley, arterioles, parametric value, lungs

Introduction
Medical terms related to the lungs after being with pulmonary as in the adjectival form: (pulmonary) or form the latinpulmonarius (of the lungs) or with pneumo-(from Greek "lungs"). The lungs are spongy lobes inside the chest, protected by the rib cage. In held air is directed of the trachea (windpipe) into two tubes (bronchi) that each service a lung. The bronchile divide into smaller tubes called bronchioles, further still into air sacs called alveoli Each alveolus has a fine mesh of capillaries where the exchange of oxygen and carbon dioxide takes place oxygen molecules dissolve and migerate across a thin film of moisture from the air sac to the blood stream. Oxygenated blood is seen to the hard and then pump around the body. Human beings exchanges gases through lungs. In this process oxygen goes in and is utilised by the body. Thus the process in which and organaism uses oxygen for its life process and gives of carbon dioxide is called respiration.

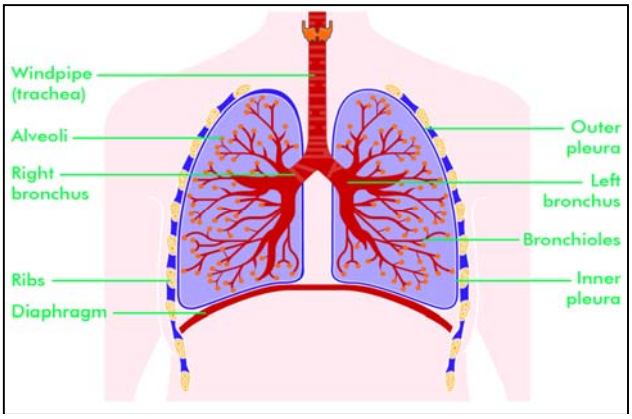


Fig 1

The main function of the lungs is (rapid) gas exchange. This is accomplished by a well-coordinated interaction of the lungs with the central nervous system, the diaphragm and

chest wall mascul at use, and the circulatory system.

Arterioles are the blood vessels in the arterial side of the vascular tree that are located proximal to the capillaries and, in conjunction to the regulation of mean arterial pressure and tissue perfusion, their wall consists or cellular and extra cellular components that have been traditionally classified as conforming three layers an intima containing end other cell sited on a basement membrane a media made of an internal elastic lamina exposed by one or two layers of smooth muscle: and adventitia composed mostly of collagen bundle, nerve endings and some fibro blasts.

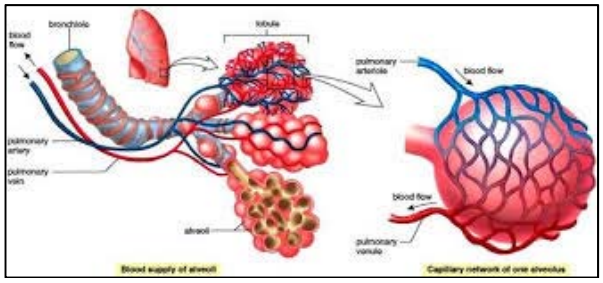


Fig 2: Pulmonary arterioles

Arterioles are defined as the primary resistance vessels that enter an organ to distribute blood flow in to capillary beds. There blood vessels very significantly in diameter depending on species, vascular bed and state of contraction. Arterioles are considered part of the resistance vasculature that provides in excess of 80% of the resistance to blood flow in the body.

Blood is a complex fluid consisting of particulate solids surpended in a non Newtonian Fluid. The particular solids are red blood cells (RBCs) White cells (WBCs) and platelates. Fluid is plasma which itself is a complex mixture of protein and other integradiant in an aqueous base 50% of the plasma and 45% of blood cells and in 45% of the blood is RBC and there is a few part of the other cells. Which are ignorable, so one phase of the blood is plasma and IInd phase of the blood is RBC$_s$. face pulmonary blood flow is a study of measuring the blood pressure. If the hemoglobin is known. The percentage of volume covered by blood cells in the whole blood is calledhematocrit. This work will focus on two phase pulmonary blood flow a lungs with the special reference to emphysema.

The shape and size of RBC very different classes of vertebrates. Human erythrocytes are7-8 cm (1 am-10-6m) in diameter and 2 am thick near the rim. Then almost entire cytoplasm is filled with haemoglobin. In the absence of cell organelles, the consumption of oxygen is every layer haemoglobin is conjugated protein which is made up of a protein called globin and Fe^{2+}porsphyric complex called heme. Thus the total count of RBC is more in man than in woman. The life of a RBC is about 120 days.

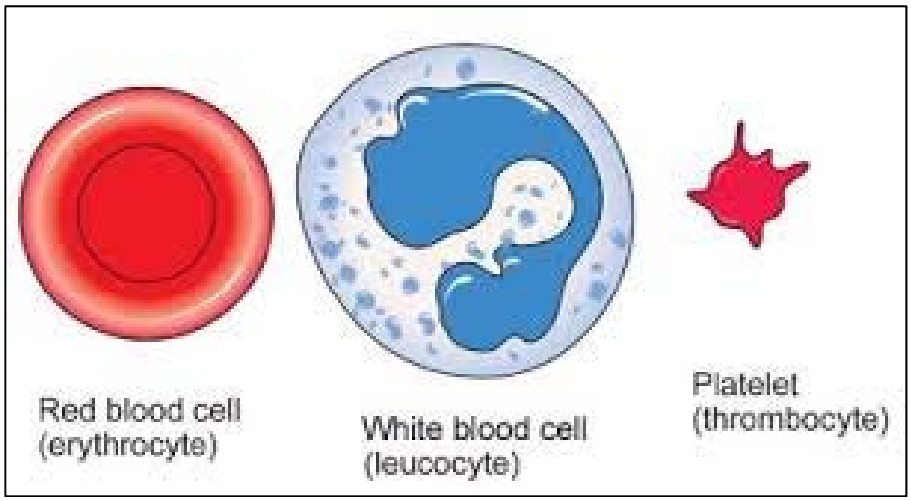


Fig 3

Blood Plasma is basically similar to a dilute solution of sea water and indeed. They share the same salt but in concentrations. Blood plasma has roughly 0.85% salt. The main group of plasma proteins are alblimin, globulin and fibrinogen. Their most important role is in the maintenance of proper osmotic pressure between the circulatory fluid and the fluid in the tissue space. The leucocytes are of two Agranulocytesand Granulocytes.

Agranulocytes indeed lymphocytes and monocytes. Granulocytes are ecsinophils basophils and neutrophils These are really cell fragment rather than true cells. Blood platelets are much smaller than both the red and white blood corpuscles. Their number harmally varies from 0.15-0.45billions per micro liter of blood when an injury is caused the blood platelets release certain chemical which are termed the plated factor (thromboplastin).

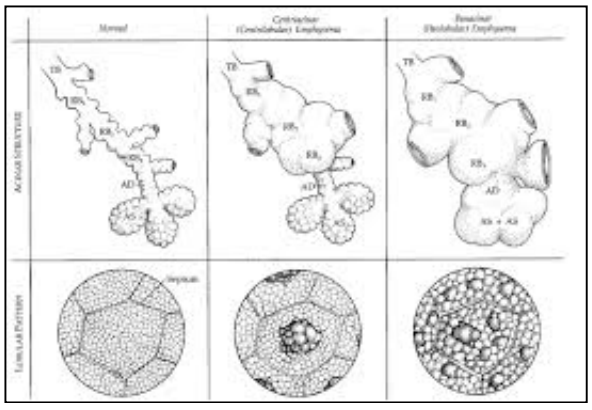


Fig 4

A person with emphysema has damaged alveoli and bronchi. The weakened and ruptured air sacs are unable to efficiently move oxygen from the air to the blood. As the disease progresses and damages more air sacs, the person may eventually feel breathless even when they are resting. Bronchitis means inflammation of the bronchi. The lungs normally produce a small amount of fluid to keep healthy, but chronic bronchitis causes an over production of fluid. This leads to frequent and productive coughing (producing mucus or phlegm).

A person with COPD is at increased risk of a number of complications, including:

- Chest infection- a common cold can easily lead to a severe infection
- Pneumonia- a lung that targets the alveoli and bronchioles
- Collapsed lung – the lung may develop an air pocket. If the air pocket bursts during a coughing fit, the lung will deflate.
- Heart problems- the heart has to work extremely hard to pump blood through the Lungs.
- Osteoporosis –where bones become thin and break more easily. Steroid use in people with COPD is thought to contribute to osteoporosis.
- Anxiety and depression-breathlessness or the fear of breathlessness can often lead to feelings of anxiety and depression.
- Oedema(fluid retentaion)-problems with blood circulation can cause fluid to pool, paticulary in the feet and ankles
- Hypoxaemia-caused by lack of oxygen to the brain. Symptoms include congnitive difficulties such as confusion, memory lapes and depression
- Risks of sedentary lifestyle-as symptoms of COPD progress, many people adjust their lifestyle to avoid symptoms. For example, they reduce their physical activitiy to avoid breathlessness on exertion.

This downward spiral of inactivity means the person is prone to a range of potentially serious health problems, such as obesity and cardiovascular disease.

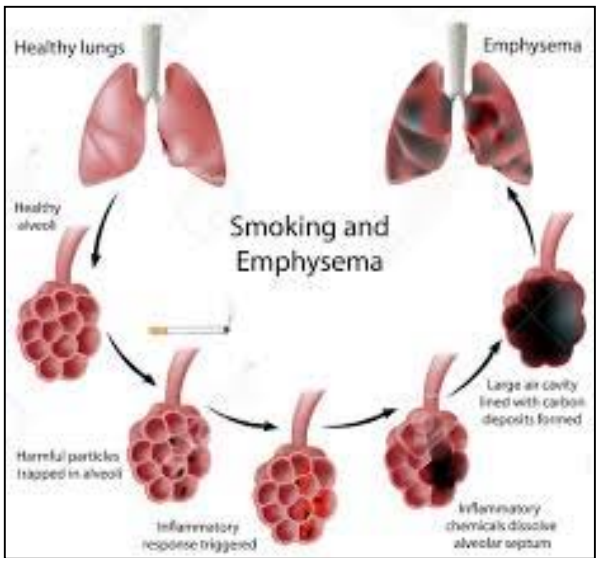


Fig. 5:

Mathematical modeling

The blood is an important and ideal fluid which is a mixture of plasma and blood cells white flowing through different vessels it changes its shapes and size Veerendra.

In aorta and pulmonary arteries the blood flows due to unusual high Reynolds no of flow as 5000 to 10000 Veerendra.

As we know that principal of conservations and momentum which is applicable to hemodynamics, hence the equation of motion based on the principle has been obtained in tensorial form. In year2007 – Christine L. Hattrup and Sandra J. Gendler; this review focuses on the three Best – characterized cell surface mucins expressed in the respiratory tract: MUC1, MUC4 MUC16.

In year 2010 Kiarash Emami and all others, a comparison between pulmonary function testing and hyperpolarized MRI metrics.

In year 2010- Proshin, A.P; Solodyannikov, yu. V. Consider a formulation of the problem of parametric identification from measurement of periodic motion.

In year 2011- Mustapha, Norzieha, Mandal, Prashant k. Abdullah llyani; Amin Norsraahaida; Hayat tasawar focus that the numerical in the investigation of the generalized Newtonian blood flow through a couple of irregular atrial stenosis.

In year 2012 Stephen Dubsky and all other, Synchrotron-based dynamic computed Tomography of tissue motion for regional lung function measurement.

In year 2013 Dina Visca, Marina Aiello and Alfredo Chetto consider a formulation of the problem of specifically addressed to cardiovascular system function.

In year 2014 Cheryl R. Laratta and Stephan van Eeden. Consider a formulation acute exacerbation of COPD. The present time, many of these events are unrecognized; despite improve tools for diagnosis and assessment.

Equation of Continuity

The flow of blood is effected by the presence of blood cells. The blood cells effect is directly proportional to the volume. Let the volume be X, this X is replaced by 1/100, where H is the hematocrit the volume % of blood cells. Then the volume portion covered by the plasma will be [1-X]. If mass ratio of cells to plasma is r then clearly.

$$r = \frac{X\rho_c}{(1-X)\rho_p} \ldots\ldots\ldots (1)$$

where ρ_c and ρ_p are densities of blood cells and plasma respectively. Usually this mass ratio is not a constant. Even then this may by supposed to constant in present context. According to the principal of Conservation of mass in pulmonary circulatory system, equation of continuity for two phase are as follows:

$$\frac{\partial X_{\rho_c}}{\partial t} + X_{\rho_c} v^i, i = 0 \ldots\ldots\ldots \quad (2)$$

And, $\frac{\partial y}{\partial t}(1-X)_{\rho_p} + (1-X)_{\rho_c} v^i, j = 0 \ldots\ldots\ldots (3)$

Where v is the common velocity of two phase blood cells and plasma. Again $X\rho_{c\,V^{i,}}$ Is covariant derivative of with respect to X^i.In the same way is the covariant derivative of (1-X) with respect to X^i.

If we define the uniform density of bloodρ_m as follows:

$$\frac{1+r}{\rho_m} = \frac{r}{\rho_0} + \frac{1}{\rho_p} \dots\dots\dots\dots\dots (4)$$

The equation 2 and 3 can be combined together as follows-

$$\frac{\partial \rho_m}{\partial t} + (\rho_m v^i), i = 0 \dots\dots\dots\dots (5)$$

Equation of motion of blood flow

The hydrodynamical pressure p between the two phases of blood can be supposed to be uniform because the both phases i.e. always in equilibrium state in blood [12]. Taking viscosity coefficient of blood cells to be $\eta_c$and applying the principle of conservation of momentum in pulmonary circulatory system, we get the equation of motion for the phase of blood cells as follows;

$$\frac{\partial V^i}{\partial t} + (X\rho_c V^i)V_j^i = -X_{p,}\, j\delta^{ij} + X_{\eta c}\left(g^{ij}v_k^i\right)j \dots\dots (6)$$

Similarly the viscosity coefficient of plasma to be η_ρ

$$(1-X)\rho_p \frac{\partial v^i}{\partial t} + \{(1-X)\rho_p v^i\}v_j^i$$
$$-(1-X)\rho, jg^{ij} + (1-X)_{\eta_c}(g^{ij}v_k^i)_j \dots\dots\dots (7)$$

Now adding equation (6) and (7) and using relation (4) the equation of motion for blood flow With the both phase will be as follows.

$$\rho_m \frac{\partial v^i}{\partial t} + \rho_m v^j v_j^i = -p, jg^{ij} + \eta_m\left(g^{ij}v_k^j\right)j \dots\dots\dots (8)$$

Where $\eta_m = X\eta_c + (1-X)\eta_p$ is the viscosity coefficient of blood as a mixture of two phases.

The pumping effects of heart on these vessels is very low. The yield stress derived from this phenomenon. Increases the blood viscosity ten times. Hence the pumping of the heart on these vessels relatively low. (Van Hirtum *et al*……2009) Secondly these vessels are relatively narrow down more rapidly. In this situation, the blood cells line up on the axis to build up rouleaux. That's why the Herschel Bulkley law hold good on this two phase blood flow through veins arterioles,

Venules and whose constitutive equation is as follows:

$$T' = \eta_m e^h e^n + T_p \,(T' > T_P)$$

And e = 0 (T" $<T_p$)

Where T is the yield stress.

When strain rate e=o (T'$<T_p$) a core region is formed whis flows just like a plug. Let the radias of the Plug be r_p .

The stress acting on the surface of the plug will be T. equating the forces acting on the plug,

We get

$$p\,\pi\, r_{p^2} = T_p\, 2\pi\, r_p$$

$$r_p = \frac{2T_p}{p} \dots\dots\dots\dots (9)$$

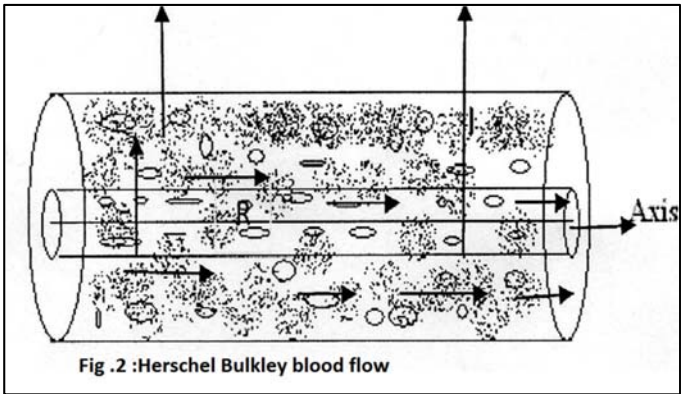


Fig .2 :Herschel Bulkley blood flow

The consecutive equation rest part of blood vessels is

$$T' = \eta_m e^h + T_p$$
$$T' - T_p = \eta_m e^h$$

Whose generalized form will be as follows.

$$T^{ij} = -pg^{ij} + T_e^{ij}$$

Where the symbols have their usual meanings.

Now we describe the basic equation for Herschel Bulkley flow as follows.

Equation of continuity

$$^{1/}\sqrt{g}\left(\sqrt{g}v^t\right) = 0$$

The equation of motion

$$\rho_m \frac{\partial v^i}{\partial \mathrm{t}} + \rho_m v^i v_j^i = -T^{ij}\ \mathrm{j} \ldots \ldots (10)$$

Where all the symbols have their usual meanings

Analysis

Since the blood vessels are cylindrical, the above governing equation are transformed into cylindrical form. As we know earlier

$x^1 = \mathrm{r}$, $x^2 = \theta$, $x^3 = z$

Matrix of metric tensor in cylindrical co-ordinate is as follow.

$$[g^{ij}] = \begin{vmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

While matrix of conjugate metric tensor is as follows

$$[g_{ij}] = \begin{vmatrix} 1 & 0 & 0 \\ 0 & {}^{1}/_{r^2} & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Whereas the Christoffels symbols of 2nd kind as follows;

$$\begin{Bmatrix} 1 \\ 2\ 2 \end{Bmatrix} = r \begin{Bmatrix} & 2 \\ 2 & \end{Bmatrix} = \{ \quad {}^{1}_{2} \} = {}^{1}/_{r}$$

Remaining other are zero.

Relation between contra variant and physical components of velocity of blood flow will be as follows;

$$\sqrt{g_{11}v^1} = v_r = v^1$$
$$\sqrt{g_{22}v^2} = v_\theta = v_\theta = rv^2$$
$$\sqrt{g_{33}v^3} = v_z = v_z = v^3$$

Again the physical components of $-p_j g^{ij}$ are $-\sqrt{g_{ij}p_j g^{ij}}$

Eq. (9) and (10) are transformed into cylindrical form so as solve as power law model to get

$$\frac{dv}{dr} = \left(\frac{p_r}{2\eta_m}\right)^{1/n}$$

Where pressure gradient

$$\frac{dp}{dz} = p$$
$$\frac{dv}{dr} = \frac{p(r - r_p)^{1/n}}{2\eta_m}$$
$$\frac{dv}{dr} = \left\{\frac{1}{2pr} - \frac{1}{2pr_p}/2\eta_m\right\}^{\frac{1}{n}}$$

From equation (9)….

$$\frac{dv}{dr} = \left\{\frac{\frac{1}{2}p_r - T_p\}^{\frac{1}{n}}}{\eta_m}\right\} \ldots\ldots(11)$$

Substituting the values of T'_0 from (7) in to (11)

$$\frac{dv}{dr} = [p_{r/2} - pr_{p/2}/\eta_m]^{1/n}$$

$$\frac{dv}{dr} = \frac{p}{2\eta_m}\ (r - r_p)^{\frac{1}{n}} \ldots\ldots(12)$$

Integrating above equation under the no slip boundry condition $v = 0, r = R$ so as to get

$$v = [p/2\eta_m]^{\frac{1}{n}}\frac{n}{n+1}\ \ [(R - r_p)^{\frac{1}{n}+1} - (r - r_p)^{\frac{1}{n}+1}] \ldots\ldots\ldots(13)$$

Which is the formula of velocity of blood flow in arteriole venules and veins putting r = r' to get the velocity of plug flow as follows.

$$v_p = \frac{n}{n+1}\left[\frac{P}{2\eta_m}\right]^{\frac{1}{n}}\left(R - r_p\right)^{\frac{1}{n}+1} \ldots\ldots\ldots\ldots\ldots(14)$$

where the value ofr_p is taken from(7)

Result and Discussion

Observation: Hematocrit and blood pressure from an authorized City Hospital and Research centre hospital Jabalpur and Associated Hospital Kanpur by Dr. Rajeeve Trivedi (M.D.)

Patient Name: Mr. Tapan Kumar

Diagnosis: - Emphysema

Table 1

Date	HB(Hemoglobin)	B.P(Blood-pressure in(mm hg)	BP drop	BP drop in (pascal)	Hematocrit
03/09/2016	13.09	120/80	20	2664.66	39.27
04/09/2016	13.09	130/90	20	2664.66	39.27
05/09/2016	14.01	150/80	35	4663.155	42.03
06/09/2016	14.09	120/70	25	3330.825	42.06

The flow flux phased blood flow in arterioles, venules and veins is

$$Q = \int_0^{r_p} 2\pi v_p dr + \int_{r_p}^{R} 2\pi r v dr$$

$$\int_0^{r_p} 2\pi r\ n_{n+1} p/_{2\eta_m}{}^{1/n} R - r_p{}^{\frac{1}{n}+\frac{1}{dr}} + \int_{\mathrm{o}}^{\mathrm{r_p}} 2\pi r\ n_{n+1} p/2\eta_m{}^{1/n}$$

Using (12) and (14)

$$\frac{2\pi n}{n+1}(p/2\eta_m)^{\frac{1}{n}}\ (R - r_p)^{\frac{1}{n}+1}\ [r^2/2]_0^{r_p} + \frac{2\pi n}{n+1}(p/2\eta_m)^{\frac{1}{n}}\ [\frac{r^2}{2}\ (R - r_p)^{\frac{1}{n}+1}\ r\left(\frac{r - r_p}{\frac{1}{n}+2}\right) - \frac{(r - r_p)^{\frac{1}{n}+3}}{\frac{1}{n}+2)(\frac{1}{n}+3)}{}_{r_p}^{\ R}$$

$$\frac{2\pi n}{n+1}(p/2\eta_m)^{\frac{1}{n}} r_p^2\ (R - r_p)^{\frac{1}{n}+1} R^2\ (R - r_p)^{\frac{1}{n}+1} - 2R\left(\frac{(r - r_p)^{\frac{1}{n}+2}}{\frac{1}{n}+2}\right) + 2\frac{(R - r_p)^{\frac{1}{n}+3}}{\frac{1}{n}+2)(\frac{1}{n}+3)} - r_p^2\ (R - r_p)^{\frac{1}{n}+1}$$

$$\frac{2\pi n}{n+1}(p/2\eta_m)^{\frac{1}{n}} R^{\frac{1}{n}+3}\ [\frac{r_p^2}{R^2}\ (1 - \frac{r_p}{R})^{\frac{1}{n}+1} + \left(1 + \frac{r_p}{R}\right)\ (1 - \frac{r_p}{R})^{\frac{1}{n}+2} - 2(1 - \frac{r_p}{R})^{\frac{1}{n}+2}/\frac{1}{n} + 2 + 2(1 - \frac{\frac{r_p}{R})^{\frac{1}{n}+3}}{\frac{1}{n}+2)(\frac{1}{n}+3)}$$

P = pressure gradient

N= parameter

V=viscosity of mixture

Now we have Q = 425ml/min R=1,$r_p = 1/3$ according to Gustafson penile R,(1980)

$\eta_p = 0.013(\ pascel - \text{sec})$

According to Glenn Elert (2010)

$\eta_m = 0.027(\ pascel - \text{sec})$

H = 24 $\eta_m = \eta_c X + \ \eta_p(1 - X)$ where

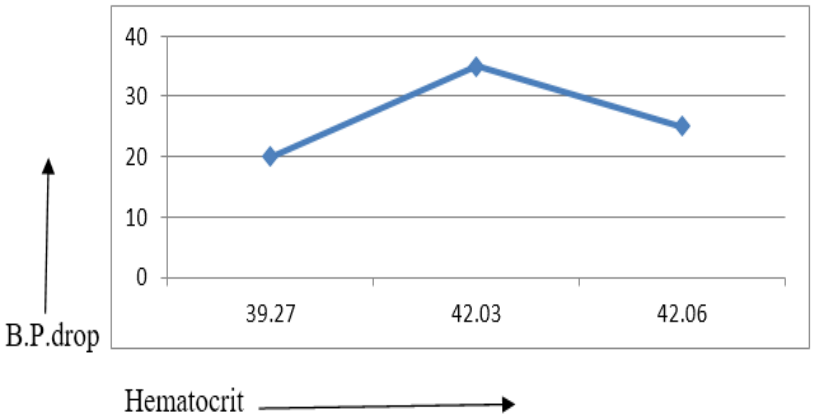
$$X = \frac{H}{100} = \frac{24}{100} = 0.24$$

$$1827.23 = (1728.40)^{\frac{1}{n}}\left[\frac{26n^3 + \ 33n^2 + 9n}{6n^3 + 11n^2 + 6n + 1}\right]$$

We get n = 4.00389

Solved by numerical method.

Hematocrit	39.27	39.27	42.03	42.06
BP drop	20	20	35	25



Conclusion

A simple survey of the graph between blood pressure and hematocrit in emphysema patient show that

1- When Hematocrit is increased then blood pressure drop is increased from $x_1$to x_2.

2 – When Hematocrit is increased then blood pressure drop is decreased from x_2 to x_3.

Acknowledgement

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