International Journal of Applied Research 2017; 3(5): 368-371

International Journal of Applied Research

**ISSN Print:** 2394-7500
**ISSN Online:** 2394-5869
**Impact Factor:** 5.2
IJAR 2017; 3(5): 368-371
www.allresearchjournal.com
Received: 11-03-2017
Accepted: 12-04-2017

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# On Nano generalized^-closed and open sets in Nano topological spaces

**R Lalitha and Dr. A Francina Shalini**

**Abstract**
This paper focuses on nano generalized^-closed sets and nano generalized^-open sets in nano topological spaces and certain properties of these are investigated. Also the converse of the theorems are proved with examples.

**Keywords:** Nano topology, Nano closed sets, Nano open sets, Nano interior, Nano closure, Nano generalized closed sets, Nano generalized^-closed sets.

## 1. Introduction

In 1970, Levine [4] introduced the concept of generalized closed sets as a generalization of closed sets in topological spaces. This concept was introduced as a generalization of closed sets in Topological spaces through which new results in general topology were introduced. Lellis Thivagar [3] introduced Nano topological space with respect to a subset X of a universe which is defined in terms of lower and upper approximations of X. The elements of Nano topological space are called Nano open sets. He has also defined Nano closed sets, Nano-interior and Nano closure of a set. He also introduced the weak forms of Nano open sets namely Nano-α open sets, Nano semi open sets and Nano pre-open sets.
In this paper, we define a new class of sets called nano generalized^- closed and open sets in nano topological spaces and study the relationships with other nano sets.

## 2. Preliminaries

**Definitiom 2.1 [6]:** A g^-closed set [20] if cl(A)⊆G whenever A⊆G and G is semi-open in (X,τ). The complement of a g^-closed set is called a g^-open set.

**Definition 2.2: [2]**
Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be the approximation space Let U be a non-empty finite set of objects called the universe and R be an equivalence relation. Let X ⊆U.
(!) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and its is denoted by $L_R(X)$
That is, $L_R(X)=\bigcup_{x\in U}\{R(x) : R(x) \subseteq X\}$ where R(X) denotes the equivalence class determined by x.
(2) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$.That is,
$U_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \cap X \neq ø\}$
(3). The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not-X with respect to R and it is denoted by, $B_R(X)$. That is, $B_R(X)= U_R(X) - L_R(X)$

**Property 2.3: [2]** If (U, R) is an approximation space and X, Y⊆U, then
(i) $L_R(X) \subseteq X \subseteq U_R(X)$
(ii) $L_R(ø) = U_R(ø)$ and $L_R(U) = U_R(U) = U$

(iii) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
(iv) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$
(v) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$
(vi) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
$L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq L_R(Y)$ whenever $X \subseteq Y$
(vii) $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$
(viii) $U_R U_R(X) = L_R U_R(X) = U_R(X)$
(ix) $L_R L_R(X) = U_R L_R(X) = L_R(X)$

**Definition 2.4:** [2]
Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \emptyset, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by property 2.3 τ R (X) satisfies the following axioms:

1. U and ø belongs to $\tau_R(X)$
2. The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$
3. The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$. That is, $\tau_R(X)$ is a topology on U called the Nano topology on U with respect to X. We call (U, $\tau_R(X)$) as the Nanotopological space. The elements of $\tau_R(X)$ are called as Nano-open sets.

**Remark: 2.5** [2] If $\tau_R(X)$ is the Nano topology on U with respect to X, then the set B ={ U, $\emptyset$, $L_R(X)$, $U_R(X)$, $B_R(X)$} is the basis for τR (X).

**Definition 2.6:** [2]
If ( U, $\tau_R(X)$ ) is a Nano topological space with respect to X where X ⊆U and if A⊆U, then the Nano interior of A is defined as the union of all Nano-open subsets of A and it is denoted by NInt(A). That is, N Int(A) is the largest Nano-open subset of A.
The Nano closure of A is defined as the intersection of all Nano closed sets containing A and it is denoted by Ncl(A). That is, Ncl(A) is the smallest Nano closed set containing A.

**Definition 2.7:** [2]
Let (U, $\tau_R$(X)) be a Nano topological space. A subset A of (U, $\tau_R$(X)) is called Nano generalized closed set (briefly Ng-closed) if Ncl(A)⊆Vwhere A⊆V and V is Nano open.

**Definition 2.8:** [2]
A subset A of (U, $\tau_R$(X)) is called Nano semi-pre generalized closed set (Nspg) if Nspcl(A)⊆V whenever A⊆V, V is Nano semi-open.

**Definition 2.9:** [2]
A subset A of (U,$\tau_R$(X)) is called nano semi-generalized closed set (briefly Nsg-closed) if Nscl(A)⊆V whenever A⊆V and V is nano semi-open in (U,$\tau_R$(X)).

## 3. Nano generalized^-closed sets

Throughout this paper (U, $\tau_R(X)$) is a Nano topological space with respect to X where X ⊆ U, R is an equivalence relation on U, U/R denotes the family of eqialence classes of U by R.

**Definition 3.1**
Let (U,$\tau_R(X)$) be a nano topological space. A subet A of (U, $\tau_R(X)$) is called Nano generalized^-closed set (briefly Ng^- closed) if Ncl(A) ⊆ V where A⊆V and V is Nano Semi-Open.

**Theorem 3.2**
Every Ng^ closed set is Nano semi-generalized closed set.

**Proof**
Let A be a Ng^-closed set.
Then Ncl(A)⊆V where A⊆V and V is Nano semi-open in U.
But Nscl(A)⊆Ncl(A) where A⊆V, Vis Nano semi-open in U.
Now we have, Nscl(A)⊆V where A⊆V and V is Nano semi-open in U.
Hence, A is Nano semi-generalized closed set.

**Remark 3.3**
The converse of the above theorem need not be true as shown by the example.

**Example 3.4**
Let U={a,b,c,d}with X={a,c} with U/R= {{c},{a,b},{d}}.
Let A={a,b,d} be a Nano semi-generalized closed set.
Here, Nscl(A)⊆V where A⊆V, V is Nano semi-open in U.
Hence, A={a,b,d}⊆U is nano semi-generalized closed set.
A⊆{a,b,c,d}, Nscl(A)⊆{a,b,c,d}
Which implies that A is Nano semi-generalized closed, but A is not a Ng^-closed.

**Theorem 3.5**
Every Ng^-closed set is Nano semi-pre generalized closed set.

**Proof**
Let A be Ng^-closed set.
Then Ncl(A)⊆V where A⊆V and V is Nano semi-open in U.
But, Nspcl(A)⊆Ncl(A) where A⊆V and V is Nano semi-open set in U.
Now, we have Nspcl(A)⊆V, where A⊆V and V is Nano semi-open in U.
Hence, A is Nano semi-pre generalized closed set.

**Remark 3.6**
The converse of the above theorem need not be true as shown by the example.

**Example 3.7**
Let U={a,b,c,d} with X={a,c} with U/R={{c},{a,b},{d}}
Let A={a,c} be Nano semi-pre generalized closed set.
Here Nspcl(A)⊆V, where A⊆V and Vis nano semi-open in U.
Hence, A={a,c}⊆U is a Nano semi-pre generalized closed set.
A⊆{a,c}, Nspcl(A)⊆{a,b,c,d}.
Which implies that A is Nano semi-pre generalized.
But, A is not a Ng^-closed set.

**Theorem 3.8**
Every Ng-closed is Ng^-closed set.

**Proof**
Let A be Ng-closed set.
Then Ncl(A)⊆V, where A⊆V and V is Nano open in U.
Since, every Nano open is Nano semi-open in U.
Now, we have Ncl(A)⊆V, where A⊆V and V is semi open in U.

Hence, A is Ng^-closed set.

**Remark 3.9**
The converse of the above theorem need not be true as shown by the example.

**Example 3.10**
Let U={a,b,c,d} with U/R= {{a},{c},{b,d}} and X={a,b}
Let A={b,c} be a Ng^-closed.
Here Ncl(A)⊆V, where A⊆V and V is Nano semi-open in U.
Hence, A={b,c} be a Ng^-closed set.
A⊆{a,b,c,d}, Ncl(A)⊆{a,b,c,d}.
Which implies that, A is Ng^-closed.
But, A is not a Ng closed.

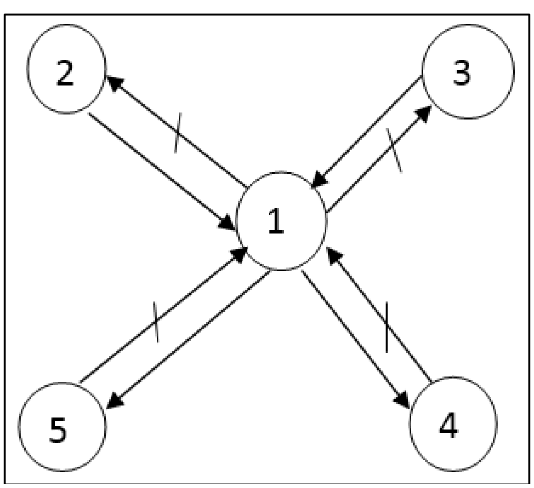
**Theorem 3.11**
Every Nano closed set is Ng^-closed set.

**Proof**
Let A⊆V and V is Nano semi-open in τ R(X).
Since, A is Nano closed, Ncl(A)⊆A.
That is, Ncl(A)⊆A⊆V.
Hence, A is Ng^-closed set.

**Remark 3.12**
The converse of the above theorem need not be true as shown by the example.

**Example 3.13**
Let u={a,b,c,d} with X={a,c} with U/R={ c},{a,b},{d}}.
$\tau_R$(X)= {U,∅,{c},{a,b,c},{a,b}}
Nano closed sets are {∅, U, {a,b,d}, {d},{c,d}}
Let A={a,d} and A⊆{a,b,c,d}, Ncl(A)⊆{a,b,c,d}
Which implies that A is Ng^-closed.
but A is not Nano closed.

**Remark 3.14**
The following diagram shows the relationships of Ng^-closed set with other known existing sets.
A $\longrightarrow$ B represents A implies B, A $\not\longrightarrow$ B represents A does not implies B.
1). Ng^-closed set.
2). Nano closed set.
3). Nano generalized closed set.
4) Nano semi generalized closed set.
4). Nano semi pre generalized closed set.




**Example 3.15**
Let U={a,b,c,d} with U/R = {{c},{a,c},{d}} and X={a,c}.
Then the Nanotopology $\tau_R(X) = \{U, \emptyset, \{c\}, \{a,b,c\}, \{a,b\}\}$.
Nano closed sets are $\{\emptyset, U, \{a,b,d\}, \{d\}, \{c,d\}\}$.
Let V={c,d} and A={c}.
Then Ncl(A)={c,d} ⊆ V.
That is A is said to be Ng^-closed in (U,τ R(X)).

**Theorem 3.16**
A subset A of (U, $\tau_R$(X)) s Ng^-closed if Ncl(A)-A contains no nonempty Ng^-closed set.

**Proof**
Suppose if A is Ng^-closed. Then Ncl(A) ⊆ V where A⊆V and V is semi-open.
Let Y be nano closed subset of Ncl (A)-A.
Then A⊆$Y^c$ and $Y^c$ is Nano semi open.
Since A is Ng^-closed, Ncl (A)⊆$Y^c$ or Y ⊆ $[Ncl(A)]^c$.
That is Y⊆Ncl (A) and Y ⊆ $[Ncl(A)]^c$ implies that Y⊆ ∅.
Therefore, Y is empty.

**Theorem 3.17**
If A and B are Ng^-closed, then A ∪ B is Ng^- closed.

**Proof**
Let A and B are Ng^-closed sets.
Then Ncl(A)⊆ V where A⊆V and V is Nano semi-open and Ncl(B)⊆V where B⊆ V and V is Nano semi-open.
Since, A and B are subsets of V, (A∪ B) is a subset of V and V is Nano semi-open.
Then, Ncl(A)∪Ncl(B) ⊆ V which implies that (A∪B) is Ng^-closed.

**Remark 3.18**
The intersection of two Ng^-closed is again an Ng^- closed set.

**Proof**
Let A and B are Ng^-closed sets.
Then Ncl(A)⊆ V where A⊆V and V is Nano semi-open and Ncl(B)⊆V where B⊆ V and V is Nano semi-open.
Since, A and B are subsets of V, (A∩ B) is a subset of V and V is Nano semi-open.
Then, Ncl(A)∩Ncl(B) ⊆ V which implies that (A∩B) is again an Ng^-closed.

**Theorem 3.19**
If A is Ng^-closed and A⊆B⊆Ncl(A), then B is Ng^-closed.

**Proof**
Let B⊆V where V is Nano semi-open in τ R(X).
Then A⊆B implies A⊆V.
Since A is Ng^-closed, Ncl(A)⊆V also B⊆Ncl(A) implies Ncl(B)⊆Ncl(A).
Thus NCl(B)⊆V and so B is Ng^-closed.

**Theorem 3.20**
A Ng^-closed set A is Nano closed if and only if Ncl(A)-A is Nano closed.

**Proof**
**(Necessary part)**
let A be Nano closed.
Then Ncl(A)-A=∅ which is Nano closed.

**(Suffiency part)**
Suppose Ncl(A)-A is Nano closed.
Then Ncl(A)-A=∅.
Since A is Nano closed. That is Ncl(A)=A or A is Nano closed.

**Theorem 3.21**
Suppose that B⊆A⊆U, B is an Ng^-closed set relative to A and that A is Ng^-closed subset of U. Then B is Ng^-closed relative to U.

**Proof**
Let B⊆V and suppose that V is Nano semi-open in U. Then B⊆A∩V.
Therefore Ncl(B)⊆A∩V and A⊆ V∪Ncl(B).
It follows that A∩Ncl(B)⊆A∩V and A⊆V∪Ncl(B).
Since, A is Ng^-closed in U, we have Ncl(A)⊆ V∪Ncl(B)..
Therefore Ncl(B)⊆Ncl(A)⊆Ncl(B) and so Ncl(B)⊆V.
Then, B is Ng^-closed relative toV.

**Corollary 3.22**
Let A be Ng^-cosed set and suppose that F is a Nano closed set. Then A∩F is an Ng^-closed set which is given in the following example.

**Example 3.23**
Let u={a,b,c,d} with X={a,c} with U/R={{c},{a,b},{d}}
$\tau_R$(X)={U,∅, {c},{a,b,c},{a,b}}
The Nano closed sets are {∅,U, {a,b,d},{d},{c,d}}
Let A={a,d} and B={b,d}.
Then A∩B={d} is an Ng^-closed set.

**Theorem 3.24**
For each a∈U, either {a} is Nano closed or $\{a\}^c$ is Nano generalized^-closed set in $\tau_R$(X).

**Proof**
Suppose {a} is not Nano closed in U.
Then $\{a\}^c$ is not Nano open and the only Nano open set containing $\{a\}^c$ is V⊆U,
That is $\{a\}^c$⊆U. Therefore Ncl($A^c$)⊆U.
Which implies $\{a\}^c$ is Ng^-closed set in $\tau_R$(X).

**4. Nano generalized^-open sets**
**Definition 4.1**
A subset A is Nano topological space u is called Nano generalized^-open (simply Ng^-open) if $A^c$ is Nano g^-closed,

**Example 4.2**
Let U={a,b,c,d} with U/R={{c},{a,b},{d}} and X={a,c}
Then, the Nano topology is defined as,
$\tau_R$(X)={U,∅,{c},{a,b,c},{a,b}}.
Let A={a,b}, then $A^c$={c,d} is a Ng^-closed, since U is the only Nano semi open set containing $A^c$.
Therefore, A is Ng^-open.
If A={a.c.d} then $A^c$={b} is not Ng^-closed.
Since,Ncl($A^c$)=Ncl{b}={a,b,d} and {a,b,d}⊈{b}.
A semi open set such that $A^c$⊆V.
Therefore A is not Ng^-open.

**Theorem 4.3**
If A and B are Ng^-open. Then A∩B is Ng^-open.

**Proof**
Let A and B are Ng^-open sets,
Then Ncl(A)⊆V where A⊆V and Vis Nano semi-open and Ncl(B}⊆V where A⊆V and V is Nano semi-open
Since A and B are subsets of V, (A∩B) is a subset of V and V is Nano semi-open.
Then Ncl(A∩B)=Ncl(A) ∪ Ncl(B)⊆V.
Which implies that (A∩B) is Ng^-open.

**Remark 4.4**
The Union of two Ng^-open sets is again an Ng^-open set.

**Proof**
Let A and B are Ng^-open sets.
Then Ncl(A)⊆ V where A⊆V and V is Nano semi-open and Ncl(B)⊆V where B⊆ V and V is Nano semi-open.
Since, A and B are subsets of V, (A∪ B) is a subset of V and V is Nano semi-open.
Then, Ncl(A)∪Ncl(B) ⊆ V which implies that (A∪B) is again an Ng^-open.

**5. References**

1. Bhuaneshwari K, Ezhilarasi A. on nano semi-geralized and nano generalized- semi closed sets in nano topological spaces International Journal of Mathematics and Computer Application Research(IJMCAR) ISSN(P):2249-6955(E):2249-8060
2. Bhuvaneshwari K, Mythili Gnanapriya K. on Nano Generalized closed sets in Nano Topological spaces. International Journal of Scientific and Research Publications, 2014; 4(5) May ISSN 2250-3153.
3. Lellis Thivagar M. Carmel Richard, On Nano forms of Weakly open sets, International Journal of Mathematics and Statistics Invention, Volume
4. Levine N. Generalised Closed sets in Topology, Rend. Cire. Math. Palermo, 1963; 19(2):89-96.
5. Shalini SB, Indirani K. on Nano generalized-semi pre closed sets and Nano semi pre-generalized closed sets in nano topological spaces –IJPMS ISSN: 2277-2111(Online) 2015; 5(3).
6. Veera Kumar MKRS. g^closed sets and G^LC-functions, Indiran J Math. Indian J Math., 2001; 43(2):231-247.