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On rgwα -Closed and rgwα -Open maps in topological spaces

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Abstract
The aim of this paper is to introduce new type of closed maps rgwα-closed maps and rgwα-open maps, rgwα*-closed maps and rgwα*-open maps. We also obtain some properties of rgwα-closed maps and rgwα-open maps.

Keywords: rgwα-closed maps, rgwα*-closed maps and rgwα-open maps, rgwα*-open maps
Mathematical subject classification (2010): 54C10

1. Introduction
Mappings play an important role in the study of modern mathematics, especially in Topology and Functional Analysis. Closed and open mappings are one such mapping which are studied for different types of closed sets by various mathematicians for the past many years. Generalized closed mappings were introduced and studied by Malghan [17]. wg-closed maps and rwg-closed maps were introduced and studied by Nagavani [21]. Regular closed maps, gpr-closed maps, rg-closed maps and rgα-closed and rgα-open maps have been introduced and studied by Long [24], Gnanambal [12], Arockiarani [13], A.Vaidivel and K.Vairamanickam [34] respectively. In this paper, a new class of maps called regular generalized weakly α-closed (briefly, rgwα-closed) maps, rgwα*- closed maps have been introduced and studied their relations with various generalized closed maps. Also we defined rgwα-open maps and rgwα*-open maps and studied some of its properties. Let us recall the following definitions which are used in our present study.

2. Preliminaries
Throughout this paper, (X, τ) and (Y, σ) (or simply X and Y) represent a topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space X, cl(A) and int(A) denote the closure of A and the interior of A respectively. X\A or A^c denotes the complement of A in X.

Definition 2.1: A subset A of a topological space (X, τ) is called
1. Semi-open set [23] if A ⊆ cl(int(A)) and semi-closed set if int(cl(A)) ⊆ A.
2. Pre-open set [18] if A ⊆ int(cl(A)) and pre-closed set if cl(int(A)) ⊆ A.
3. α-open set [14] if A ⊆ int(cl(int(A))) and α -closed set if cl(int(cl(A)))⊆ A.
4. Semi-pre open set [1] (=β-open) if A ⊆ cl(int(cl(A)))) and a semi-pre closed set (=β-closed) if int(cl(int(A))) ⊆ A.
5. Regular open set [32] if A = int(clA)) and a regular closed set if A = cl(int(A)).
6. Regular semi open set [10] if there is a regular open set U such that U ⊆ A ⊆ cl(U).
7. Regular α-open set [34] (briefly, rα-open) if there is a regular open set U s.t U ⊆ A ⊆ αcl (U).

Definition 2.2: A subset A of a topological space (X, τ) is called
1. W-closed set [31] if cl(A) ⊆ U whenever A⊆ U and U is semi-open in X.
2. Wα- closed set [8] if αcl(A) ⊆ U whenever A⊆ U and U is w-open in X.
3. Generalized closed set(briefly g-closed) [22] if cl(A) ⊆ U whenever A ⊆ U and U is open in X.

4. Generalized semi-closed set(briefly gs-closed) [4] if scl(A) ⊆ U whenever A ⊆ U and U is open in X.
5. Generalized pre regular closed set(briefly gpr-closed) [12] if pcl(A)⊆U whenever A⊆U and U is regular open in X.
6. Regular generalized α-closed set (briefly, rgα-closed) [34] if αcl (A) ⊆ U whenever A⊆ U and U is regular α-open in X.
7. α-generalized closed set (briefly αg -closed) [15] if αcl(A)⊆U whenever A⊆U and U is open in X.
8. Generalized α-closed set (briefly gα-closed)if αcl(A)⊆U whenever A⊆U and U is α-open in X.
9. Weakly generalized closed set (briefly, wg-closed) [21] if cl(int(A))⊆U whenever A⊆U and U is open in X.
10. Regular weakly generalized closed set (briefly, rwg-closed) [21] if cl(int(A)) ⊆ U whenever A⊆U and U is regular open in X.
11. Generalized pre closed (briefly gp-closed) set [12] if pcl(A) ⊆U whenever A⊆U and U is open in X.
12. regular w-closed (briefly rw -closed) set [9] if cl(A) ⊆ U whenever A⊆U and U is regular semi-open in X.
13. Generalized regular closed (briefly gr–closed) set [7] if rcl(A)⊆ U whenever A⊆ U and U is open in X.
14. Regular generalized weak (briefly rgw-closed) set [19] if cl(int(A)) ⊆ U whenever A⊆ U and U is regular semi open in X.
15. Generalized weak α-closed (briefly gwα-closed) set [30] if αcl(A) ⊆U whenever A⊆U & U is wα- open in X.
16. Generalized star weakly α-closed set (briefly g*wα-closed) [29] if cl(A) ⊆U whenever A⊆U & U is wα-open in X.
17. Regular generalized weakly α-closed (briefly rgwα-closed) [26] if rαcl(A)⊆U whenever A⊆U & U is wα-open in X.

The compliment of the above mentioned closed sets are their open sets respectively.

Definition 2.3: A map f: (X, τ) →(Y, σ) is said to be

i) regular-continuous(r-continuous) [3] if $f^{-1}(V)$ is r-closed in X for every closed subset V of Y.
ii) completely-continuous [3] if $f^{-1}(V)$ is regular closed in X for every closed subset V of Y.
iii) strongly-continuous [32] if $f^{-1}(V)$ is clopen (both open and closed) in X for every subset Vof Y.
iv) g-continuous [6] if $f^{-1}(V)$ is g-closed in X for every closed subset V of Y.
v) w-continuous [6] if $f^{-1}(V)$ is w-closed in X for every closed subset V of Y.
vi) α-continuous [14] if $f^{-1}(V)$ is α-closed in X for every closed subset V of Y.
vii) wα-continuous [8] if $f^{-1}(V)$ is wα-closed in X for every closed subset V of Y.
viii) αg-continuous [24] if $f^{-1}(V)$ is αg-closed in X for every closed subset V of Y.
ix) wg-continuous [10] if $f^{-1}(V)$ is wg-closed in X for every closed subset V of Y.
x) rwg-continuous [10] if $f^{-1}(V)$ is rwg-closed in X for every closed subset V of Y.
xi) gs-continuous [23] if $f^{-1}(V)$ is gs-closed in X for every closed subset V of Y.
xii) gpr-continuous [12] if f^{-1} (V) is gpr-closed in X for every closed subset V of Y.
xiii) rgα-continuous [34] if $f^{-1}(V)$ is rgα-closed in X for every closed subset V of Y.
xiv) gr-continuous [7] if $f^{-1}(V)$ is gr-closed in X for every closed subset V of Y.
xv) rw-continuous [9] if $f^{-1}(V)$ is rw-closed in X for every closed subset V of Y.
xvi) rgw-continuous [19] if $f^{-1}(V)$ is rgw-closed in X for every closed subset V of Y.
xvii) rgwα-continuous [27] if $f^{-1}(V)$ is rgwα-closed in X for every closed subset V of Y.

Definition 2.4: A map f: (X, τ) →(Y, σ) is said to be

i) irresolute [24] if $f^{-1}(V)$ is semi- closed in X for every semi-closed subset V of Y.
ii) α-irresolute [18] if $f^{-1}(V)$ is α-closed in X for every α-closed subset V of Y.
iii) contra irresolute [18] if $f^{-1}(V)$ is semi-open in X for every semi-closed subset V of Y.
iv) contra r-irresolute [7] if $f^{-1}(V)$ is regular-open in X for every regular-closed subset V of Y.
v) contra continuous [11] if $f^{-1}(V)$ is open in X for every closed subset V of Y.
vi) rw*-open(resp rw*-closed) [31] map if f(U) is rw-open (resp rw-closed) in Y for every rw-open (resp rw-closed) subset U of X.
vii) β*-quotient map if f is β-irresolute and $f^{-1}(V)$ is an β-open set in (X,τ) implies V is an open set in (Y,τ).

Lemma 2.5: see [26]

1. Every close (resp. regular-closed, w-closed, α-closed and β-closed)set is rgwα-closed set in X.
2. Every rw-closed (resp. rs-closed, rα-closed, wα-closed, gα-closed, rgα-closed, gwα-closed and g*wα-closed) set is rgwα-closed set in X.
3. Every rgwα-closed set is gβ-closed set
4. The set g-closed (resp. wg-closed, rg-closed, gr-closd, gpr-closed, rgw-closed, rwg-closed and αg closed) set is independent with rgwα-closed set.

Lemma 2.6: see [26] If a subset A of a topological space X, and

1. If A is weak-open and rgwα-closed then A is α-closed set in X.
2. If A is both weak α-open and rgwα-closed then it is rα-closed set in X
3. If A is weak-open and rα-closed then A is rgwα-closed set in X
4. If A is both open and g-closed then A is rgwα-closed set in

Definition 2.7: A topological space (X, τ) is called

1. an α-space [14] if every α-closed subset of X is closed in X.
2. $\tau_{½}$ space [17] if every g-closed set is closed.
3. $_{½}T\alpha$-space [15] if every αg-closed set is α-closed.
4. $\beta\tau_{1/2}$-space [35] if every gα-closed and gβ-closed set is β-closed.
5. $\tau_{\alpha g}$-space [36] if every αg-closed set is gα-closed.
6. $\tau_{rgw\alpha}$ space [27] if every rgwα-closed set is closed.

Definition 2.8: A map f : (X, τ) → (Y, σ) is said to be

1. α-closed [14] if f(F) is α-closed in Y for every closed subset F of X.

2. αg-closed [15] if f(F) is αg-closed in Y for every closed subset F of X.
3. wg-closed [21] if f(V) is wg-closed in Y for every closed subset V of X.
4. rwg-closed [21] if f(V) is rwg-closed in Y for every closed subset V of X.
5. gs-closed [4] if f(V) is gs-closed in Y for every closed subset V of X.
6. gp-closed [16] if f(V) is gp-closed in Y for every closed subset V of X.
7. gpr-closed [12] if f(V) is gpr-closed in Y for every closed subset V of X.
8. wα-closed [8] if f(V) is wα-closed in Y for every closed subset V of X.
9. g-closed [6] if f(V) is g-closed in Y for every closed subset V of X.
10. w-closed [31] if f(V) is w-closed in Y for every closed subset V of X.
11. rgα- closed [34] if f(V) is rgα-closed in Y for every closed subset V of X.
12. gr-closed [7] if f(V) is gr-closed in Y for every closed subset V of X.
13. rw-closed [9] if f(V) is rw–closed in Y for every closed subset V of X.
14. rgw-closed [19] if f(V) is rgw–closed in Y for every closed subset V of X.
15. regular-closed [32] if f(F) is closed in Y for every regular closed set F of X.
16. Contra- closed [5] if f(F) is closed in Y for every open set F of X.
17. Contra regular-closed [32] if f(F) is r-closed in Y for every open set F of X.
18. Contra semi-closed [20] if f(F) is s-closed in Y for every open set F of X.

Definition 2.9: A map f: (X, τ) → (Y, σ) is said to be
1. g-open [6] if f(U) is g-open in (Y, σ) for every open set U of (X, τ),
2. w-open [31] if f(U) is w-open in (Y, σ) for every open set U of (X, τ),
3. wg-open [21] if f(U) is wg-open in (Y, σ) for every open set U of (X, τ),
4. rwg-open [21] if f(U) is rwg-open in (Y, σ) for every open set U of (X, τ),
5. gpr-open [12] if f(U) is gpr-open in (Y, σ) for every open set U of (X, τ),
6. regular- open [32] if f(U) is open in (Y, σ) for every regular open set U of (X, τ).

3 rgwα-Closed and rgwα-Open Maps

Definition 3.1: A map f**:** (X, τ) → (Y, σ) is said to be regular generalized weakly α-closed map (briefly, rgwα-closed map) if the image of every closed set in(X, τ)is rgwα-closed in (Y, σ).

Theorem 3.2: Every closed map is rgwα-closed map, but not conversely.

Proof: The proof follows from the definitions and fact that every closed set is rgwα-closed.

Theorem 3.3: Every α-closed map is rgwα-closed map but not conversely.

Proof: The proof follows from the definitions and fact that every α-closed set is rgwα-closed.

The converse of the above Theorem need not be true, as seen from the following example.

Example 3.4: Let X=Y={a,b,c,d}, τ ={X, ϕ, {a}, {b}, {a,b},{a,b,c} } and σ ={Y, ϕ, {a},{b}, {a,b}, {a,b,c}}. Let map f:X→Y defined by f(a)=a, f(b)= c, f(c)=d, f(d)=a, then f is rgwα-closed map but not closed map and not α-closed map, as image of closed set F= {d} in X, then f(F)={a} in Y, which is not α-closed, not closed set in Y.

Theorem 3.5: If f: (X, τ) → (Y, σ) is contra-w-closed and rgwα-closed map then f is α-closed map.

Proof: Let V be a closed set in (X, τ). Then f (V) is weak-open and rgwα-closed By Lemma 2.6, f (V) is α-closed. Thus f is α-closed map.

Theorem 3.6: If a map f: X→Y is closed, then the following holds.
- i) If f is w-closed map, then f is rgwα-closed map.
- ii) If f is r-closed map (resp. β-closed, rw-closed, rα-closed, wα-closed, gα-closed, rs-closed, rgα-closed, gwα-closed and g*wα-closed map) then f is rgwα-closed map.
- iii) If f is rgwα-closed map, then is gβ-closed map.

Proof:
- (i) The proof follows from the definitions and fact that every w-closed set is rgwα-closed.
- (ii) Similarly we can prove (ii).
- (iii) The proof follows from the definitions and fact that every rgwα-closed set is gβ-closed

The converse of the above Theorem need not be true, as seen from the following example.

Example 3.7: Let X=Y= {a,b,c,d,e}, τ= {X, ϕ, {a},{d},{e}, {a,d},{a,e},{d,e},{a,d,e}} σ ={Y, ϕ,{a}, {d},{e}, {a,d},{a,e},{d,e},{a,d,e}}, Let map f: X→Y defined by f(a)=b, f(b)=c, f(c)=d, f(d)=e, f(e)=a, then f is rgwα-closed map, but not w-closed, r-closed, β-closed, rw-closed, rα-closed, wα-closed, gα-closed, rs-closed, rgα-closed, gwα-closed and g*wα-closed map, as image of closed set f{b,c}= {c,d} which is not w-closed, r-closed β-closed, rw-closed, rα-closed, gα-closed, rgα-closed, gwα-closed and g*wα-closed, the image of closed set f{a,c,e}= {a,b,d} is not wα-closed and the image of closed set f{b,c,d,e}= {a,c,d,e} is not rs-closed set.

Example 3.8: Let X=Y= {a,b,c}, *τ*={X, ϕ, {a}, {b,c}} and *σ*={Y, ϕ, {a}}. Let map f: X→Y defined by f(a)=b, f(b)=a, f(c)=c, then f is gβ-closed map, but not rgwα-closed map, as the image of closed set f{b,c}= {a,c} which is not rgwα-closed set.

Remark 3.9: The following examples show that rgwα-closed maps are independent of g-closed, wg-closed, αg-closed, rg-closed, gr-closed, gpr-closed, rwg-closed and rgw-closed maps.

Example 3.10: Let X=Y= {a,b,c}, τ = {X, ϕ,{a},{b,c}} σ ={Y, ϕ,{a}}. Let map f: X→Y defined by f(a)=b, f(b)=a, f(c)=c, then f is g-closed, wg-closed, αg-closed, rg-closed, gr-closed, gpr-closed, rwg-closed and rgw-closed maps, but not rgwα-closed map, as the image of closed set f{a}= {b} which is not rgwα-closed set.

Example 3.11: Let X=Y= {a,b,c,d}, τ={X, ϕ,{a},{b},{a,b},{a,b,c}} and σ ={Y, ϕ, {a},{b},{a,b},{a,b,c}}. Let map f: X→Y defined by f(a)=a, f(b)= c, f(c)=d, f(d)=a then f is rgwα-closed map but not g-closed, wg-closed, αg-closed, rg-closed, gr-closed, gpr-closed, rwg-closed and rgw-closed maps, as the image of closed set F={d}={a}which is rgwα-closed set but not g-closed, wg-closed, rg-closed, gr-closd, gpr-closed, rgw-closed, rwg-closed and αg- closed set.

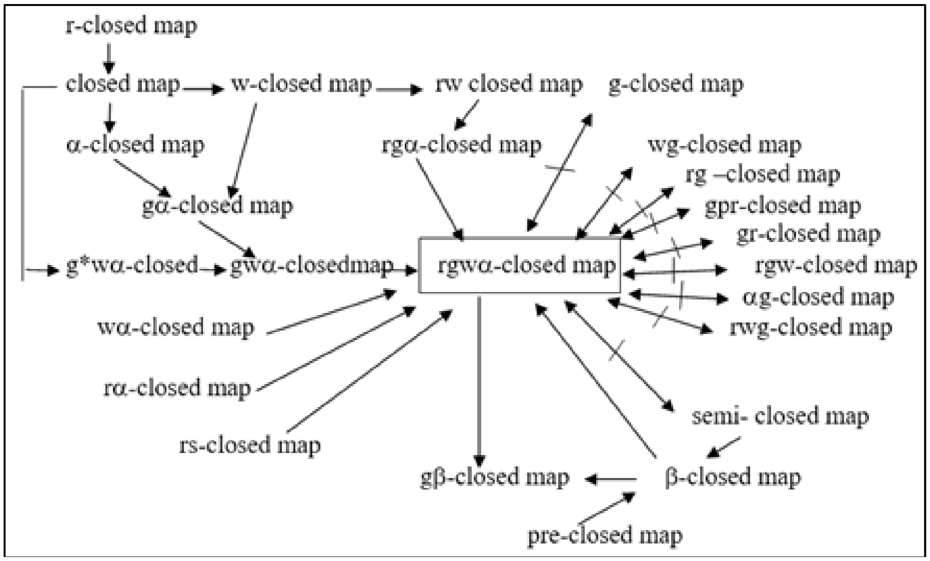

Fig 1: By A →B we mean A implies B but not conversely and A↮B means A and B are independent of each other.

Theorem 3.13: If f: (X, τ) → (Y, σ) is contra closed and gβ-closed map then f is rgwα-closed map.

Proof: Let V be a closed set in (X, τ). Then f(V) is open and gβ-closed. f(V) is rgwα-closed. Thus f is rgwα-closed map.

Theorem 3.14: If f: (X, τ) → (Y, σ) is αg-continuous and (Y, σ) is $\tau_{\alpha g}$-space then f is rgwα-closed map.
Proof: Let V be a closed set in (X, τ). Since f is αg-continuous f(V) is αg-closed set and (Y, σ) is $\tau_{\alpha g}$-space so f(V) is gα-closed hence rgwα-closed. Therefore f is rgwα-closed map.

Theorem 3.15: If a mapping f: (X, τ) → (Y, σ) is rgwα-closed, then rgwαcl(f(A)) ⊆ f(cl(A)) for every subset A of (X, τ).

Proof: Suppose that f is rgwα-closed and A ⊆X. Then cl(A) is closed in X and so f(cl(A)) is rgwα-closed in (Y, σ). We have f(A) ⊆ f(cl(A)), by Theorem 5.2(iv) in [28], rgwαcl(f(A)) ⊆ rgwαcl(f(cl(A))) → (i). Since f(cl(A)) is rgwα-closed in (Y, σ), rgwαcl(f(cl(A))) = f(cl(A)) → (ii), by the Theorem 5.3 in [28]. From (i) and (ii), we have rgwαcl(f(A))⊆ f(cl(A)) for every subset A of (X, τ).

Corollary 3.16: If a mapping f: (X, τ) → (Y, σ) is a rgwα-closed, then the image f(A) of closed set A in (X, τ) is $\tau_{rgw\alpha}$ -closed in (Y, σ).

Proof: Let A be a closed set in (X, τ). Since f is rgwα-closed, by above Theorem 3.15, rgwαcl(f(A))⊆ f(cl(A))→(i). Also cl(A)=A, as A is a closed set and so f(cl(A)) = f(A) →(ii). From (i) and (ii), we have rgwαcl(f(A)) ⊆ f(A). We know that f(A) ⊆ rgwαcl(f(A)) and so rgwαcl(f(A)) =f(A). Therefore f(A) is $\tau_{rgw\alpha}$ -closed in (Y, σ).

Theorem 3.17: Let (X, τ) be any topological spaces and (Y, σ) be a topological space where "rgwαcl(A) = αcl(A) for every subset A of Y " and f : (X, τ) →(Y, σ) be a map, then the following are equivalent.

(i) f is rgwα-closed map.
(ii) rgwαcl(f(A)) ⊆ f(cl(A)) for every subset A of (X, τ).

Proof: (i) ⇒ (ii) Follows from the Theorem 3.14.
(ii) ⇒ (i) Let A be any closed set of (X, τ). Then A=cl(A) and so f(A)=f(cl(A))⊆ rgwαcl(f(A)) by hypothesis. We have f(A) ⊆ rgwαcl(f(A)), by Theorem 5.2(ii) in [28]. Therefore f(A) = rgwαcl(f(A)). Also f(A) = rgwαcl(f(A)) = αcl(f(A)), by hypothesis. That is f(A) = αcl(f(A)) and so f(A) is α-closed in (Y, σ). Thus f(A) is rgwα-closed set in (Y, σ) and hence f is rgwα-closed map.

Theorem 3.18: A map f : (X, τ) → (Y, σ) is rgwα-closed if and only if for each subset S of (Y, σ) and each open set U containing $f^{-1}(S)$ ⊆U, there is a rgwα-open set V of (Y, σ) such that S ⊆V and $f^{-1}(V)$ ⊆ U.

Proof: Suppose f is rgwα-closed. Let S ⊆ Y and U be an open set of (X, τ) such that $f^{-1}(S)$ ⊆U. Now X − U is closed set in(X, τ). Since f is rgwα-closed, f(X − U) is rgwα-closed set in (Y, σ). Then V = Y − f(X − U) is a rgwα-open set in (Y, σ). Note that $f^{-1}(S)$ ⊆ U implies S ⊆ V and $f^{-1}(V)$ =X – f^{-1}(f(X −U)) ⊆ X − (X − U) = U. That is $f^{-1}(V)$ ⊆ U. For the converse, let F be a closed set of (X, τ). Then $f^{-1}((f(F))^c)$ ⊆ F^c and F^c is an open in (X, τ). By hypothesis, there exists a rgwα-open set V in (Y, σ) such that $f(F)^c \subseteq V$ and $f^{-1}(V) \subseteq F^c$ and so $F \subseteq (f^{-1}(V))^c$. Hence $V^c \subseteq f(F) \subseteq f(((f^{-1}(V))^c) \subseteq V^c$ which implies $f(F) = V^c$. Since V^c is rgwα-closed, f(F) is rgwα-closed. Thus f(F) is rgwα-closed in (Y, σ) and therefore f is rgwα-closed map.

Remark 3.19: The composition of two rgwα-closed maps need not be rgwα-closed map in general and this is shown by the following example.

Example 3.20: Let X = Y = Z = {a, b, c}, τ={X, ϕ, {a}, {a,b},{a,c}}, σ = {Y, ϕ, {a}, {c}, {a,c}} and η = {Z, ϕ, {a}, {b}, {a, b}}. Define f:(X, τ) → (Y, σ) by f(a)=b, f(b)=a, f(c)=c and g: (Y, σ) → (Z, η) by g(a)=b, g(b)=c, g(c)=a. Then f and g are rgwα-closed maps, but their composition g ∘f : (X, τ) → (Z, η) is not rgwα-closed map, because F = {b,c} is closed in (X, τ), but g ∘ f(F) =g ∘ f({b,c}) = g(f({b,c})) = g({a,c})= {a,b} which is not rgwα-closed in (Z, η).

Theorem 3.21: If f: (X, τ) → (Y, σ) is closed map and g: (Y, σ) → (Z, η) is rgwα-closed map, then the composition g ∘ f: (X, τ) → (Z, η) is rgwα-closed map.

Proof: Let F be any closed set in (X, τ). Since f is closed map, f(F) is closed set in (Y, σ). Since g is rgwα-closed map, g(f(F)) is rgwα-closed set in (Z, η). That is g ∘ f(F) = g(f(F)) is rgwα-closed and hence g ∘ f is rgwα-closed map.

Remark 3.22: If f: (X, τ) → (Y, σ) is rgwα-closed map and g:(Y, σ) → (Z, η) is closed map, then the composition need not be rgwα-closed map as seen from the following example.

Example 3.23: Let X = Y = Z = {a, b, c}, τ={X, ϕ, {a}, {a,b},{a,c}}, σ = {Y, ϕ, {a}, {c}, {a,c}} and η = {Z, ϕ, {a}, {b}, {a, b}}. Define f:(X, τ) → (Y, σ) by f(a)=b, f(b)=a, f(c)=c and g: (Y, σ) → (Z, η) by g(a)=b, g(b)=c, g(c)=a. Then f is rgwα-closed map and g is closed map, but their composition g ∘f : (X, τ) → (Z, η) is not rgwα-closed map, because F = {b,c} is closed in (X, τ), but g ∘ f(F) =g ∘ f({b,c}) = g(f({b,c})) = g({a,c})= {a,b} which is not rgwα-closed in (Z, η).

Theorem 3.24: If f: (X, τ) → (Y, σ) and g: (Y, σ) → (Z, η) is rgwα-closed maps and (Y, σ) be a $\tau_{rgw\alpha}$-space then g ∘ f: (X, τ) → (Z, η) is rgwα-closed map.

Proof: Let A be a closed set of (X, τ). Since f is rgwα-closed, f(A) is rgwα-closed in (Y, σ). Then by hypothesis, f(A) is closed. Since g is rgwα-closed, g(f(A)) is rgwα-closed in (Z, η) and g(f(A)) = g ∘ f(A). Therefore g ∘ f is rgwα-closed map.

Theorem 3.25: If f: (X, τ) → (Y, σ) is g-closed, g: (Y, σ) → (Z, η) be rgwα-closed and (Y, σ) is $\tau_{1/2}$-space then their composition g ∘ f: (X, τ) → (Z, η) is rgwα-closed map.

Proof: Let A be a closed set of (X, τ). Since f is g-closed, f(A) is g-closed in (Y, σ). Since (Y, σ) is $\tau_{1/2}$-space, f(A) is closed in (Y, σ). Since g is rgwα-closed, g(f(A)) is rgwα-closed in (Z, η) and g(f(A)) = g ∘ f(A). Therefore g ∘ f is rgwα-closed map.

Theorem 3.26: Let f: (X, τ) → (Y, σ) and g: (Y, σ) → (Z, η) be two mappings such that their composition g∘f: (X, τ) → (Z, η) be rgwα-closed mapping. Then the following statements are true.

(i) If f is continuous and surjective, then g is rgwα-closed.
(ii) If g is rgwα-irresolute and injective, then f is rgwα-closed.
(iii) If f is g-continuous, surjective and (X, τ) is a $\tau_{1/2}$-space, then g is rgwα-closed.
(iv) If g is strongly rgwα-continuous and injective, then f is rgwα-closed.

Proof:

(i) Let A be a closed set of (Y, σ). Since f is continuous, $f^{-1}(A)$ is closed in (X, τ) and since g ∘ f is rgwα-closed, (g ∘f)(f^{-1}(A)) is rgwα-closed in (Z, η). That is g(A) is rgwα-closed in (Z, η), since f is surjective. Therefore g is rgwα-closed.
(ii) Let B be a closed set of (X, τ). Since g ∘ f is rgwα-closed, g ∘ f(B) is rgwα-closed in (Z, η). Since g is rgwα-irresolute, g^{-1}(g ∘ f(B)) is rgwα-closed set in (Y, σ). That is f(B) is rgwα-closed in (Y, σ), since f is injective. Therefore f is rgwα-closed.
(iii) Let C be a closed set of (Y, σ). Since f is g-continuous, f^{-1}(C) is g-closed set in (X, τ). Since (X, τ) is a $\tau_{1/2}$-space, f^{-1}(C) is closed set in (X, τ). Since g ∘ f is rgwα-closed, (g∘f) (f^{-1}(C)) is rgwα-closed in (Z, η). That is g(C) is rgwα-closed in (Z, η), since f is surjective. Therefore g is rgwα-closed.
(iv) Let D be a closed set of (X, τ). Since g ∘ f is rgwα-closed, (g ∘ f)(D) is rgwα-closed in (Z, η). Since g is strongly rgwα-continous, g^{-1}((g ∘f)(D) is closed set in (Y, σ). That is f(D) is closed set in (Y, σ), since g is injective. Therefore f is closed.

Theorem 3.27: If f: (X, τ) → (Y, σ) is an open, continuous, rgwα-closed surjection and cl(F) = F for every rgwα-closed set in (Y, σ), where X is regular, then Y is regular.

Proof: Let U be an open set in Y and p ∈ U. Since f is surjection, there exists a point x ∈X such that f(x) = p. Since X is regular and f is continuous, there is an open set V in X such that x ∈ V ⊆cl(V) ⊆ f^{-1}(U). Here p ∈ f(V) ⊆ f(cl(V))⊆ U → (i). Since f is rgwα-closed, f(cl(V)) is rgwα-closed set contained in the open set U. By hypothesis, cl(f(cl(V))) = f(cl(V)) and cl(f(V)) = cl(f(cl(V))) → (ii). From (i) and (ii), we have p ∈ f(V) ⊆ cl(f(V)) ⊂ U and f(V) is open, since f is open. Hence Y is regular.

Theorem 3.28: If a map f: (X, τ) → (Y, σ) is rgwα-closed and A is closed set of X, then f_A: (A, τ_A) → (Y, σ) is rgwα-closed.

Proof: Let F be a closed set of A. Then F = A ∩ E for some closed set E of (X, τ) and so F is closed set of (X, τ). Since f is rgwα-closed, f (F) is rgwα-closed set in (Y, σ). But f(F) = f_A(F) and therefore f_A: (A, τ_A) → (Y, σ) is rgwα-closed.

Theorem 3.29: If f: (X, τ) →(Y, σ) is a continuous, rgwα-closed map from a normal space (X, τ) onto a space (Y, σ) then (Y, σ) is rα-normal.

Proof: Let A and B be two disjoint closed sets of (Y, σ). Then f^{-1}(A) and f^{-1}(B) are disjoint closed sets of (X, τ), since f is continuous. Therefore there exists open sets U and V such that $f^{-1}(A) \subseteq U$ and $f^{-1}(B) \subseteq V$, since X is normal. Using theorem 3.18, there exists rgwα-open sets C, D in (Y, σ) such that A ⊆ C, B ⊆ D, $f^{-1}(C) \subseteq U$ and $f^{-1}(D) \subseteq V$. Since A and B are closed, A and B are rα-closed and wα-closed. By the definition of rgwα-open, C is rgwα-open if and only if A ⊆ rαint(C) whenever A ⊆ C and A is wα-closed, we get A ⊆ rαint(C). Thus A ⊆ rαint(C) and B ⊆ rαint(D). Hence Y is rα-normal.

Theorem 3.30: If f: (X, τ) →(Y, σ) is an β-irresolute, (Y, σ) is a $\beta\tau_{1/2}$ -space then f is an rgwα-irresolute map.

Proof: Let U be rgwα-closed in (Y, σ) then U is gα-closed and gβ-closed. Since (Y, σ) is a $\beta\tau_{1/2}$ -space, U is β-closed. Since f is β-irresolute, $f^{-1}(U)$ is β-closed. Hence $f^{-1}(U)$ is rgwα-closed. Thus f is rgwα-irresolute map.

Theorem 3.31: If f: (X, τ) →(Y, σ) is an gβ-irresolute where (X, τ) is a discrete space then f is an rgwα-irresolute map.

Proof: Let U be rgwα-closed in (Y, σ). Then U is gβ-closed. Since f is gβ-irresolute and (X, τ) is discrete, $f^{-1}(U)$ is gβ-closed and open. Hence $f^{-1}(U)$ is rgwα-closed. Thus f is rgwα-irresolute map.

Theorem 3.32: If f: (X, τ) →(Y, σ) is a β*-quotient map and (Y, σ) is $\beta\tau_{1/2}$-space then (Y, σ) is a $\tau_{rgw\alpha}$ -space.

Proof: Let U be rgwα-closed in (Y, σ). Then U is gβ-closed in (Y, σ). Since (Y, σ) is $\beta\tau_{1/2}$-space then U is β-closed in (Y, σ), Since f is β-irresolute, $f^{-1}(U)$ is β-closed in (X, τ). Since f is β*-quotient map, then U is closed in (Y, σ). Thus (Y, σ) is a $\tau_{rgw\alpha}$ -space.

Analogous to rgwα-closed maps, we define rgwα-open map as follows.

Definition 3.33: A map f: (X, τ) → (Y, σ) is called a regular generalized weakly α-open map (briefly rgwα-open map) if the image f(A) is rgwα-open in (Y, σ) for each open set A in (X, τ).

From the definitions we have the following results.

Theorem 3.34:

(i) Every open map is rgwα-open but not conversely.
(ii) Every α-open (resp. r-open, w-open, rα-open, wα-open, gα-open, β-open, rs-open, rw-open, rgα-open, gwα-open, g*wα-open) map is rgwα-open but not conversely.
(iii) Every rgwα-open map is gβ-open but not conversely.
(iv) g-open (resp. wg-open, rg-open, gr-open, αg-open, rwg-open, rgw-open, gpr-open) map is independent with rgwα-closed map.

Theorem 3.35: For any bijection map f: (X, τ) → (Y, σ), the following statements are equivalent:

(i) f^{-1}: (Y, σ) → (X, τ) is rgwα-continuous.
(ii) f is rgwα-open map and
(iii) f is rgwα-closed map.

Proof: (i) ⇒ (ii) Let U be an open set of (X, τ). By assumption, $(f^{-1})^{-1}(U)$ =f(U) is rgwα-open in (Y, σ) and so f is rgwα-open.

(ii) ⇒ (iii) Let F be a closed set of (X, τ). Then F^c is open set in (X, τ). By assumption, $f(F^c)$ is rgwα-open in (Y, σ). That is $f(F^c) = f(F)^c$ is rgwα-open in (Y, σ) and therefore f(F) is rgwα-closed in (Y, σ). Hence f is rgwα-closed.

(iii) ⇒ (i) Let F be a closed set of (X, τ). By assumption, f(F) is rgwα-closed in (Y, σ). But $f(F) = (f^{-1})^{-1}(F)$ and therefore f^{-1} is continuous.

Theorem 3.36: If a map f: (X, τ) → (Y, σ) is rgwα-open, then f(int(A)) ⊆ rgwαint(f(A)) for every subset A of (X, τ).

Proof: Let f: (X, τ) → (Y, σ) be a open map and A be any subset of (X, τ). Then int(A) is open in (X, τ) and so f(int(A)) is rgwα-open in (Y, σ). We have f(int(A)) ⊆ f(A). Therefore by Theorem 5.15 (iii) in [28], f(int(A)) ⊆ rgwαint(f(A)).

Theorem 3.37: If a map f: (X, τ) → (Y, σ) is rgwα-open, then for each neighborhood U of x in (X, τ), there exists a rgwα-neighborhood W of f(x) in (Y, σ) such that W ⊆ f(U).

Proof: Let f: (X, τ) → (Y, σ) be a rgwα-open map. Let x ∈X and U be an arbitrary neighborhood of x in (X, τ). Then there exists an open set G in (X, τ) such that x ∈ G ⊆ U. Now f(x) ∈f(G) ⊆ f(U) and f(G) is rgwα-open set in (Y, σ), as f is a rgwα-open map. By Theorem 6.7 in [28], f(G) is rgwα-neighborhood of each of its points. Taking f(G) = W, W is a rgwα-neighborhood of f(x) in (Y, σ) such that W ⊆ f(U).

Theorem 3.38: A map f: (X, τ) → (Y, σ) is rgwα-open if and only if for any subset S of (Y, σ) and any closed set of (X, τ) containing $f^{-1}(S)$, there exists a rgwα-closed set K of (Y, σ) containing S such that $f^{-1}(K) \subseteq F$.

Proof: Suppose f is rgwα-open map. Let S ⊆ Y and F be a closed set of (X, τ) such that $f^{-1}(S) \subseteq F$. Now X −F is an open set in (X, τ). Since f is rgwα-open map, f(X − F) is rgwα-open set in (Y, σ). Then K = Y − f(X − F) is a rgwα-closed set in (Y, σ). Note that $f^{-1}(S) \subseteq F$ implies S ⊆ K and $f^{-1}(K) = X - f^{-1}(X - F) \subseteq X - (X - F) = F$. That is $f^{-1}(K) \subseteq F$.

For the converse, let U be an open set of (X, τ). Then $f^{-1}((f(U))^c) \subset U^c$ and U^c is a closed set in (X, τ). By hypothesis, there exists a rgwα-closed set K of (Y, σ) such that $(f(U))^c \subseteq K$ and $f^{-1}(K) \subseteq U^c$ and so $U \subseteq (f^{-1}(K))^c$. Hence $K^c \subseteq f(U) \subseteq f((f^{-1}(K)))^c \subseteq K^c$ which implies $f(U) = K^c$. Since K^c is a rgwα-open, f(U) is rgwα-open in (Y, σ) and therefore f is rgwα-open map.

Theorem 3.39: If a function f: (X, τ) → (Y, σ) is rgwα-open, then $f^{-1}(rgw\alpha cl(B)) \subseteq cl(f^{-1}(B))$ for each subset B of (Y, σ).

Proof: Let f: (X, τ) → (Y, σ) be a rgwα-open map and B be any subset of (Y, σ). Then $f^{-1}(B) \subseteq cl(f^{-1}(B))$ and $(f^{-1}(B))$ is closed set in (X, τ). By above Theorem 3.42., there exists a rgwα-closed set K of (Y, σ) such that B ⊆ K and $f^{-1}(K) \subseteq cl(f^{-1}(B))$. Now rgwαcl(B) ⊆ rgwαcl(K) = K, by Theorems 5.2 and 5.3 in [28], as K is rgwα-closed set of (Y, σ). Therefore $f^{-1}(rgw\alpha cl(B)) \subseteq f^{-1}(K)$ and so $f^{-1}(rgw\alpha cl(B)) \subseteq f^{-1}(K) \subseteq cl(f^{-1}(B))$. Thus $f^{-1}(rgw\alpha cl(B)) \subseteq cl(f^{-1}(B))$ for each subset of B of(Y, σ).

We define another new class of maps called rgwα*-closed maps which are stronger than rgwα-closed maps.

Definition 3.40: A map f: (X, τ) → (Y, σ) is said to be regular generalized weakly α*-closed map (briefly rgwα*-closed map if the image f(A) is rgwα- closed in (Y, σ) for every rgwα-closed set A in (X, τ).

Theorem 3.41: Every rgwα*-closed map is rgwα-closed map but not conversely.

Proof: The proof follows from the definitions and fact that every closed set is rgwα-closed.
The converse of the above Theorem is not true in general as seen from the following example.

Example 3.42: Let X = {a, b, c, d, e}, Y= {a, b, c. d} τ = {X, ϕ {a}, {d}, {e}, {a,d}, {a,e}, {d,e}, {a,d,e}} and σ = {Y, ϕ, {a}, {b}, {a, b}, {a,b,c}}. Let f: (X, τ) → (Y, σ) be defined as f(a)=b, f(b)=c, f(c)=d, f(d)=a, f(e)=d. Then f is rgwα-closed map but not rgwα*-closed map, since for the rgwα-closed set {a,b,d} in (X, τ), f({a,b,d}) = {a,b,c} which is not rgwα-closed set in (Y, σ).

Theorem 3.43: If f: (X, τ) → (Y, σ) and g: (Y, σ) → (Z, η) are rgwα*-closed maps, then their composition g ∘ f: (X, τ) → (Z, η) is also rgwα*-closed.

Proof: Let F be a rgwα-closed set in (X, τ). Since f is rgwα*-closed map, f(F) is rgwα-closed set in (Y, σ). Since g is rgwα*-closed map, g(f(F)) is rgwα-closed set in (Z, η). Therefore g ∘f is rgwα*-closed map.

Analogous to rgwα*-closed map, we define another new class of maps called rgwα*-open maps which are stronger than rgwα-open maps.

Definition 3.44: A map f: (X, τ) → (Y, σ) is said to be regular generalized weakly α*-open map (briefly rgwα*-open map) if the image f(A) is rgwα-open set in (Y, σ) for every rgwα-open set A in (X, τ).

Remark 3.45: Since every open set is a rgwα-open set, we have every rgwα*-open map is rgwα-open map. The converse is not true in general as seen from the following example.

Example 3.46: Let X = {a, b, c, d, e}, Y= {a, b, c. d} τ = {X, ϕ {a}, {d}, {e}, {a,d}, {a,e}, {d,e}, {a,d,e}} and σ = {Y, ϕ, {a}, {b}, {a, b}, {a,b,c}}. Let f: (X, τ) → (Y, σ) be defined as f(a)=b, f(b)=c, f(c)=d, f(d)=a, f(e)=d. Then f is rgwα-open map but not rgwα*-open map, since for the rgwα-open set {c,e,} in (X, τ), f({c,e}) = {d} which is not rgwα-open set in (Y, σ).

Theorem 3.47: If f: (X, τ) → (Y, σ) and g: (Y, σ) → (Z, η) are rgwα*-open maps, then their composition g ∘ f:(X, τ) → (Z, η) is also rgwα*-open.

Proof: Proof is similar to the Theorem 3.43.

Theorem 3.48: For any bijection map f: (X, τ) →(Y, σ), the following statements are equivalent:
i) f^{-1}: (Y, σ) → (X, τ) is rgwα-irresolute.
ii) f is rgwα*-open map
iii) f is rgwα*-closed map.

Proof: Proof is similar to that of Theorem3.35.

4. Conclusion
In this paper we have introduced and studied the properties of rgwα-Closed and rgwα-Open maps. Our future extension is rgwα- Closed and rgwα-Open maps in Fuzzy Topological Spaces.

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